

**EVALUATING
ACADEMIC READINESS
FOR APPRENTICESHIP TRAINING**
Revised for
ACCESS TO APPRENTICESHIP

**MATHEMATICS SKILLS
OPERATIONS WITH PERCENTAGES**

**AN ACADEMIC SKILLS MANUAL
for**

The Construction Trades (Structures)

This trade group includes the following trades:
Drywall & Acoustical Applicator, General Carpenter,
Mason (Brick & Stone and Restoration), Reinforcing Rod Worker, Roofer,
Terrazzo, Tile & Marble Mechanic

*Workplace Support Services Branch
Ontario Ministry of Training, Colleges and Universities*

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In preparing these Academic Skills Manuals we have used passages, diagrams and questions similar to those an apprentice might find in a text, guide or trade manual.

This trade related material is not intended to instruct you in your trade. It is used only to demonstrate how understanding an academic skill will help you find and use the information you need.

MATHEMATICS SKILLS

OPERATIONS WITH PERCENTAGES

*An academic skill required for the study of the
Construction Trades: Structures*

INTRODUCTION

In your training and on the job in construction you will come across statements such as:

“Nearly 20% of the lost time injuries in construction are related to the back. More than 50% of these injuries result from lifting excessive weight, or lifting incorrectly.”

“Passive solar designs have advantages over conventional heating. A properly designed system can supply up to 50% of the heating requirements.”

“CHBA claims that using control joints in a cast-in-place wall can minimize shrinkage by 90%.”

All these statements use a mathematical shorthand called percent. This skills sheet looks at different ways to use percent. It covers the following topics:

- ◆ The meaning of percent
- ◆ Writing a number as a percent
 - changing whole numbers, decimals and fractions to percent,
 - rounding off
- ◆ Changing a percent to a decimal
- ◆ Problems involving percent, including
 - finding a percent of a number
 - finding what percent one number is of another
 - finding a number when a percent of it is known

THE MEANING OF PERCENT

A **percent** is an amount expressed out of a total of one hundred.

Example: 25% means 25 parts out of a total of 100 parts.

Example: When your course states that you need to achieve 65% on your tests to pass, this means you need to get 65 marks out of every 100 possible marks.

Example: You put a 40% antifreeze solution in your truck.

- The % (percent sign) tells you that the solution has been divided into a total of 100 parts.
- 40 tells us that forty of those parts consist of antifreeze,
- It also means that the other 60 of the parts ($100 - 40 = 60$), or 60%, consists of other things, let's say water.

The relative amounts of antifreeze and water are easier to picture as percents than if you read that the solution is $\frac{2}{5}$ antifreeze and $\frac{3}{5}$ water.

Percent is often used to calculate financial amounts.

Examples:

A 10% increase in heating costs this year means that for every dollar you spent on heating costs last year, you now need to spend \$.10 more.

You pay 13% HST on all parts and materials which you forward on to your customers when you calculate the bill.

If you want to start your own business and you need a loan, the amount of interest on the loan is given as a percent such as 6.5%. To find out how much interest you will pay on an \$85,000 loan at 6.5% interest per year, you need to know how to do calculations with percent.

A Percent is a Special Kind of Fraction

A percent expresses an amount out of a total of 100 hundred equal parts. Since the 100 parts represent one whole amount, any percent less than 100 is a partial amount. A partial amount can be written as a percent, a fraction or a decimal.

Fraction: A fraction tells how many equal parts a whole amount is divided into; it also tells you how many of those parts you have.

Example: In the fraction $\frac{19}{100}$, the bottom number tells you how many parts, in this case 100, that the total amount is divided into. The top number tells you how many of these parts, in this case 19, you have.

- The top part of the fraction, representing the partial amount 19, is called the *numerator*.
- The bottom part of the fraction, representing the total amount 100, is called the *denominator*.
- The numerator and denominator are divided by the *fraction line*.

Percentages are fractions too: A percent is a kind of fraction whose denominator, like the fraction $\frac{19}{100}$, is 100.

- When we write a percent, we only write the numerator.
- The numerator is written with the percent (%) symbol after it.
- The % symbol represents the fraction line and indicates that the unwritten denominator is 100.

Example: To express the fraction $19/100$ as a percent:

- The numerator 19 is written with the % sign after it.
- The denominator 100 is not shown.
- Nineteen percent is written as 19%.

WRITING A NUMBER AS A PERCENT

Your course description might state that you must attend 90 hours out of a total of 100 course hours. Or it might state that you must attend 90% of the course. 90 out of 100 is the fraction $90/100$. The fraction $90/100$ and 90% both indicate the same time amount expressed in different ways.

You can convert any fraction, whole number or decimal number to a percent using the following rule:

To write a number as a percent, multiply it by 100 and add the % sign.

Remember: A quick way to multiply a number by 100 is to move the **decimal point** two places to the **right**. If the decimal point isn't shown at the end of a whole number, it is assumed to be after the last digit in the number. (Digits are the symbols for numbers. The digit for four is 4.)

We will look at changing a whole number, a decimal number and a fraction to a percent. There is one method each for converting whole numbers and decimals to percent and two methods for changing fractions to percent.

To Change a Whole Number to a Percent

1. Multiply the number by 100.
2. Add the percent sign.

Example: Express 3 as a percent.

$$\begin{array}{l} 3 \times 100 \\ = 300 \\ = 300\% \end{array} \quad \begin{array}{l} \\ \\ \text{write the \% sign} \end{array}$$

To Change a Decimal Number to a Percent

1. Multiply the number by 100.
2. Move the decimal point two places to the right, using zero as a place holder if necessary.
3. Add the % sign.

Example: Express .725 as a percent.

$$\begin{array}{l} .725 \times 100 \\ = 72.5 \\ = 72.5\% \end{array} \quad \begin{array}{l} \text{move the decimal point two places to the right} \\ \text{write the \% sign} \\ \end{array}$$

Example: Express .5 as a percent.

$.5 \times 100$	move the decimal point two places to the right
$= 50$	use zero as a place holder
$= 50\%$	write the % sign

Changing a Fraction to a Percent

There are two methods for changing fractions to percentages.

Method One: (This method is handy if the denominator of the fraction divides evenly into 100.)

1. Multiply the fraction by 100.
2. Reduce the answer to its lowest terms
3. Write the percent sign with the answer.

Example: Express $\frac{2}{5}$ as a percent.

$\frac{2}{5} \times 100$	5 divides evenly into 100
$= 40$	you can cross out before multiplying
$= 40\%$	write the % sign

Method Two: (This is the most common way to convert a fraction to a percent. This method is handy if the denominator of the fraction does not divide evenly into 100.)

1. Change the fraction to a decimal.
 - a. Divide the denominator of the fraction into the numerator. The answer is a decimal number.
2. Multiply the decimal by 100 and write the percent sign.

Example: Express $\frac{3}{8}$ as a percent.

$\frac{3}{8}$	change the fraction to a decimal
$= 3 \div 8$	
$= .375$	
$.375 \times 100$	multiply by 100
$= 37.5$	write the percent sign
$= 37.5\%$	

Methods One and Two result in the same answer. You can use either method with any fraction to convert it to a percent.

Examples:

- a) Express .25 as a percent.
 $.25 \times 100 = 25\%$
- b) Express 1 as a percent.
 $1 \times 100 = 100\%$
- c) Express $\frac{1}{4}$ as a percent.
 $\frac{1}{4} \times 100 = 25\%$
- d) Express 65 out of 100 as a percent.
 $\frac{65}{100} \times 100 = 65\%$
- e) Express $\frac{5}{8}$ as a percent.
 $5 \div 8 = .625$
 $.625 \times 100 = 62.5\%$
- f) Express $\frac{1}{3}$ as a percent.
 $\frac{1}{3}$
 $= 1 \div 3$
 $= .33\dots$
 $= .33\dots \times 100$
 $= 33.33\dots \%$

To round off an answer:

1. Look at the digit one place to the right of the place you are rounding off to.
2. If that digit is five or more, change the digit you are rounding off to one more than it is.
3. If the digit to the right is less than five, leave the digit you are rounding off the same.
4. Discard any digits after the one you have rounded off.
5. If the number you are rounding off to is 9 and the digit one past it is five or greater, the 9 becomes 10 and the place value where the 9 was becomes 0. The one from the 10 is added to the digit to the left of the 9, which becomes one larger in value.

Example: Round .2769 to two decimal places.

To round off to two decimal places, look at the digit in the third decimal place, which is a 6. Since it is five or larger, the 7 becomes 8 and the other digits are dropped.

.2769 rounded to two decimal places is .28.

Example: Express $\frac{1}{3}$ as a percent with the percent rounded to two decimal places.

$\frac{1}{3}$ expressed as a percent is 33.3333...%.
33.3333... % rounded to two places is 33.33%.

Example: Express $\frac{2}{3}$ as a percent.

$\frac{2}{3}$ expressed as percent is 66.666...%. Rounded off, it becomes 66.67%.

Sometimes you are told how many places to round off to in the question and sometimes you are not. If you are not told, answers are generally rounded off to two places, although there are often exceptions. One eighth ($\frac{1}{8}$) is 12.5% as a percent and .125 as a decimal number. When a decimal number ends evenly in three or four places, we don't round it off.

CHANGING A PERCENT TO A DECIMAL

If you are laying terrazzo tile for a sunroom; after calculating the quantity of tiles you need for the area of floor, you know that you must add 10% to that amount for wastage and cutting. To do this, you need to convert 10% to a decimal and then multiply that decimal number by the quantity of tiles. However first need to know how to change a percent to a decimal.

Decimals Are Also a Type of Fraction

Like percent, decimals are a kind of fraction. A *decimal is the numerator of a fraction which has a denominator that is a power of ten.* (A power of ten is a number that starts with 1 followed by any number of zeros such as 10, 100, 1000.) However, decimals show no denominator. A decimal point placed in front of the numerator replaces the denominator and the fraction line.

You have already learned how to convert a decimal number to a percent: multiply by 100 and add the % sign. To change a percent to a decimal, reverse the process.

To change a percent to a decimal:

- Remove the percent sign and divide by 100.

Remember: To divide by 100 quickly, move the decimal point two places to the left. When dividing by 100, the decimal point moves in the opposite direction than when multiplying by 100. If there are empty spaces between the decimal point and the digits of the number after you have moved the decimal point to the left, fill the spaces with zeros.

Example: Divide 3.67 by 100 the quick way.

Move the decimal point two places to the left and fill the empty space with a zero.
The answer is .0367.

Example: Change 5% to a decimal.

5% = .05 move the decimal 2 places to the left, use zero as a place holder, remove the % sign

Example: Change 33.3% to a decimal.

33.3% = .333 move the decimal 2 places to the left, remove % sign

Example: Express 10% as a decimal.

$10\% = .10$ or $.1$ move the decimal point two places to the left , remove the % sign

Example: After calculating that you need 140 tiles for the area of the sunroom floor, you know that you must add 10% for wastage and cutting. How many extra tiles must be ordered?

$10\% = .1$ change 10% to a decimal.

$.1 \times 140 = 14.$ multiply the number of tiles needed by $.1$ (10%)

The number of extra tiles needed is 14. You will order a total of 154 tiles.

PROBLEMS INVOLVING PERCENT

Sometimes you have to solve problems which involve percent. For example, you have to use percent to find the amount of sales tax owed or to find the interest on a loan.

Note: When doing percent problems, a calculator can do the calculations accurately, but you first have to decide what steps to use and in what order. Once you have figured out what you are asked to find and what method to use, you can use your calculator.

We will look at three main types of percent problems so you will know what method to choose for different situations. The three types of problems are:

1. Finding the percent of a number.
2. Finding what percent one number is of another.
3. Finding a number when you know a percent of it.

Problem Type #1: To Find a Percent of a Number

The most common kind of percent problem involves finding a percentage of a number.

Example: In Ontario, you pay 13% HST for many goods and services you buy. If you want to know the total price of an object or service before buying it, you need to calculate the amount of tax (13%) and add that to the cost. If the price of a truck is \$22 000, you have to add 13% harmonized sales tax to find the total cost.

To find the percent of a number:

1. First change the percent to a decimal.
2. Then multiply the number by the decimal.

We looked at how to change a percent to a decimal in the last section. Drop the percent sign and move the decimal point two places to the left.

Example: Find 5% of 20.

First change 5% to a decimal.

5% = .05 Drop the percent sign and move the decimal point two places to the left.
You need to use a zero as a place holder.

Now multiply .05 x 20.

$$\begin{array}{r} 20 \\ \times .05 \\ \hline 100 \\ \underline{000} \\ 1.00 \end{array}$$

5% of 20 = 1

Example: Find 25% of \$150.

25% = .25 change 25% to a decimal and multiply.

$$\begin{array}{r} 150 \\ \times .25 \\ \hline 750 \\ 3000 \\ \hline 37.50 \end{array}$$

$$25 \% \text{ of } 150 = 37.5$$

Most problems that ask you to find a percentage of a number are worded something like this:

- If a saw costs \$49.99 and the HST is 13%, what is the total amount you will have to pay?
- Find the amount of interest you must pay in a year on a loan of \$3000 if the interest rate is 12% per annum (per year).
- Find the total amount you pay for a truck that sells for \$22 00 if you pay 13% HST.

In each case, you must first find a percentage of the given number. In the first problem, you find 13% of \$49.99, in the second, 12% of \$3000 and in the third problem, 13% of \$22,000.

Example 1: If a saw costs \$49.99 and the sales tax is 13%, what is the total amount you have to pay?

First find the amount of tax paid. Change 13% to the decimal .13 and multiply.

$$\begin{array}{l} .13 \times \$49.99 \\ = \$6.4987 \text{ rounded to } \$6.50 \end{array}$$

Add the amount of tax paid to the cost of the saw.

$$\begin{array}{l} \$49.99 + \$6.50 \\ = \$56.49 \end{array}$$

The total amount paid is \$56.49.

Example 2: Find the amount of interest you must pay in a year on a loan of \$3000 if the interest rate is 12% per annum.

Change 12% to the decimal .12 and multiply.

$$\begin{array}{l} .12 \times \$3000 \\ = \$360 \end{array}$$

The yearly interest paid is \$360.

Example 3: A truck sells for \$22,000. You will also you pay 13% HST. Find the total cost of the truck.

Change 13% to .13 and multiply by \$22,000 to find the amount of tax you have to pay.

$$\$22,000 \times .13 = \$2,860$$

Add \$2,860 to the cost of the truck.

$$\$22,000 + \$2,860 = \$24,860$$

The total cost for the truck is \$24,860.

TO FIND A PERCENT OF A NUMBER

1. Change the number with the percent to a decimal by dropping the percent sign and moving the decimal **two** places to the **left**.
2. Multiply the two numbers together.

Solve the following questions. Round off money values to two decimal places. **Answers are at the end of this skills manual.**

3. a) Find 50% of 200. b) Find 33.3% of 150. c) Find 4.25% of 30.

d) Find 12% of \$48.50. e) Find 15% of \$1.99. f) Find .25% of 12.

g) What is 100% of 75? h) What is 150% of 68? i) What is 9% of \$10.75?
4. If the sales tax is 13%, what is the tax on a set of wrenches that costs \$45.98? What is the total cost?
5. 80% of the students in a course passed their test. If there are 40 students in the class, how many failed the test? (Hint: If 80% passed, what % failed?)
6. How much interest will you pay in a year on a loan of \$6,000 if the interest rate is 6% per year?

Problem Type #2: To Find What Percent One Number Is of Another

This type of problem allows us to use percent to compare things in a way we can easily understand

Example: you need to decide which brand of shingles to use on a job. You learn that Brand A costs \$20.00 per bundle for 25 year shingles compared to \$25.00 per bundle for Brand B. Both shingles have similar ratings and colour choice. The difference in price is \$5.00 per bundle. When you talk to your employer, you might state that Brand A is 20% cheaper than Brand B.

To express the difference in cost as a percentage, you expressed the savings, \$5, as a percent out of \$25. Questions that require you to do this can be worded in two ways:

“5 is what percent of 25?” *or* “What percent of 25 is 5?”

To find what percentage one number is of another, follow these three steps:

First step: Make a fraction. When one number is expressed out of another, it can be written as a fraction.

Example: 150 out of 500 is the fraction 150/500.

You have to know which number is the numerator and which number is the denominator to make the fraction.:

- ◆ The number representing the total, or *the number following “of”*, is the **denominator**. It forms the bottom part of the fraction.
- ◆ The other number is the **numerator**. It forms the top part of the fraction.

Remember: To make a fraction, write “is” over “of”, or is/of.

Second step: Make the fraction into a decimal number. *The numerator is divided by the denominator.*

Note: *You make the fraction in order to see what part is the numerator and what part is the denominator. This lets you correctly divide the numerator by the denominator. You can divide without making a fraction if you know to divide the number associated with “is”, the numerator, by the number following “of”, the denominator.*

Third step: Change the decimal answer to a percent:

- ◆ Multiply it by 100.
- ◆ Add the percent sign.

Remember: To multiply by 100 move the decimal point two places to the right.

Example: 15 is what percent of 50?

Write the fraction.

$$\frac{\text{"15 is" over}}{\text{"of 50"}} = \frac{15}{50}$$

Change the fraction to a decimal.

$$15 \div 50 = .3$$

Change the decimal to a percent.

$$.3 \times 100 = 30\%$$

In Brief

To find what percent one number is of another number:

1. Make a fraction by writing the number with the word *is* over the number with the word *of*.
2. Convert the fraction to a decimal by dividing the numerator by the denominator.
3. Change the decimal to a percent by multiplying by 100 and adding the % sign.

If you are dividing with a calculator, always key in the numbers so that the dividend (the number being divided into) is keyed in first, then the division sign and then the divisor (the number you divide by). To divide 75 by 25 using a calculator, key in the dividend 75 first and then the divisor 25.

Example: 620 is what percent of 1000?

$$\frac{620}{1000} \quad \frac{\text{is}}{\text{of}} \quad \text{Write the fraction.}$$

$$620 \div 1000 \quad \text{Change the fraction to a decimal by dividing.}$$
$$= .62$$

$$.62 \times 100 \quad \text{Change the decimal to a percent by multiplying and adding the \% sign.}$$
$$= 62\%$$

620 is 62% of 1000

Example: What percent of 50 is 25?

$$\frac{25}{50} \quad \text{Remember "is" over "of."}$$
$$= 25 \div 50$$
$$= .5$$

$$.5 \times 100 = 50\%$$

Rounding off: If the division answer doesn't come out evenly or is a repeating decimal, round off the answer. If the answer will be in percent, round off to four places so you have two decimal places in your percent.

Example: 1 is what percent of 3?

$$1 \div 3$$
$$= .3333... \quad \text{round off to four places}$$
$$.3333 \times 100$$
$$= 33.33\%$$

Example: 200 is what percent of 300?

$$200 \div 300$$

$$= .6666... \text{ rounded to four places is } .6667$$

$$.6667 \times 100 = 66.67\%$$

TO FIND WHAT PERCENT ONE NUMBER IS OF ANOTHER

1. Make a fraction. Use the number representing the total or the number following “of” as the denominator. Make the other number the numerator.
2. Change the fraction to a decimal number by dividing the numerator by the denominator.
3. Multiply this division answer by 100 and add the percent sign.

Answer the following questions. Answers are at the end of this skills manual.

7. a) 3 is what percent of 12? b) 28 is what % of 32? c) What percent of 36 is 9?
- d) What % of 40 is 15? e) What % of 1 is 1? f) 250 is what % of 200?
- g) 25 is what % of 75? h) What % of \$3.99 is \$1.10?
8. If the normal price of a reciprocating saw is \$500 and you only pay \$440, what % savings do you get? (Your savings are the amount you don't pay. The discount is \$60. What % is 60 of 500?)
9. If an average of 9 out of 100 people in your city plan to renovate this year, what percent of the city's population plans to renovate?
10. A construction company estimates that it will take 40 days to frame in a house. What percent of the job still needs to be done after 8 days? (Subtract 40 - 8 to find how many days work are left.)

Problem Type #3: To Find a Number When a Percent of It Is Known

To find an unknown number when a percent of it is given:

1. Change the percent to a decimal number by dropping the percent sign and moving the decimal point two places to the left.
2. Divide the given number by the decimal number. The division answer is the unknown number.

Example: An examination of construction costs reveals that windows cost 6% of the total construction costs. If a company spent \$18,000 on windows for a building, how much did the total construction cost?

$$6\% = .06$$

Change 6% to a decimal.

$$\begin{aligned} \$18,000 \div .06 \\ = \$300,000 \end{aligned}$$

Divide the cost of the windows by the decimal number.
The answer is the cost of the whole construction.

The building costs \$300,000 to construct.

This type of question can be worded in two different ways, but each is solved in the same way.

Example: 45% of what number is 90?

$$90 \div .45 = 200$$

Change 45% to .45 and divide

The unknown number is 200.

Example: 12 is 24% of what number?

$$12 \div .24 = 50$$

Change 24% to .24 and divide

The unknown number is 50.

ANSWER PAGE

WRITING PERCENT

1. a) 10% b) 20% c) 100% d) 200% e) 85% f) 275%
g) 5 % h) 33.33% i) 100%

CHANGING PERCENT TO A DECIMAL

2. a) .28 b) .567 c) .07 d) 1 e) .085 f) 2.5
g) .15 h) .002 i) .625

FINDING A PERCENT OF A NUMBER

3. a) 100 b) 49.95 c) 1.275 d) \$5.82 e) \$.30 f) .03
g) 75 h) 102 i) \$.97

4. Sales tax is $.13 \times 45.98 = \$5.98$ The total cost is $\$45.98 + \$5.98 = \$51.96$.

5. 20% of the students didn't pass the test, so $.2 \times 40 = 8$. 8 students failed.

6. $.06 \times 6,000 = \$360$. You will pay \$360 interest.

FINDING WHAT PERCENT ONE NUMBER IS OF ANOTHER

7. a) 25% b) 87.5% c) 25% d) 37.5% e) 100% f) 125%
g) 33.33% h) 27.57

8. $60 \div 500 = .12$
 $.12 \times 100 = 12\%$
The saving is 12%.

9. $9 \div 100 = .09$
 $.09 \times 100 = 9\%$

10. $32 \div 40 = .8$
 $.8 \times 100 = 80\%$. 80% of the job is still left to be done.

FINDING A NUMBER WHEN A PERCENT OF IT IS KNOWN

11. a) 500 b) 50 c) 80 d) 24 e) 70 f) 31.5

12. $\$1250 \div .25 = \1666.67
The total bill will be \$16667.66

13. \$17.16

14. 60% is being used, 40% is left over
15. 6% interest on \$19,000 is \$1140, total cost is \$24,140
16. \$127.50 (find the cost of 200, then find 75% of that cost)