EVALUATING
ACADEMIC READINESS
FOR APPRENTICESHIP TRAINING
Revised for
ACCESS TO APPRENTICESHIP

MATHEMATICS SKILLS
CALCULATION OF PERIMETER, AREA
& VOLUME OF GEOMETRIC FIGURES

AN ACADEMIC SKILLS MANUAL
for
The Construction Trades (Structures)

This trade group includes the following trades:
Drywall & Acoustical Applicator, General Carpenter,
Mason (Brick & Stone and Restoration), Reinforcing Rod Worker, Roofer,
Terrazzo, Tile & Marble Mechanic

Workplace Support Services Branch
Ontario Ministry of Training, Colleges and Universities

Revised 2011

In preparing these Academic Skills Manuals we have used passages, diagrams and questions similar to those an apprentice might find in a text, guide or trade manual. This trade related material is not intended to instruct you in your trade.

It is used only to demonstrate how understanding an academic skill will help you find and use the information you need.

MATHEMATICS SKILLS: CALCULATION OF PERIMETER, AREA & VOLUME OF GEOMETRIC FIGURES

An academic skill required for the study of the Construction Trades: Structures

INTRODUCTION

An important part of the job of a skilled construction tradesperson involves making measurements based on instructions such as blueprints and then building based on those measurements. Before you begin construction, one of the challenges may be to take those measurements and to make calculations such as perimeter, area and volume.

For example, to make a window frame, a glazier must calculate the perimeter around the glass in order to know how much trim will be needed. A reinforcing rod worker would need to calculate the total area of concrete coverage in order to determine the number of reinforcing rods to use.

This skill sheet reviews the steps in finding the perimeter, area and volume of simple two and three dimensional geometric figures, including:

- Two dimensional figures
- Finding the perimeter
- Finding the area
- Calculating the cost of covering an area
- Three dimensional figures
- Finding the surface area
- Finding the volume

TWO DIMENSIONAL GEOMETRIC FIGURES

A simple, closed, two dimensional (flat) figure with three or more straight sides is called a **polygon**.

- Triangles, squares, rectangles, and parallelograms (figures with 2 pair of opposite sides parallel) are all examples of polygons.

A **circle** is also a flat, closed figure but it is a curve, consisting of points that are all the same distance from the center.

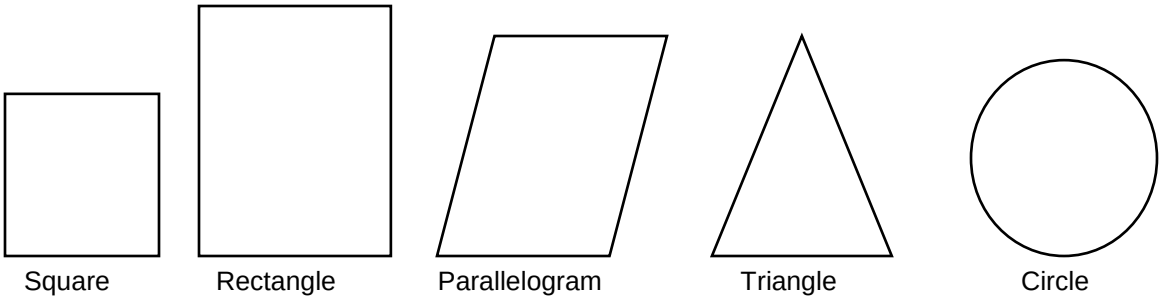


FIGURE 1: Some Simple Geometric Shapes

These figures can be measured in different ways.

- Whenever we use measurements to make calculations with geometric figures, all measurements must be in the same linear units.
  - The units might be meters or centimeters, but they can't be a mix of meters and centimeters.

FINDING THE PERIMETER

*The **perimeter** (P) of any polygon is the distance around its boundary*. Perimeter is found by adding together the lengths of the sides.

Perimeter of a Rectangle

*A **rectangle** is a polygon with four 90°angles (right angles) and with each pair of parallel sides the same length* (see Figure 1).

- This means that we can find the perimeter of a rectangle by adding the lengths of the two long side to the lengths of the two shorter side.

The perimeter of a rectangle equals twice the length (*l*) added to twice the width (*w*). The formula is written in two forms:

$P = 2l + 2w$ where P is the perimeter, l is the length and w is the width of the rectangle.

or $P = 2(l + w)$

***Remember:** When finding perimeter, all units must be the same. If the length is measured in feet and the width in yards, one unit must be changed to that of the other.*

Example: Find the perimeter of a house that is 30 m long and 16 m wide.

$P = 2l + 2w$
$= 2(30\text{ m}) + 2(16\text{ m})$
$= 60\text{ m} + 32\text{ m}$
$= 92\text{ m}$

The perimeter is 92 m.

Example: Find the amount of fencing required to close in a space that is 400 yd wide and 1500 ft long.

Known:
l = 1500 ft
w = 400 yd = 1200 ft 400 yd x3 = 1200 ft

Find perimeter (P)

P = 2(*l* + *w*)
= 2 (1500 ft + 1200 ft)
= 2(2700 ft)
= 5400 ft

The space will require 5400 ft of fencing.

Perimeter of a Square
A square is a rectangle with all four sides the same length.

To find the perimeter of a square, multiply the length by 4.

Perimeter of a square = 4*l*

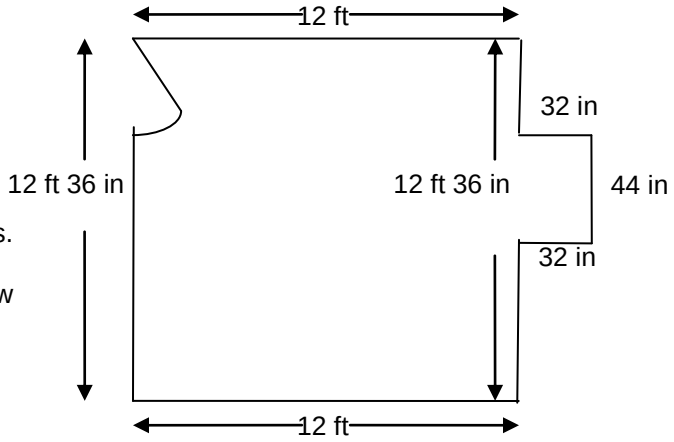
Example: How much baseboard trim is required for a bedroom that is 12 ft square? (If a room is 12 ft square, it measures 12 ft by 12 ft.)

P = 4*l*
= 4(12)
= 48 ft

48 ft of trim is required.

Often a space is more complicated than a simple square.

Example: If the bedroom has two door openings, each measuring 36 in. and a closet with two sides measuring 32 in. and a back length of 44 in, how much trim will be required now?



A diagram can help you with these calculations. When you have to find the perimeter or area and a diagram is not shown, it is helpful to draw one.

36 in÷12 = 3 ft First convert the 36 in door openings to feet.

32 in + 32 in + 44 in = 108 in	Add the widths of the closet
108 in ÷ 12 = 9 ft	Convert the inches to feet
48 ft - 2(3 ft) + 9 ft = 48 ft - 6 ft + 9 ft = 51 ft	Subtract the door openings and add the closet

51 ft of trim is needed.

To find the perimeter of an irregular shape, you basically add all the lengths together. Just make sure all the measurements are in the same units.

Finding the Length of an Unknown Side When the Perimeter Is Known

If you know the perimeter of a rectangle and the length of one side, you can find the other side.

1. Manipulate (or rearrange) the variables in the formula for perimeter so the letter for length or width is by itself on the left side.
2. Solve to find the unknown side.
3. *Remember, whatever you do to one side of the formula, you need to do to the numbers and letters on the other side.*

Example: The perimeter of a window is 144 inches. The height of the window is 42 inches. What is the width?

Known:
P = 144 in
l = 42 in

Find w

P = 2*l* + *2 w* 144 = 2(42) + *2w* 144 = 84 + 2*w*	Fill in the quantities you are given.
84 + 2*w* = 144	Reverse the equation.
84 – 84 + 2*w* = 144 – 84	Subtract 84 from both sides.
2*w* = 60	Divide both sides by 2.
w = 30	Write in the units, inches.

The width is 30 inches.

FINDING THE AREA

*The **area** of a polygon is the measure of the surface inside the boundary*. The units of area are squared units.

Area of a Rectangle

The area of a rectangle is the amount of surface enclosed within its boundaries of ***length*** and ***width***.

Example: The area of a room is the amount of floor space it has.

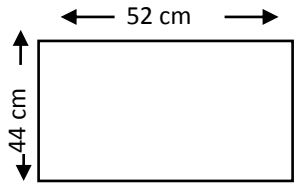
Area is calculated by multiplying the length of the rectangle times its width.
The formula for area is:

$A = lw$

Remember: *When finding the area of a rectangle, the units used to measure the length and the width must be the same. If the length is in meters, the width must also be in meters. If the units are different, one must be converted to the other before you can multiply.*

Example: Find the area of a rectangle that is 52 cm long and 44 cm wide.
(The units are the same so we don't have to convert.)

Draw the rectangle



Known:	*Find:*
l = 52 cm	Area
w= cm	Use A = lw

A = *lw*
= 52 cm x 44 cm
= 2288 cm^2

Note: When two of the same units are multiplied together, such as the centimeters in our example, they become square units. Instead of writing square centimeters, you can use the short form of cm^2 or sq cm. (Sq is the short form for square.) Four square feet is written 4 sq ft or 4 ft^2.

Example: Find the area of a space with length 5 m and width 142 cm.

We must convert one of the units so both are the same.

Known:	*Find:*
l = 5 m	area
w = 142 cm	Use A = lw
w = 1.42 m	

A = *lw*
A = 5 m x 1.42 m
A = 7.1 m^2

Example: Find the floor space of a box that measures 60 inches long by 40 inch wide by 20 inches high.
(The information on height is not needed to answer this question.)

Known: l = 40 in
w = 20 in

Find: A
Use A= lw

A = *lw*
= 60 x 40
= 2400 sq in

Example: In order to calculate the quantity of terrazzo tile for a family room you need to calculate the area of the floor. If the family room measures 5 m by 3.5 m, what is the floor space to be covered?

Known: l = 5m
W = 3.5 m

Find: A

A = *lw*
= 5 x 3.5
= 17.5 m^2

The floor space to be covered is 17.5 m^2

Area of a Square

The four sides of a square are all the same length. To find the area of a square, square the length. (To square a number, multiply it by itself. Three squared is 3 x 3 = 9.)

Example: Find the area of a square with sides 15 ft long.

Known: l = 15 ft
w = 15 ft

Find A

A = *lw* or l^2
A = 15 ft x 15 ft
A = 225 sq ft

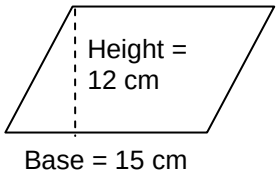
Area of a Parallelogram

The area of a parallelogram is equal to the altitude or height times the base. The formula is:

A = *ab* or *bh*

Example: Find the area of a parallelogram with a height of 12 cm and a base of 15 cm.

Draw and label a parallelogram



Known: b = 15 cm
h = 12 cm

Find A

A = *bh*
= 15 cm x 12 cm
= 180 cm^2

Finding the Length of an Unknown Side When the Area Is Known

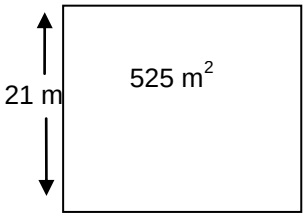
If you know the area of a rectangle and the width of one side, you can find the length of the other side.

1. Manipulate the variables in the formula for area so the letter for length ends up by itself on the left side.
2. Substitute the known measurements for area and width.
3. Solve to find the unknown side.

You can substitute the given measurements either before or after you manipulate the formula.

Example: If the area of a rectangle is 525 m^2 and the width is 21 m, what is the length?

Draw and label the rectangle



Known :
w = 21 m
A = 525 m^2

Find l

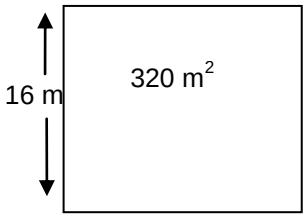
A = *lw*

525m^2 = *l* x 21m	First fill in the given quantities.
l x 21 m = 525m^2	Reverse the equation.
$\frac{1}{\cancel{21m}} \times \frac{25}{\cancel{21m}} = \frac{\cancel{525m^2}}{\cancel{21m}}$	Divide both sides by 21 m to isolate *l* on the left.
$\frac{1}{1}$	When you divide a squared unit by a linear unit, such as square meters by meters, the meters on the bottom cancel one of the units on the top, leaving meters in the answer.
l = 25 m	

The length is 25 meters.

Example: Find the height of a concrete wall that has an area of 320 m^2 and a width of 16 m.

Draw and label the wall



Known:
w = 16 m
A = 320 m^2

Find l

First rearrange the letters of the formula so *l* is by itself on the left.

A = *lw*

lw = A Reverse the equation.

l = A/*w* Divide both sides by *w.*

l = $\frac{320 \text{ m } (\cancel{\text{m}})}{16 \cancel{\text{m}}}$ Fill in the given amounts. Divide. Cancel the units where possible.

l = 20 ft

The length of the wall is 20 m.

CALCULATING THE COST OF COVERING AN AREA

Once you have found the area of a space, you might have to calculate the cost of covering it with a material like steel, or flooring, or paint. The cost of the material is usually given as a rate, such as $16.95 per square meter, or $12.45 per can where a can of paint covers 150 sq ft.

To find the cost of covering an area:

1. First calculate the area to be covered.
2. Then multiply the area by the cost per unit area.

Example: Find the cost of replacing the shingles on a roof measuring 9 m long and 5 m high. The shingles come in bundles costing $64.99. Each bundle covers an area of 5 m^2.

Known:
Roof l = 9 m
w = 5 m
Cost of shingles 64.99 per 5 m^2

Find:
Area of the roof (A = lw)
Number of bundles to cover the roof (Area of roof ÷ 5 m^2)
Cost to cover the roof (number of bundles x $64.99/$m^2$)

First find the area:
A = *lw*
= 9 m x 5 m
= 45 m^2

Next, find the number of bundles

$$\frac{45\ m^2}{5\ m^2} = 9 \text{ bundles}$$

Find the total cost:

9 x $64.99 = $584.91

The cost of shingles for the roof is $584.91

THREE DIMENSIONAL FIGURES

A closed, solid geometric figure has three dimensions. It has length, width and height or depth. Some solid figures are the cube, the rectangular solid, the cylinder, the cone and the sphere.

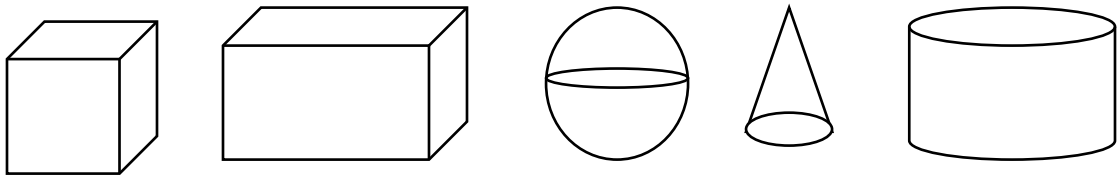


FIGURE 2: Solid Geometric Figures

SURFACE AREA OF THREE DIMENSIONAL FIGURES

The surface area of a three dimensional figure is the combined areas of all the outside surfaces or faces of the figure. When finding the surface area, all measurements must be in the same linear units. The answer will be in square units.

Finding the surface area of a rectangular solid

To find the total area of the outside surface of a rectangular solid, we have to find the areas of each face of the figure.

1. First find the area of the front surface by multiplying the length times the height.
   - The back surface is the same area, so multiply that answer by 2.
2. Next find the area of one side by multiplying the width times the height.
   - Since the opposite side is the same, multiply the answer by 2.
3. Now find the base by multiplying the length times the width.
   - The top is the same as the base, so multiply that answer by 2 also.

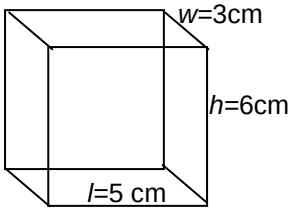
The formula is:

$A = 2lh + 2wh + 2lw$
or $A = 2(lh + wh + lw)$

or $A = 2(lh + wh + lw)$

Example: Find the total area of the outside surface of a rectangular solid 5 cm long, 3 cm wide and 6 cm high.

Draw and label the solid



Known:
l = 5 cm
w = 3 cm
h = 6 cm

Find:
Outside surface area of the solid A = 2(lh + wh + lw)

A = 2 (*lh* + *wh* + *lw*)
= 2(5cm x 6cm + 3cm x 6cm + 5cm x 3cm)
= 2(30 cm^2 + 18 cm^2 + 15 cm^2)
= 2(63 cm^2)
= 126 cm^2

Finding the surface area of a cube

A cube is made of six identical squares. Each edge is the same length, each side has the same area.

To find the area of a cube:

1. Find the area of one side (l^2) and multiply it by 6.

The formula is:

$A = 6 (l^2)$

Example: Find the total surface area of a cube whose edges measure 10 in.

Known:
Edges of cube = 10 in

Find:
Surface area of cube $A = 6(l^2)$

A = 6 (l^2)
= 6(10^2)
= 6(100)
= 600 sq in.

Finding the surface area of a cylinder

The surface area of a cylinder consists of the outside curved surface, which is actually a rectangle if it is straightened, and the circular areas at the top and bottom.

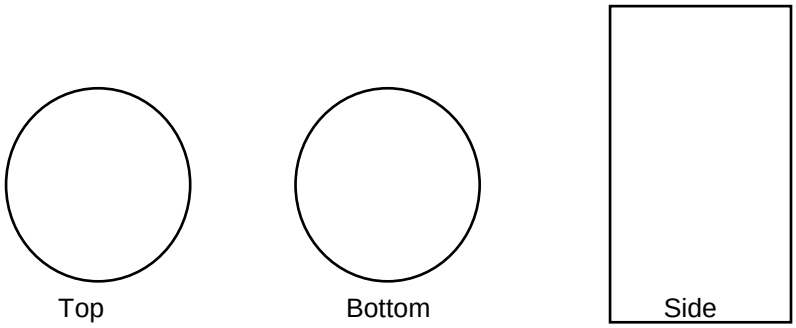


FIGURE 3: Finding the surface area of a cylinder

To find the surface area of a cylinder:

1. Find the area of each of the top and bottom circles.
2. Find the area of the rectangular side:
3. Add the areas together.

1. To find the area of the top and bottom: Use the formula $A = \pi r^2$. A cylinder has two circles (the top and the bottom), so we need to find the two areas, $2\pi r^2$.

Remember: $\pi = 3.14$

2. To find the area of the side of the cylinder (a rectangle): Multiply the length times the width.

The formula is: $A = lw$.

This rectangle has a width equal to the height of the cylinder so substitute height (h) for the width.

The formula is now: $A = 2\ lh$.

The length of the rectangle is the same as the perimeters of the circles at the top and bottom. We find the perimeter of a circle using the formula $P = 2\pi r$. Substitute this formula for the length of the rectangle.

The formula becomes $A = 2\pi rh$.

3. To find the area of the cylinder add the areas of the top and bottom ($2\pi r^2$) to the area of the rectangle ($2\pi rh$).

$A = 2\pi r^2 + 2\pi rh$.

4. The formula is rearranged to become:

$A = 2\ \pi r(r + h)$

Remember: $\pi = 3.14$

Example: Find the surface area of a cylinder when its radius is 8 ft and its height is 20 ft.

Known:
r of cylinder = 8 ft
h of cylinder = 20 ft

Find the surface area of the cylinder

$A = 2\ \pi r(r + h)$
$= (2 \times 3.14 \times 8)(8 + 20)$
$= (50.24)\ (28)$
$= 1406.72$ sq ft

Finding the surface area of a sphere
A sphere is a ball. The surface area of a sphere is equal to 4 times π times the radius squared. The formula is:

$A = 4\ \pi\ r^2$

Example: Find the surface area of a sphere with a radius of 5 cm.

Known
r = 5 cm

Find the surface area of the sphere

$A = 4\ \pi\ r^2$
$= 4 \times 3.14 \times 5^2$
$= 314\ cm^2$

To find the cost of covering the outside surface of an object:

1. Find the surface area, and
2. Multiply it by the cost per unit area.

Example: It takes 3 cans of spray paint to cover the outside of a metal box. Each can covers approximately 10 sq ft. What is the approximate surface area of the box? What will it cost to cover the box if each can costs $4.99.

Known:
1 can covers 10 ft^2
It takes 3 cans to cover the box
1 can paint costs $4.99

Find:
The approximate surface area of the box.
The cost to cover the box.

If one can of paint covers 10 ft^2, 3 cans will cover three times as much.

3 x 10 = 30 ft^2

The surface area of the box is about 30 sq ft.

If 1 can costs $4.99, 3 cans will cost:

3 x $4.99 = $14.97

VOLUME OF THREE DIMENSIONAL GEOMETRIC FIGURES

The ***volume*** *or capacity* of a solid figure is the amount of space contained within its boundaries. To calculate volume, multiply length times width times depth. Since each linear measurement has a unit, the units in the answer become cubic units. For example, meters x meters x meters equal cubic meters. The short form for cubic units such as cubic inches is in^3 or cu in.

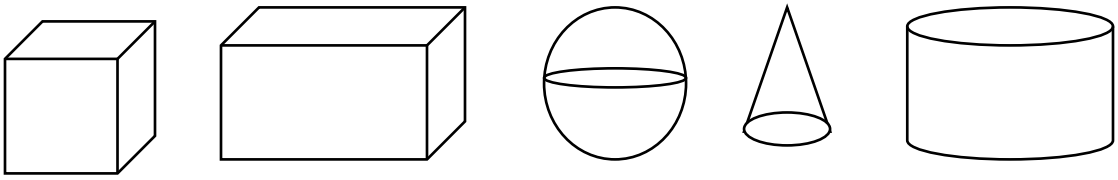


FIGURE 2: Solid Geometric Figures

Volume of a rectangular solid

The volume of a rectangular solid equals the length times the width times the height.

$V = lwh$

Example: Find the volume of a rectangular solid 9 m long, 4 m wide and 3 m high.

$V = lwh$
$= 9 \times 4 \times 3$
$= 108\ m^3$

Volume of a cube

The volume of a cube equals the length of one edge cubed. The formula is:

$V = l^3$

Example: Find the volume of a cube whose length measures 2 m.

$V = l^3$
$= 2^3$
$= 8\ m^3$

Volume of a cylinder

The volume of a cylinder equals π times the square of the radius of the base times the height. The formula is:

$$V = \pi r^2 h$$

Example: Find the volume of a cylinder with a radius of 12 ft and a height of 72 in.

72 in ÷ 12 = 6 ft Change the units of height to feet by dividing by 12.

Now use the formula.

$V = \pi r^2 h$
$= 3.14 \times 12^2 \times 6$
$= 2713$ cu ft

Volume of a sphere

The volume of a sphere equals 4/3 times π times the cube of the radius. The formula is:

$$V = \frac{4\pi r^3}{3}$$

Example: Find the volume of a sphere with a radius of 10 inches.

$$V = \frac{4\pi r^3}{3}$$

$$V = \frac{4(3.14)(10^3)}{3}$$

$= 4186.67$ cu in

Answer the following questions about geometric figures. Answers are at the end of this manual.

1. Find the number of centimeters of metal trim needed to go around a window that is 45 cm by 25 cm.

2. Find the amount of flashing needed to encircle a flat roof that that is 30 feet by 10 yards.

3. Find the area of a rectangle that is 18 meters long and 400 centimeters wide.

4. Find the area of a parallelogram that is 7.2 m long and 4.7 m wide.

5. Find the area of a square with sides that are 16 yards long.

6. Find the cost of installing terrazzo tile on a floor that is 8.4 m long and 5.1 m wide if the tiles cost $34.95 per square meter.

7. Welded wire fabric is available in 150 ft rolls that are 5 ft wide. How many rolls are needed for a concrete foundation that is 600 ft by 100 ft?

8. If a sheet of drywall measures 4 ft by 8 ft, how many sheets are needed to cover the walls of a room if two walls measure 8 ft by 12 ft and the other two walls measure 8 ft by 10 ft?

9. If one ton of stone makes a wall 3 ft long by 3 ft high by 2 ft thick, how many tons are required for a wall that is 8 ft long by 4 ft high by 2 ft thick?

10. Find the volume of air in a room that measures 14 ft by 12 ft with 8 ft ceilings.

11. Find the volume of a cube whose sides measure 8 yards.

ANSWERS

1. P = 2 L + 2 W
   = 2(45 cm) + 2(25 cm)
   = 90 cm + 50 cm
   = 140 cm

2. Change feet to yards. 30 ft. = 10 yd.
   P = 2(10 yd.) + 2(10 yd.)
   = 40 yd.

3. Change cm to m. 400 cm = 4 m
   18 x 4 = 72 m^2

4. A = 7.2 m x 4.7 m
   = 33.84 m^2

5. 256 sq. yd

6. A = 8.4 m x 5.1 m= 42.84 m^2
   Cost = 42.84 m^2 x \$34.95/ m^2 = \$1497.26

7. Area covered by one roll = *l* x *w* = 150 x 5 = 750 sq ft
   Area of concrete = *l* x *w* = 600 x 100 = 60 000 sq ft
   Number of rolls needed = 60 000 sq ft ÷ 750 sq ft = 80

8Area covered by 1 sheet = 4 x 8 = 32 sq ft
   Area of first two walls = 2 (8 ft x 12 f.)
   = 2 x 96 sq ft
   = 192 sq ft
   Area of other two walls = 2 (8 ft x 10 f.)
   = 2 x 80 sq ft
   = 160 sq ft
   Total area = 352 sq ft
   Number of sheets = 352 ÷ 32 = 11

9. Volume of one ton of stone = *lwh =3 x 3 x 2 =18 sq ft*
   Volume of required wall = *lwh =8* x 4 x 2 = 64 sq ft
   Number of tons of stone = 64 sq ft÷18 sq ft = 3.6 tons

10. Vol = *lwh*
    = 14 ft x 12 ft x 8 ft
    = 1344 cu ft

11. V = l^3
    = 8^3
    = 512 cu. yd.