EVALUATING
ACADEMIC READINESS
FOR APPRENTICESHIP TRAINING
Revised for
ACCESS TO APPRENTICESHIPS

MATHEMATICS SKILLS
OPERATIONS WITH FRACTIONS AND MIXED NUMBERS

AN ACADEMIC SKILLS MANUAL
for
The Metal Work Trades

This trade group includes the following trades:
Heat & Frost Insulator, Iron Worker,
Precision Metal Fabricator, Sheet Metal Worker, and
Welder & Fitter

Workplace Support Services Branch
Ontario Ministry of Training, Colleges and Universities

Revised 2011

In preparing these Academic Skills Manuals we have used passages, diagrams and questions similar to those an apprentice might find in a text, guide or trade manual.

This trade related material is not intended to instruct you in your trade. It is used only to demonstrate how understanding an academic skill will help you find and use the information you need

MATHEMATICS SKILLS
OPERATIONS WITH FRACTIONS AND MIXED NUMBERS

An academic skill required for the study of the Metal Work Trades

INTRODUCTION

If you have an eight foot sheet of metal and you need half of it, you cut it into two equal pieces that are four feet long. The new pieces are, of course, smaller than the original piece. Each is *one-half* as large as the original one. The expression one-half is used to indicate a partial amount of a whole. Expressions like one-half, two-thirds and three-quarters are called ***fractions.*** *Fractions indicate a part of a whole*.

If you have one-half of a sheet of metal, you have one of the two equal parts that make up the whole sheet. If you have one-quarter of the sheet, you have one out of four equal pieces that make up the whole. If you have three-thirds, you have all three parts that make up the whole sheet. You have one complete sheet, even though it may be cut into three pieces.

Measurements are often expressed as fractions, especially in the imperial system. A certain length of wire might be expressed as 81½ feet or 3 1/4 inches. You need to be able to add, subtract, multiply and divide fractions so that you can do operations with measurements that include fraction.

This skills manual will cover the following information about fractions:

- What is a fraction?
- Equivalent fractions and lowest terms
- Adding fractions
  - fractions with the same denominator
  - fractions with different denominators
  - adding more than two fractions
  - adding mixed numbers
- Subtracting fractions
- Multiplying fractions
- Dividing fractions
- Using a calculator

WHAT IS A FRACTION?

Terms such as one half and three quarters indicate partial amounts of an original whole. They are called fractions. *A **fraction** is a mathematical shorthand used to indicate a part of a whole.*

Let's examine what fractions are by looking at the inch divisions on a ruler. See Figure 1.

Each 1 inch division can be divided into smaller, equal spaces. If an inch is divided into two equal spaces, the spaces are ***one half*** of an inch. Two one half spaces equal one inch.

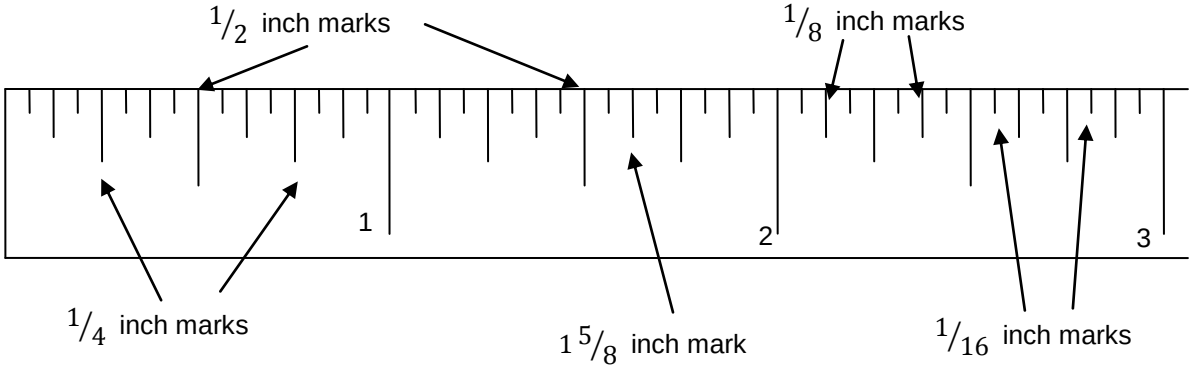


Figure 1: Imperial Ruler

If a one inch division is divided into four equal spaces, each space is ***one fourth,*** or ***one quarter*** of an inch. So 4 one quarter inch spaces equal one inch. A measurement that is three of the quarter inch divisions is called three quarters of an inch.

Fractions are used in many different calculations.

Example: You need 8 even spaces marked on a 32 ½ foot bar. To find the spacing, you must divide 32 ½ feet by 8.

Example: You work 5 ½ hours one day, 7 hours the next, 6 3/4 the next, 7 1/4 the next and 8 hours the last day of the week. To submit your total hours for the week, you have to add numbers with fractions.

Writing Fractions

Fractions can be written in several ways. Here are different ways to write the fraction three quarters.

- three quarters
- 3/4
- 3 out of 4
- three fourths
- $\frac{3}{4}$

*A **fraction** is written to indicate a part of a whole.*

1. Numerically, a fraction is written with two numerals separated by a fraction line.
2. *The digit under the fraction line is the **denominator. It tells how many parts the whole thing was divided into.***
    - In the fraction ¾ the denominator, 4, tells how many parts the whole thing is divided into.

3. *The digit above the fraction line is the* ***numerator****.* It tells how many of the total parts we have.
   - In the fraction ¾ the numerator, 3, tells how many of the total parts we have.
4. We have 3/4 of something, or three parts of the total of four.

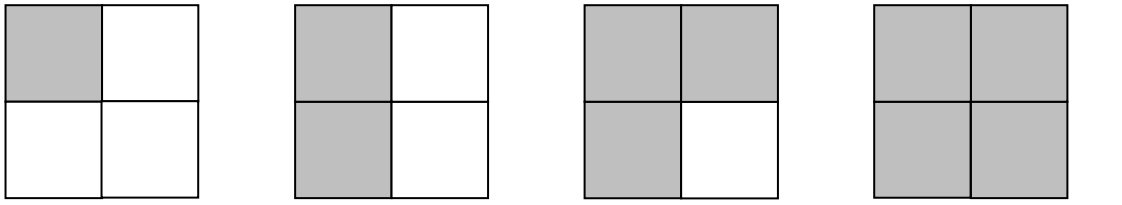
Remember that the **d**enominator is **d**own under the fraction line.

Fractions are part of something whole
A fraction represents *part* of an original whole that has been divided into equal pieces.

> **Example:** You cut a piece of wire into five equal pieces and you use two pieces. In other words, you use two pieces out of five or 2/5 of the wire. In the fraction 2/5, the ***denominator*** 5 tells you that the original whole wire has been divided into 5 equal parts. The ***numerator*** 2 tells that you use or have 2 of those parts.

Look at Figure 2 as you read the following. Consider a sheet of metal that is divided into 4 equal pieces. If you use 2 out of the 4 pieces, you use 2/4 of the total. You can see that this is the same amount as 1/2 of the sheet. Two pieces out of four is the same as one out of two. You can write 2/4 as 1/2.

Four pieces out of four are equal to all of the original amount or one whole. 4/4 equals all of the sheet or 1. Even though the sheet is now cut into 4 pieces, you still have the same total amount.



1 part out of 4 = 1/4

2 parts out of 4= 2/4
2/4 = 1 part out of 2 = 1/2

3 parts out of 4= 3/4

4 parts out of 4 = 4/4 or 1 whole

FIGURE 2: A Fraction Is A Part Of A Whole

Naming fractions: 1/2 is written in words as one half. 1/3 is written as one third. 1/4 is written as one quarter or one fourth. The other fractions are written by adding ***th*** to the name of the denominator. If we divide a whole sheet into four pieces, we say the sheet is divided into fourths. If it is divided into twenty equal pieces, we say the sheet is divided into twentieths. If we have eleven out of the twenty pieces, we have eleven twentieths.

Fractions Mean Something is Divided
Division is a way of splitting a number into *equal, smaller* numbers. For example, a group of 12 nails can be divided into 3 smaller groups of 4 nails each. 12÷3 = 4 (Sometimes when we divide a number we have a leftover amount known as a remainder. 13 ÷3 = 4 R1)

Fractions are a way of indicating a division question in which the numerator is divided by the denominator.

3/4 is the same as 3 ÷ 4
½ is the same as 1 ÷ 2
12/3 is the same as 12 ÷ 3
8/5 is the same as 8 ÷ 5

When you use a number as a fraction, you don't actually do the division. If you do the division, the answer will be one of these: a whole number, a whole number with a remainder or a decimal number.

When you use a formula to find an unknown number, division is indicated by the fraction form.

Example: To find the length (l) of a room when you know the area (A) and the width (w), you use the formula $l = A/w$. The fraction A/w tells you to divide the area by the width.

Mixed Numbers and Improper Fractions
You can have one or more whole objects along with part of an object.

Example: You might have three whole pounds and one half pound of sealant. This is written 3 ½ lb and read as three and a half pounds.

*A **mixed number** is a combination of a whole number and a fraction.*

- A mixed number such as 3 ½ can be written as the fraction 7/2, *with the whole number included in the numerator*.
  - The 3 whole numbers are each divided into 2 halves, just like the inch division on a ruler.
  - This gives a total of 6 halves.
  - If you add the 6 halves to the 1 half, you have 7 halves.
  - Seven halves can be written 7/2.

*An **improper fraction** is a fraction with the numerator larger than the denominator.*

- The form 7/2, in which the numerator is larger than the denominator, is an improper fraction.

You can change a mixed number like 3 ½ to an improper fraction like 7/2. You can also change an improper fraction into a mixed number.

- In fact, in order to multiply or divide with numbers that include fractions, you must ***change all mixed numbers to improper fractions before multiplying or dividing.***

- On the other hand, when you get an improper fraction as the answer to a question, ***the improper fraction is usually changed to a mixed number***.

You need to know how to change a mixed number such as 1 1/4 to the improper fraction 10/8. You also need to know how to change an improper fraction such as 45/7 to the mixed number 6 3/7.

To change a mixed number to an improper fraction:

1. Multiply the *whole number* by the *denominator* of the fraction.
2. Add this *answer* to the *original numerator* to get the *new numerator*.
3. Write the *new numerator* over the fraction line and the *original denominator* under it. Notice the denominator stays the same.

Example: With 3 ½ pounds of sealant, our denominator is 2. We make the whole number 3 into a fraction with the denominator 2.

- When we do that, we divide each pound into two pieces, giving us 3 x 2 or 6 halves.
- When we put all the halves together (six halves plus the original one half), we will have 7 of them.
- Write the numerator 7 on top of a fraction line and the denominator 2 underneath. This gives the improper fraction 7/2.

Example: Change 3 2/5 to an improper fraction:

3 x 5 = 15	Multiply the whole number 3 by the denominator 5..
15 + 2 = 17	Add 15 to the numerator to get the new numerator of the improper fraction.
	Replace the original numerator with the new numerator. Put 17 over the original denominator 5 to get the improper fraction 17/5.

The mixed number 3 2/5 is the improper fraction 17/5.

To change an improper fraction to a mixed number:

1. Divide the numerator by the denominator.
2. Your answer becomes the whole number.
3. The remainder forms the numerator of the fractional part.
4. It is written over the fraction line above the original denominator.

Example: Change 14/3 to a mixed number:

14 ÷ 3 = 4	with remainder 2	4 becomes the whole number. remainder 2 becomes the numerator. the denominator 3 remains the same

The improper fraction 14/3 is the mixed number 4 2/3.

Note: If the fraction is not already in lowest terms, it should be reduced to lowest terms. We will look at how to do this later.

There are a few unusual cases that you should recognize.

1. An improper fraction will end up as a whole number if the denominator divides evenly into the numerator.

   Example: 9/3 = 3

2. Any fraction with the same numerator and denominator is equal to one.

   Example: 4/4 = 1

Note: Multiplying or dividing any number by 1 does not change the value of the number. So multiplying or dividing a number by 1 expressed as a fraction such as 4/4 does not change the value of the number.

3. Any whole number may be written as a fraction by putting it over 1. This step will be used later when dividing a fraction by a whole number.

   Example: 8 = 8/1

EQUIVALENT FRACTIONS AND LOWEST TERMS

Equivalent Fractions

When you cut a sheet of metal down the middle, dividing it evenly into two pieces, each piece is half (½) of the whole sheet. Look at the diagram.

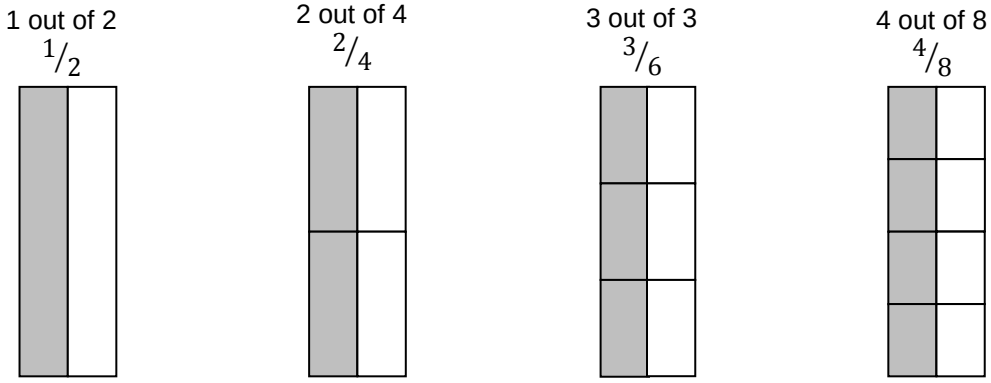


FIGURE 3: $^{1}/_{2}, ^{2}/_{4}, ^{3}/_{6}, ^{4}/_{8}$ are equivalent fractions.

The shaded areas all represent the same amount.

You see that if you cut the sheet into 4 equal pieces and use two fourths (2/4), you would, again, use ½ of the sheet. If you cut the sheet into six equal parts, you see that 3 of those 6 pieces, or 3/6 of the whole sheet, is the same as ½ of the sheet. If you cut the sheet into 8 equal pieces, you see that 4 of the 8 pieces, 4/8, is also equal to 1/2.

The fractions 2/4, 3/6 and 4/8 each represent the same amount as 1/2. They are equivalent fractions; that is, they are the same as or equal to ½. They represent the same partial amount of the whole. Because 4/8, 3/6, 2/4 and 1/2 represent the same amount, they are called ***equivalent fractions.***

Making an Equivalent Fraction
Let's take a closer look at equivalent fractions, starting with 1/2 and 2/4.

- Notice that by multiplying both the numerator and the denominator of 1/2 by 2, we get 2/4.

  $\underline{1}$ x 2 = $\underline{2}$
  2 x 2 = 4

- And, if we divide both the numerator and the denominator of 2/4 by 2, we get 1/2.

  $\underline{2}$ ÷ 2 = $\underline{1}$
  4 ÷ 2 = 2

Look next at the equivalent fractions 1/2 and 3/6.

- If we multiply both the numerator and the denominator of 1/2 by 3, we get 3/6.

  $\underline{1}$ x 3 = $\underline{3}$
  2 x 3 = 6

- And if we divide both the numerator and the denominator of 3/6 by 3, we get 1/2.

  $\underline{3}$ ÷ 3 = $\underline{1}$
  6 ÷ 3 = 2

Now look at ½ =4/8

$$\frac{1 \times 4}{2 \times 4} = \frac{4}{8}$$

And

$$\frac{4 \div 4}{8 \div 4} = \frac{1}{2}$$

In each case, we changed one fraction into an equivalent fraction by multiplying or dividing.

To make an equivalent fraction with a larger denominator:

1. Multiply both the denominator and the numerator of the fraction by the same number.

2. Use the answers from the multiplication to form the numerator and denominator of the equivalent fraction.

Example: Using the fraction 2/5, make an equivalent fraction by multiplying by 3/3.

$\frac{2}{5} \times \frac{3}{3}$ Multiply both the numerator and denominator by 3.
In other words, multiply the fraction by 3/3.

$= \frac{6}{5}$ The equivalent fraction is 6/15.

Example: Change 2/3 to an equivalent fraction by multiplying by 4/4.

2/3 = 8/12 Multiply both the numerator 2 and the denominator 3 by 4.

Example: Change 3/4 to an equivalent fraction by multiplying by 5/5.

3/4 =15/20

You also will be given a fraction and only the denominator of a new, equivalent fraction. You have to find the numerator of the second fraction.

Example: You might need to change the fraction 2/3 to an equivalent fraction with a denominator of 12. Write the second fraction as ?/12 until you find the numerator.

To find an equivalent fraction when one fraction and the denominator of an equivalent fraction are known:

1. Divide the larger denominator by the smaller denominator.

2. Multiply the numerator and denominator of the original fraction by the division answer.

3. The resulting answer is the equivalent fraction.

Example: Find an equivalent fraction to 2/3 whose denominator is 12.
In other words, 2/3 = ?/12.

12 ÷ 3 = 4 Divide 12 by 3.

$\frac{2}{3} \times \frac{4}{4}$ Multiply the numerator 2 and the denominator 3 by the division answer 4.

The fraction that is equivalent to 2/3 and that has a denominator of 12 is 8/12.

Example: 3/4 = ?/20

20 ÷ 4 = 5 Divide 20 by 4.

$\frac{3}{4} \times \frac{5}{5}$ Multiply the numerator 3 and the denominator 4 by the division answer 5.

3/4 = 15/20

Example: 1/6 = ?/18

18 ÷ 6 = 3 Divide 18 by 6.

$\frac{1}{6} \times \frac{3}{3}$ Multiply the numerator 1 and the denominator 6 by the division answer 3.

1/6 = 3/18

Fractions in Their Lowest Terms

When you finish a question, change any improper fractions to mixed numbers. Next, write the fraction answer as the ***smallest equivalent fraction***. This is called ***reducing to lowest terms***.

Any number that will divide evenly into both the numerator and denominator is a ***common factor:***

- 5 divides into both 10 and 15, so it is a common factor of 10 and 15.

A common factor could be one of the numbers *in the fraction:*

- 6 divides into 6 and 18, so 6 is a common factor.

To reduce a fraction to lowest terms:

1. Look for a common factor:
   - if there is none, the fraction is already in its lowest terms.
2. If you see a common factor, divide both the numerator and denominator by that number.
3. Write the division answers as the new numerator and denominator.
4. Look for another common factor:
   - if there is one, repeat the steps.
5. If there are no more common factors, the fraction is in its lowest terms.

If you aren't sure if there are any common factors, try **two, three or five**. If one of them divides evenly into both the numerator and the denominator, start with that.

Example: Reduce 16/24 to lowest terms:

$\frac{16 \div 2}{24 \div 2}$ 2 is a common factor; divide the numerator and denominator by 2

= 8/12 Try to do it again.

$\frac{8 \div 4}{12 \div 4}$ It is now easy to see that 4 will divide evenly into 8 and 12

= 2/3 The fraction is in its lowest terms.

You might have seen immediately that 8 is a common factor and divided by it, getting the lowest term fraction 2/3 in one step. The larger the common factor you use, the fewer steps you need to use, but the final answer will be the same.

Example: Reduce 15/35 to lowest terms.

$\frac{15 \div 5}{35 \div 5}$ 5 is the common factor.

= 3/7

Example: Reduce 18/24 to lowest terms.

$= \frac{18 \div 6}{24 \div 6}$ (6 is the common factor)

= 3/4

TO CHANGE A MIXED NUMBER TO AN IMPROPER FRACTION
1. Multiply the whole number by the denominator.
2. Add the answer to the numerator.
3. Place this new numerator over the original denominator.

1. Change the following mixed numbers to improper fractions. **Answers are at the end of this skills manual.**

a) 3 2/7 = b) 1 5/6 = c) 5 1/4 =

d) 2 7/8 = e) 10 2/3 = f) 6 1/5 =

TO CHANGE AN IMPROPER FRACTION TO A MIXED NUMBER

1. Divide the numerator by the denominator. Your answer becomes the whole number.
2. The remainder forms the new numerator. Write it over the original denominator.
3. Reduce the fraction to its lowest terms.

2. Change the following improper fractions to mixed numbers and reduce to lowest terms.

a) 12/9 = b) 15/5 = c) 23/7 =

d) 99/10 = e) 6/1 = f) 10/4 =

3. Find the equivalent fraction:

a) 3/5 = ?/40 b) 1/3 = ?/9 c) 10/3 = ?/18

d) 5/8 = ?/24 e) 1/4 = ?/12 f) 2/7 = ?/21

ADDING FRACTIONS

In order to add fractions, all of the fractions must have a common (the same) denominator. To add fractions that do not have a common denominator, we have to make all of the fractions into equivalent fractions with the same denominators.

First let's look at adding fractions which already have common denominators.

Adding Fractions with Common Denominators

1. Add all the numerators together.
2. Write the sum over the original denominator.
3. Change any improper fraction to a mixed number.
4. Reduce the answer to lowest terms.

Example: Add 5/9 + 7/9

5 + 7 =12	1. Add the numerators together.
5/9 + 7/9 = 12/9	2. Place the new numerator 12 over the original denominator 9.
12/9 = 1 3/9	3. Change the improper fraction to a mixed number:
1 3/9 = 1 1/3	4. Divide by the common factor 3 to reduce to lowest terms.

Example: 3/8 + 1/8 =

$\frac{3}{8} + \frac{1}{8} = \frac{4}{8}$	add the numerators put the answer over the denominator 8
= 1/2	reduce to lowest terms

Example: Add 5/16 +3/16 + 1/16 =

$$\frac{5}{16} + \frac{3}{16} + \frac{1}{16} = \frac{9}{16}$$

Example: Add 5/12 + 11/12 + 1/12

$$^5/_{12} + {}^{11}/_{12} + {}^1/_{12} = {}^{17}/_{12}$$ improper fraction

$$= 1\,{}^5/_{12}$$ change to a mixed number

Adding Mixed Numbers

Adding mixed numbers follows the same steps as adding fractions, with the additional step of adding the whole numbers.

To add mixed numbers:

1. First add the whole numbers.
2. Then add the fractions.
3. Combine both answers.
4. If the fraction part of the answer is an improper fraction, change it to a mixed number.
5. Add the two whole numbers. Combine the answer with the new fraction.

Example: 3 2/5 + 1 1/5

3 + 1 = 4	1.Add the whole numbers
2/5 + 1/5 = 3/5	2. Add the fractions.
4 + 3/5 = 4 3/5	3. Combine the answers.

Example: 1 5/8 + 2 7/8

1 + 2 = 3	1. Add the whole numbers
5/8 + 7/8 = 12/8	2. Add the fractions.
12/8 = 1 4/8	3. Change the improper fraction to a mixed number and
1 4/8 = 1 ½	4. Reduce the fraction to its lowest terms.
3 + 1 ½ = 4 ½	5. Combine the whole numbers and write the answer as a mixed number with its fraction.

Adding Fractions That Have Different Denominators

Remember, we can add fractions only if they have the same denominators. *If fractions have denominators that are different, we cannot add them until they are all changed into* ***equivalent fractions*** *that have the same denominator, called a* ***common denominator.*** A common denominator is one which can be evenly divided by any of the denominators in the addition question.

To change denominators to a common denominator follow these steps:

1. Figure out what the common denominator is going to be.
2. Change the fractions to equivalent fractions with that common denominator.
3. Add the equivalent fractions by adding the numerators and putting the answer over the common denominator.
4. Change any improper fraction to a mixed number. Reduce to lowest terms if necessary.

Step 1: Find a common denominator

Finding a common denominator is mostly a case of trying out different methods until you get a number that will work.

Method 1: *If one denominator will divide evenly into the other, use the larger number as the common denominator*.

Example: Find a common denominator of 3/4 and 5/8. The denominators are 4 and 8.
4 will divide evenly into 8.
We can use 8 as a common denominator

Method 2**:** *A* ***common factor*** *is a number that will divide evenly into two or more other numbers.*

Examples:
2 is a common factor of 6 and 8.

3 is a common factor of 9 and 15.

If fractions such as 3/10 and 4/15 have denominators with a common factor, you can search for the smallest number that each denominator will divide into evenly. *Try multiplying the* ***larger denominator*** *by 2, 3, 4... and look at each answer to see if the smaller denominator will divide evenly into it. The first number that the smaller denominator will divide into can be used as a common denominator*.

Example: Find a common denominator of 3/10 and 4/15.
The denominators are 10 and 15.
15 x 2 = 30
You can divide 10 into 30.
30 can be used as a common denominator.

Example: Find a common denominator of 1/6 and 9/10.
10 x 2 = 20 6 will not divide into 20. Try again.
10 x 3 = 30 6 will divide into 30
So 30 can be used.

Note: This method gives the smallest possible common denominator but it can be time consuming.

Method 3: *If the denominators have no common factor, multiply the two denominators together and use the answer as the common denominator*. This method will always get a common denominator you can use, although it might not be the smallest one possible.

Example: Find the common denominator of 2/3 and 4/5.
The denominators are 3 and 5.
Multiply 3 x 5 = 15
15 is a common denominator.

One of these three methods will get you a common denominator. Once you have found a common denominator, continue with the next steps.

Step 2: Change the original fractions to equivalents with common denominators and add.

Using the common denominator we have found, we change the original fractions to ***equivalent fractions*** with that denominator. We will find equivalent fractions using the examples with their common denominators from this and the previous page. Then we will do the addition.

Example 1: We found the common denominator of 3/4 and 5/8 to be 8. Now we have to find an equivalent fraction for 3/4 with 8 as the denominator.

3/4 = ?/8 4 x 2 = 8 and 3 x 2 = 6
= 6/8

We don't have to do anything to change 5/8.
Now we can add.

6/8 + 5/8
= 1 1/8 Change to a mixed number
= 1 3/8

Example 2: The common denominator of 2/3 and 4/5 is 15. Now we find equivalent fractions with 15 as the denominator.

2/3 = ?/15
= 10/15

4/5 = ?/15
= 12/15

Now we add:
10/15 + 12/15
= 22/15 Change to a mixed number
= 1 7/15

Example 3: The common denominator of 3/10 and 4/15 is 30. Next we find equivalent fractions.

3/10 = ?/30
= 9/30
4/15 = ?/30
= 8/30

Now we add:
9/30 + 8/30 = 17/30 17/30 is in lowest terms

Adding More than Two Fractions Together
Before three or more fractions can be added, they all have to have common denominators. If they don't already have common denominators, change them to equivalent fractions with common denominators.

Example: Add 3/4 + 3/8 + 11/16.
See if you can use the largest denominator as the common denominator. 4 and 8 will divide evenly into 16, so 16 can be used as the common denominator.

3/4 = ?/16
3/4 = 12/16
3/8 = ?/16
3/8 = 6/16
11/16 already has 16 as its denominator.

Now add:

12/16 + 6/16 + 11/16 12 + 6 + 11 = 29
= 29/16

Change 29/16 to a mixed number.

29/16 = 1 13/16

Example: Add 1/4 + 2/5 + 3/8.

If you can't use the largest denominator as a common denominator, find a number that all the denominators will divide into evenly. Multiply the largest denominator by 2, 3, 4 and so on, each time seeing if the other denominators will divide evenly into the multiplication answer. When you get a number that each denominator will divide into evenly, use that as your common denominator. We have written down all steps but you can do the multiplying and dividing in your head.

8 is the largest denominator.
8 x 2 = 16 4 will go into 16 but 5 won't
8 x 3 = 24 this won't work either
8 x 4 = 32 won't work
8 x 5 = 40 denominators 4, 5 and 8 will all divide into 40

Now find equivalent fractions with 40 as the denominator.

1/4 = ?/40
1/4 = 10/40

2/5 = ?/40
2/5 = 16/40

3/8 = ?/40
3/8 = 15/40

Add the equivalent fractions.

10/40 + 16/40 + 15/40 10 + 16 +15 = 41
= 41/40 Change to a mixed number
= 1 1/40

Example: Add 2 3/8 + 4 1/6.

Add the whole numbers.

2 + 4 = **6**

Add the fractions 3/8 and 1/6.

3/8 + 1/6 — 24 is a common denominator
= 9/24 + 4/24 — 9 + 4 = 13
= 13/24

Combine the whole number and the fraction.

6 + 13/24 = 6 13/24 — do this step in your head

Thus:
2 3/8 + 4 1/6 = 6 13/24

You can add the whole numbers in the same step that you add the fractions, as long as you keep the two operations separate.

Example: Add 5 2/3 + 4 1/4.

5 ?/12 + 4 ?/12
= 5 8/12 + 4 3/12
= 9 11/12

If the fraction part of the answer is an improper fraction, it must be changed to a mixed number. Then add the two whole numbers. Combine this answer with the new fraction.

Example: Add 4 2/7 + 9 6/7.

4 2/7 + 9 6/7
= 13 8/7

8/7 — change improper fraction to a mixed number.
8/7 = 1 1/7

13 + 1 1/7 — Add the two whole numbers 13 + 1= 14
= 14 1/7

Example: Add 5 4/5 + 2 2/3.

5 4/5 + 2 2/3
= 5 12/15 + 2 10/15
= 7 22/15 — 22/15 = 1 7/15
= 7 + 1 7/15
= 8 7/15

TO ADD FRACTIONS OR MIXED NUMBERS WITH THE SAME DENOMINATOR

a) Add all the whole numbers, if there are any, together.
b) Add all the numerators together.
c) Write the sum of the numerators over the original denominator.
d) If the sum is an improper fraction, change to a mixed number and add any whole numbers together.
e) Reduce the fraction to lowest terms.

4. **Add**: (Answers are on the last page.)

a) 2/7 + 4/7 b) 3/4 +1/4 c) 5/8 + 7/8

d) 6 2/5 + 1/5 e) 9 2/3 + 4 2/3 f) 4 1/6 + 8 5/6

g) 1/8 + 5/8 + 3/8 h) 4 1/12 + 5/12 + 3 5/12 i) 5 1/10 + 2 3/10 + 7/10

TO ADD FRACTIONS OR MIXED NUMBERS WITH DIFFERENT DENOMINATORS

1. First find a common denominator using one of the methods below:
   A. See if the smaller denominators will divide evenly into the largest one.
   B. Multiply the denominators together if there are no common factors.
   C. If the denominators have a common factor, you can search for a smaller common denominator by multiplying the largest denominator by 2,3,4 ... and seeing if the others will divide evenly into the answer. If they do, that answer can be used as the smallest common denominator.
   D. You can also find a common denominator by identifying in your head any number that both denominators will divide into evenly.
2. After finding a common denominator, change the fractions to equivalent fractions with that denominator.
3. Add the numerators of the equivalent fractions and place the sum over the common denominator.
4. Change any improper fractions to mixed numbers, add all whole numbers together and reduce the fraction to lowest terms.

5. **Add**:

a) 1/8 + 3/4 b) 9 1/3 + 2/5 c) 4/5 + 2/7

d) 4 1/3 + 5/12 e) 8 1/2 + 2/3 f) 2 3/8 + 4 5/6

g) 4/9 + 5/27 h) 3/10 + 5/6 i) 7/11 + 1/3

j) 3/4 + 3/8 + 1/2 k) 3/4 + 1/8 + 4/5 l) 2 5/6 + 4 2/3 + 6 5/8

6. If you poured 1 1/3 liters of oil into a can that already contained 3 1/4 liters, how much oil would be in the can?

7. In one week, Ian worked 7 1/2 hours, 5 1/4 hours, 9 hours, 7 3/4 hours and 8 1/2 hours. What was the total number of hours he worked?

8. An ironworker needs three pieces of pipe measuring 4 1/3 in, 8 2/5 in, and 6 7/15 in. What size of pipe does he need to cut all three pieces?

SUBTRACTING FRACTIONS

To subtract fractions with the same denominator:
1. Write the two fractions so the one you are subtracting from is written first.
2. Subtract one numerator from the other.
3. Put the answer over the original denominator.
4. Reduce the answer to lowest terms if necessary.

Example: Subtract 5/8 - 1/8.

5/8 - 1/8	Subtract the numerators. 5 - 1 = 4
5/8 - 1/8 = 4/8	Put the subtraction answer as the new numerator over the original denominator.
4/8 = 1/2	Reduce to lowest terms.

To subtract fractions if the denominators are *not* the same:
1. Change the fractions to equivalent fractions with a common denominator.
2. Subtract as above.

Example: 1/2 – 1/6

1/2 = 3/6 The common denominator is 6. Change 1/2 to an equivalent fraction with the denominator 6.

3/6 - 1/6 Now subtract. (3 - 1 = 2)
= 2/6 Reduce to lowest terms
= 1/3

To subtract mixed numbers:

1. Subtract the whole numbers.
2. Subtract the fractions.
3. Borrow if necessary.

Example: 7 5/8 – 4 3/8

7 – 4 = 3 Subtract the whole numbers.

5/8 – 3/8 Subtract the fractions
= 2/8 reduce to lowest terms
= 1/4

7 5/8 – 4 3/8 Put the whole number answer with the fraction answer.

= 3 1/4

You could do both steps in the same line.

7 5/8 – 4 3/8
= 3 2/8
= 3 1/4

Example: 4 5/8 – 2 1/4

1/4 = 2/8 The common denominator is 8.

4 5/8 - 2 2/8
= 2 3/8

Example: 10 4/9 – 1/6

4/9 = 8/18 The common denominator is 18.

1/6 = 3/18

10 8/18 – 3/18
= 10 5/18

Borrowing in Subtraction of Fractions

When you subtract mixed numbers, the numerator of the first fraction may be smaller than the numerator of the second one. An example is 8 1/3 - 6 2/3. You can't subtract the fractions as they are because 1/3 is smaller than 2/3.

To borrow for subtraction of fractions:

1. ***Borrow*** 1 from the first whole number.
   - This reduces the whole number to one less than before.
   - If the first whole number was 8, it now becomes 7.

2. ***Convert*** the borrowed 1 to a fraction with the same denominator as the first fraction.
   - Remember, the number 1 can be written as a fraction with any denominator, as long as it has the same numerator and denominator.
   - For example, 1 = 3/3.

3. ***Add*** the 1, now in fraction form, to the first fraction.
   - The first fraction now has a larger numerator than the second fraction.

4. ***Subtract***.

Example: Subtract 8 1/3 – 6 2/3.

Step 1: Borrow or take away 1 from the 8.
8 – 1 = 7.

Step 2: Change the borrowed 1 to a fraction.
Since any number written over itself is one, 1 = 3/3

Step 3: Add the borrowed 1 to the fraction.
8 1/3 = 7 + 3/3 +1/3
8 1/3 = 7 4/3

Step 4: Now subtract, following the rules for subtraction of fractions:
8 1/3 − 6 2/3
= 7 4/3 − 6 2/3
= 1 2/3

Borrowing from a whole number

You will also have to borrow when you subtract a fraction from a whole number.

Example: Subtract 6 – 1/4.

6 – 1 = 5 Borrow 1 from the 6.

Change the borrowed 1 to a fraction with the same denominator as 1/4. Since the denominator is 4, the 1 becomes the 4/4. The 6 becomes a mixed number. Substitute 5 4/4 for the 6. Subtract.

6 – 1/4
= 5 4/4 – 1/4 4 - 1 = 3
= 5 3/4

Here is how to subtract a mixed number from a whole number:

Example: Subtract 9 – 4 2/3.

9 - 1 = 8 Borrow 1 from 9.

1 = 3/3 Change the borrowed 1 to a fraction with a denominator of 3.

9 – 4 2/3
= 8 3/3 – 4 2/3 The 9 becomes the mixed number 8 3/3.
= 4 1/3

Note: *If fractions do not have common denominators, you can only tell if you have to borrow <u>after</u> they are given common denominators*.

Example: 5 ¼ – 3 2/5

5 5/20 – 3 8/20 Common denominator is 20.
The numerator 5 is smaller than the numerator 8.
Borrow 1 from the whole number 5. (5 becomes 4.)
Change the borrowed 1 to a fraction with the new denominator, 20.
The fraction is 20/20.
Add it to existing fraction 5/20 to get 25/20.

5 5/20 = 4 25/20

Now subtract.

5 5/20 – 3 8/20 =
4 25/20 – 3 8/20 25 - 8 = 17
= 1 17/20

TO SUBTRACT FRACTIONS

1. If the fractions have the same denominators, subtract one numerator from the other, and place the answer over the same denominator.
2. If the fractions have different denominators, change to equivalent fractions with a common denominator, and subtract as above.
3. Reduce to lowest terms.

TO SUBTRACT MIXED NUMBERS

1. Subtract the whole numbers separately from the fractions.
2. If the second fraction is larger than the first, you can borrow from the first whole number to make the first fraction larger than the second. Then subtract as above.
3. If the first number is a whole number, borrow 1 from it and change that into a fraction equal to 1 with the same denominator as the fractional part of the other number. Then subtract.
4. Reduce to lowest terms.

9. Subtract: (Answers are on the last page.)

a) 7/9 – 5/9	b) 3/4 – 1/3	c) 5/8 – 1/4
d) 2/3 – 5/12	e) 6 4/5 – 2 3/5	f) 4 1/2 – 3/8
g) 7 9/10 – 3 2/5	h) 15 3/4 – 10 2/3	i) 9 11/12 – 3/8
j) 5 1/3 – 2 2/3	k) 9 1/4 – 3/4	l) 5 1/2 – 2 3/4
m) 7 – 1/5	n) 8 – 2 1/3	o) 12 – 11 2/3

10. A worker cuts 12 3/4 ft of metal from a roll 20 ft long. How much of the roll is left?

11. A piece of piping needs to be cut on an angle. The longer side of the pipe measures 7 1/4 ft. If the shorter side needs to be 5/8 ft less than the longer side, what will the shorter side measure?

12. An welder works 1 3/4 hours on one job, 2 3/4 hours on another and 1 1/2 hours on a third. He has 1 hour for lunch and two 15 minute breaks. (15 minutes is the same as 1/4 hour.) How much time does he have left to work if he works an eight hour day?

13. A piece of copper tubing measures 2 5/6 cm on the outside diameter. If the wall of the tube is 1/3 cm wide, what is the inside diameter of the tube? (Remember to add both measurements of the wall before subtracting.)

MULTIPLICATION OF FRACTIONS

To multiply two fractions:

1. Write the two fractions beside each other with a multiplication sign between them.
2. Reduce terms if possible and if desired.
3. Multiply the numerators together to get the new numerator.
4. Multiply the denominators together to get the new denominator.
5. Write your answer with the new numerator and denominator.
6. *Reduce the answer to lowest terms* and change any improper fractions to mixed numbers.

Special cases:

1. To multiply a mixed number and a fraction, or two mixed numbers,
   - Change all mixed numbers to improper fractions before multiplying.

2. To multiply a fraction by a whole number,
   - Place the whole number over 1 to make it an improper fraction.
     - This is not necessary but might make it easier for you in some cases.

Example: 3/4 x 1/2

$\frac{3}{4} \times \frac{1}{2}$ $\frac{3 \times 1 = 3}{4 \times 2 = 8}$ multiply numerators together, then denominators

$= \frac{3}{8}$

Example: 2/3 x 7

Place the whole number 7 over 1 to make an improper fraction. Then multiply the numerators and denominators together.

$\frac{2}{3} \times \frac{7}{1}$ $\frac{2x7=14}{3x1=3}$ multiply numerators together, then denominators

$= \frac{14}{3}$ change improper fraction to mixed number

$= 4\frac{2}{3}$

Example: 1 4/5 x 7/8

9/5 x 7/8 Change the mixed number 1 4/5 to the improper fraction 9/5. Then multiply.

$\frac{9}{5} \times \frac{7}{8}$

$= \frac{63}{40}$

$= 1\frac{23}{40}$

Example: 2 1/3 x 2 2/5

= 7/3 x 12/5
= 84/15
= 5 9/15
= 5 3/5

You can ***reduce terms*** before you multiply. This will give you smaller numbers to work with. Divide any common factor into any two numbers on ***opposite*** sides of the fraction line. Look again at the third step in the last example.

3 and 12 are on opposite sides of the fraction line and have 3 as a common factor. Divide like this:

$\frac{7 \times \cancel{12}^{4}}{\cancel{3}_{1} \times 5}$ 12 ÷ 3 = 4, 3 ÷ 3 = 1

$= \frac{7 \times 4}{1 \times 5}$ you don't need to write this step

$= \frac{28}{5}$

= 5 3/5 change to a mixed number

You get the same answer but the numbers you multiplied were smaller. Reducing terms isn't essential but smaller numbers are easier to work with

TO MULTIPLY FRACTIONS

1. First change mixed numbers to improper fractions and place any whole numbers over 1.
2. Reduce any terms on opposite sides of the fraction line that have common factors.
3. Multiply the numerators together to get the new numerator.
4. Multiply the denominators together to get the new denominator.
5. Write the answer, reducing it to lowest terms.

14. Multiply: (Answers are on the last page.)

a) 4 x 3/5 b) 2/7 x 3 c) 1/4 x 3/8

d) 11/15 x 2/3 e) 2 3/4 x 1/2 f) 3 1/5 x 7

g) 6 x 2 4/5 h) 4/9 x 1 1/3 i) 7/10 x 3/4

15. An ironworker is paid $28 per hour and gets time and a half for overtime. What is the hourly rate for overtime?

16. A piece of plastic measures 2 1/2 feet long by 1 1/4 feet wide. What is its area in square meters? The formula for area is length x width ($A = l \times w$).

17. A used truck costs $9450. The purchaser can put 1/3 down and pay the rest in 12 equal monthly payments. What would be the amount of the monthly payments?

DIVISION OF FRACTIONS

Dividing fractions is fairly easy now that you can multiply them.

To divide fractions:

1. ***Invert the fraction you are dividing by and then multiply.***
   - When a fraction is **inverted**, it is turned upside down.

An inverted fraction is the **reciprocal fraction** of the original fraction.

Here are some reciprocal fractions:
3/4 inverted is 4/3
4/1 inverted is 1/4
1/3 inverted is 3/1

To divide a whole number, a mixed number or a fraction by a fraction:

1. Change any mixed numbers to improper fractions.
2. If any number is a whole number, write the whole number over 1 to make it a fraction.
3. Invert the fraction that comes *after the sign.*
4. *Change the division sign to a multiplication sign.* You can reduce terms now.
5. Multiply the two fractions
6. Write the answer in lowest terms.

Example: 4 ÷ 1/4

4 ÷ 1/4 invert the 2nd fraction 1/4 to 4/1,

= 4/1 x 4/1 change the ÷ sign to a x sign

= 16/1 write the whole number as a fraction over 1, then multiply

= 16

Example: 8 ÷ 2 1/3

8 ÷ 2 1/3 change 2 1/3 to 7/3

= 8 ÷ 7/3

= 8/1 x 3/7 invert 7/3 to 3/7, change the ÷ sign to a x sign

= 24/7 multiply

= 3 3/7 change to a mixed number

Example: 2/5 ÷ 2/3

= 2/5 x 3/2 invert 2/3 to 3/2, change the ÷ sign to a x sign

= 6/10 multiply

= 3/5 reduce to lowest terms

Example: 3/4 ÷ 6

= 3/4 ÷ 6/1 write the whole number 6 as a fraction over 1

= 3/4 x 1/6 invert 6/1 to 1/6, change the ÷ sign to a x sign

$= \frac{^{1}\cancel{3} \times 1}{4 \times \cancel{6}_{2}}$ reduce terms, then multiply

= 1/8

Example: 1 1/5 ÷ 2 1/½

1 1/5 ÷ 2 ½ change 1 1/5 to 6/5 and 2 1/2 to 5/2
= 6/5 ÷ 5/2
= 6/5 x 2/5
=12/25

Example: 4 3/12 ÷ 8 5/6

4 3/12 ÷ 8 5/6

= 51/12 ÷ 53/6

= 51/12 x 6/53

$= \frac{51 \times \cancel{6}^{1}}{{}_{2}\cancel{12} \times 53}$

= 51/106

TO DIVIDE FRACTIONS

1. Change any mixed numbers to improper fractions.
2. Invert the second fraction (place it over 1 before inverting if it is a whole number).
3. Change the division sign to a multiplication sign.
4. Reduce any terms with common factors on opposite sides of the fraction line after inverting.
5. Multiply the numerators together and multiply the denominators together.
6. Write the new numerator over the new denominator, reducing the answer to lowest terms.

18. Divide:

a) 5 ÷ 1/2 b) 1/3 ÷ 3/4 c) 9/10 ÷ 3

d) 2/5 ÷ 8/25 e) 5 2/5 ÷ 3/10 f) 4 2/3 ÷ 7

g) 15 ÷ 1 1/9 h) 2 1/6 ÷ 3 1/4 i) 4/5 ÷ 4/5

19. A piece of wire that is 8 3/4 feet long is to be cut into 3 equal pieces. How long will each of the cut pieces be?

20. A 5 liter container of caulking material is 1/2 full. If the caulking in the container is to be evenly divided into 4 smaller guns, approximately how much of it will be in each of the caulking guns? (To find out how much caulking is in the large container, either multiply by 1/2 or divide by 2.)

21. A precision metal fabricator must produce pieces of nickel metal 1/16 inch thick from a larger piece that is 2 inches thick. How many pieces can he produce from the larger one?

22. The inside diameter of pipe insulation is 6 1/4 cm. The outside diameter is 8 cm. How thick is the wall of insulation? (First subtract the inside diameter from the outside diameter and divide the answer by 2.)

FRACTIONS WITH A CALCULATOR

Most calculators do not work directly with fractions. To do operations with fractions use the calculator to change fractions into decimals before you can add, subtract, multiply, or divide.

Changing a fraction to a decimal with a calculator

To change any fraction to a decimal divide the numerator by its denominator. Key in the numerator followed by the division sign and then key in the denominator.

Example: Change the fraction 1/4 to a decimal.
Divide the numerator 1 by the denominator 4.
Key in 1, the division sign, then 4.
The answer will be .25

Addition: Change all fractions to decimals by dividing the numerator by the denominator. The answer will be a decimal number. Add the decimal numbers.

Example: You have to keep track of your hours. In one day, you worked 3 ½ hours at one job, then 1 3/4 hours at a second and 2 1/4 hours at a third job. How many hours did you work?

Change the mixed numbers to decimals. To do this leave the whole number as it is and divide the numerator of the fraction by the denominator. This changes the fraction part to a decimal. Attach the decimal to its whole number.

3 1/2 = 3.5	1 ÷ 2 = .5
1 3/4 = 1.75	3 ÷ 4 = .75
2 1/4 = 1.25	1 ÷ 4 = .25

Use your calculator to add. 3.5 + 1.75 + 1.25 = 6.5.

You worked 6.5 or 6 ½ hours that day.

Subtraction: Change any fraction to a decimal number by dividing the numerator by the denominator. Then subtract the decimal numbers with the calculator.

Multiplication: Change all fractions to decimals, then multiply.

Division: Change all fractions to decimals, then divide.

ANSWER PAGE

CHANGING MIXED NUMBERS TO IMPROPER FRACTIONS

1. a) 23/7 b) 11/6 c) 21/4 d) 23/8 e) 32/3 f) 31/5

CHANGING IMPROPER FRACTIONS TO MIXED NUMBERS

2. a) 1 1/3 b) 3 c) 3 2/7 d) 9 9/10 e) 6 f) 2 1/2

3. a) 24/40 b) 3/9 c) 60/18 d) 15/24 e) 3/12 f) 6/21

ADDING FRACTIONS OR MIXED NUMBERS WITH THE SAME DENOMINATOR

4. a) 6/7 b) 4/4 = 1 c) 1 ½ d) 6 3/5 e) 14 1/3 f) 13

g) 1 1/8 h) 7 11/12 i) 8 1/10

ADDING FRACTIONS OR MIXED NUMBERS WITH DIFFERENT DENOMINATORS

5. a) 7/8 b) 9 11/15 c) 1 3/35 d) 4 3/4 e) 9 1/6 f) 7 5/24

g) 17/27 h) 1 2/15 i) 32/33 j) 1 5/8 k) 1 27/40 l) 14 1/8

6. 4 7/12 liters
7. 38 hours
8. 19 1/5 in.

SUBTRACTING FRACTIONS

9. a) 2/9 b) 5/12 c) 3/8 d) 1/4 e) 4 1/5 f) 4 1/8

g) 4 ½ h) 5 1/12 i) 9 13/24 j) 2 2/3 k) 8 ½

l) 2 ¾ m) 6 4/5 n) 5 2/3 o) 1/3

10. 7 1/4 ft
11. 7 1/4 - 5/8 = 6 5/8 ft
12. He has worked 7 ½ hours already, so he has ½ hour left out of an 8 hour day.
13. 2 5/6 - 2/3 = 2 1/6 cm

MULTIPLYING FRACTIONS

14. a) 2 2/5 b) 6/7 c) 3/32 d) 22/45 e) 1 3/8 f) 22 2/5

g) 16 4/5 h) 16/27 i) 21/40

15. $42 per hour

16. 3 1/8 square feet

17. 1/3 x $9450 = $3150
$9450 - $3150 = $6300
$6300 ÷ 12 = $525 per month payment

DIVIDING FRACTIONS

18. a) 10 b) 4/9 c) 3/10 d) 1 ¼ e) 18 f) 2/3

g) 13 ½ h) 2/3 i) 1

19. 8 3/4 ÷ 3
= 35/4 x 1/3
= 35/12
= 2 11/12 feet

20. 5 ÷ 2 = 2 1/2
2 1/2 ÷ 4/1
= 5/2 x 1/4
= 5/8 liters

21. 1/2 ÷ 1/6
= 1/2 x 6/1
= 3 pieces of nickel.

22. 8 − 6 1/4 = 1 3/4 cm
1 3/4 ÷ 2
= 7/4 x 1/2 = 7/8 cm
The wall of insulation is 7/8 cm thick.