EVALUATING
ACADEMIC READINESS
FOR APPRENTICESHIP TRAINING
Revised for
ACCESS TO APPRENTICESHIP

MATHEMATICS SKILLS
PROPERTIES OF CIRCLES

AN ACADEMIC SKILLS MANUAL
for
The Metal Work Trades

This trade group includes the following trades:
Heat & Frost Insulator, Iron Worker,
Precision Metal Fabricator, Sheet Metal Worker, and
Welder & Fitter

Workplace Support Services Branch
Ontario Ministry of Training, Colleges and Universities

Revised 2011

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In preparing these Academic Skills Manuals we have used passages, diagrams and questions similar to those an apprentice might find in a text, guide or trade manual.

This trade related material is not intended to instruct you in your trade. It is used only to demonstrate how understanding an academic skill will help you find and use the information you need.

MATHEMATICS SKILLS:
PROPERTIES OF CIRCLES

An academic skill required for the study of the Metal Work Trades

INTRODUCTION

In your work, if you have to cut a circle out of a piece of metal, there are several things you will need to know about a circle: its diameter, its radius, its circumference and its area. This skills manual looks at the properties of a circles. It covers the following topics:

- ♦ terms used to describe a circle, including
  - degrees in a circle
  - circumference of a circle
  - diameter and radius
  - meaning of pi
- ♦ finding the circumference of a circle
- ♦ finding the area of a circle

TERMS USED TO DESCRIBE A CIRCLE

Degrees in a Circle

When the minute hand of a clock makes a complete circle around the face, moving from 12 around to 12 again, it travels 60 minutes or 1 hour. The clock face is divided into 60 divisions of one minute each. The face of a clock is usually in the shape of a circle.

In the same way that a clock is divided into different sections:

- ♦ A circle is divided into **degrees**.
- ♦ *A circle is divided into* ***360*** *equal parts that measure 1 degree each.*
- ♦ This means that a degree is 1/360th of the outside boundary or the *circumference* of a circle.
- ♦ To indicate the unit degree, we write the symbol·°
- ♦ Each degree is further divided into 60 equal parts called minutes.

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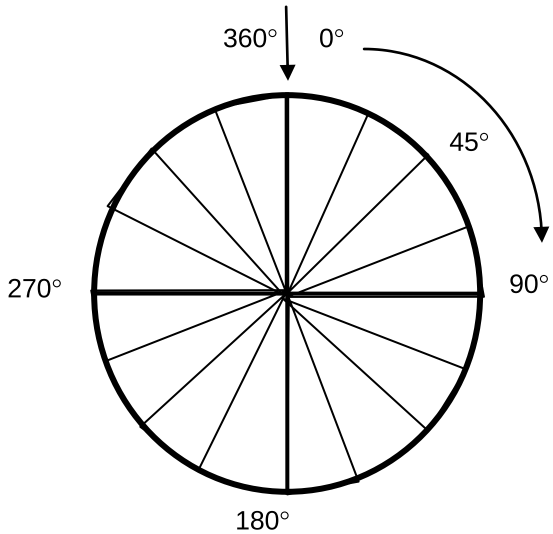


FIGURE 1: Degrees in a Circle
Some of the angles formed by dividing a circle into 360°

Circumference of a Circle

The ***circumference*** *(C) of the circle is the outside boundary of the circle.*

- Any partial section of the circumference is called an ***arc.***
- A circle has 360 arcs of equal size, each measuring one degree.
- T*he actual circumference of a circle in meters, inches, centimeters, etc, varies with the size of the circle.*
  - A larger circle will have a greater distance around its circumference than a smaller circle.

Examine Figure 1, and note the following:

- One-fourth of the circumference of a circle is equal to an arc of 90°.
  - If you draw lines from the end points of a 90° arc to connect in the centre of the circle, the lines form a 90° (right) angle.

- One-half the circumference of a circle is equal to an arc of 180°.
  - If you draw lines from the end points of a 180° arc to connect in the centre of the circle, the lines form an angle of 180°, which is also a straight line.

- An arc covering three-quarters of the circle is equal to 270°.
  - Lines drawn from the ends of this arc to the centre form an angle of 270°.

- If you travelled twice around the outside of a circle, you would travel 360° x 2 = 720° .
  - If you travelled three times around, you would travel 360° x 3 = 1080°.

Every circle is divided into 360 degrees around its circumference. However, *the actual distance in meters, inches, centimeters, etc, around the outside of a circle varies with the size of the circle*

A larger circle will have a greater distance around its circumference than a smaller circle.

- This distance could be measured by placing a flexible measuring tape around the circumference of the circle however, it is difficult to measure accurately this way.

- We can calculate circumference using the length of lines which can be drawn through the center of a circle because they can be measured accurately.

Chord and Tangent of a Circle

A line drawn through the circle from one point on the circumference to another point on the circumference is called a ***chord.*** A ***tangent*** to a circle is a line which touches the circle at only one point on the circumference.

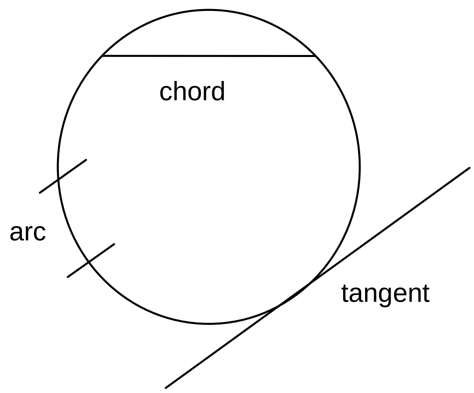


FIGURE 2: Arc, Chord And Tangent Of A Circle

Diameter and Radius

The ***diameter*** *(d) of a circle is a line that goes from one edge of the circle to the other edge and passes through the centre.*

- It is easy to measure the diameter of a circle accurately because it is a straight line, as you can see in Figure 3.

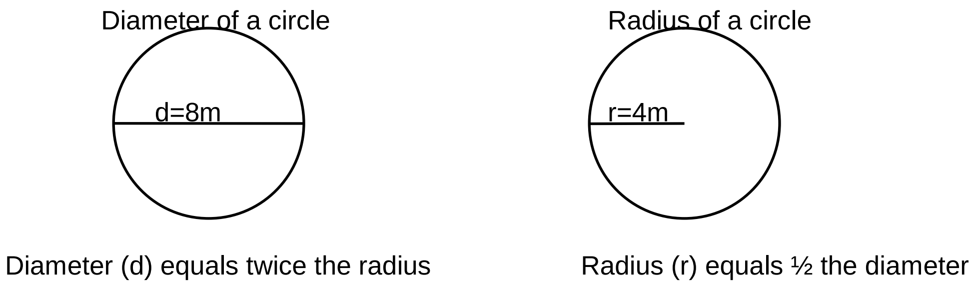


FIGURE 2: The Diameter And Radius Of A Circle

The ***radius*** *(r) of a circle is a line that goes from the centre of the circle to the circumference.*

- It is equal to one half the length of the diameter.

If you know either the diameter or the radius, you can find the other:

$r = {}^{1}/_{2}\, d$ <u>and</u> $d = 2r$

Example: If the diameter is 14 inches, what is the radius?

r = 1/2 d
= 1/2 x 14
= 7 in

If you place a circle inside a square so that the edges of the circle touch the square in four places, the length of the side of the square is equal to the diameter of the circle. So if you know the length of a side of the square, you also know the diameter of the circle.

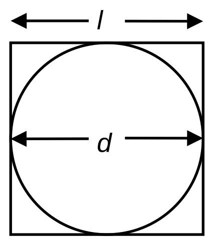


FIGURE 4: The length of a square is the same as the diameter of a circle placed inside it.

Conversely, if you know the radius of the circle, you can find the length of the sides of the square. Multiplying the radius by two gives the diameter. We know that the diameter is equal to the length of the sides. See Figure 4

Example: If the length of a side of a square is 16 cm, what is the diameter of a circle that is inside the square and touching all four sides, as in the diagram above? What is the radius?

The diameter is the same as the length of the side of the square, which is 16 cm.
The radius is one-half of the diameter,

½ x 16 = 8 cm

The Meaning of Pi (π)

We use what we know about the diameter and radius of circles to find the circumference of a circle and to find its area. In order to do these calculations we use a ratio called pi (which sounds like pie).

Pi is *written using the symbol* ***π***

- *π =22/7* when it is written as a fraction.
- *π =3.14159…* When it its written as a decimal, the decimal places go on forever.
  - it is usually rounded off to two decimal places.
  - *π =3.14* rounded off to two decimal places

FINDING THE CIRCUMFERENCE OF A CIRCLE

The circumference is the distance around the boundary of a circle. To find the circumference, you must know either the diameter or the radius.

Circumference (C) is equal to π times the diameter (d). The formula for finding circumference is:

$C = \pi d$

or, since the diameter is twice the radius (r),

$C = 2 \pi r$

Example: Find the circumference of a circle with a radius of 32 cm.

$C = 2 \pi r$
C = 2 x 3.14 x 32 cm
C = 200.96 cm

You might round the answer off to
C = 201 cm

Example: Find the circumference of a circle with a diameter of 8 m.

$C = \pi d$
= 3.14 x 8 m
= 25.12 m

Example: If a piece of wire is to be put around the outside of a can that has a diameter of 20 cm, how much wire will be used?

The amount of wire used will be equal to the circumference of the can.
$C = \pi d$
= 3.14 x 20
= 62.8 cm
You need 62.8 cm of wire to circle the can.

FINDING THE AREA OF A CIRCLE

The area of a circle is the amount of space enclosed within the boundary of the circle.

***Area** (A) is equal to pi (π) times the radius squared.* The formula for finding the area of a circle is:

$A = \pi r^2$

Squaring a number: To find the area of a circle, you need to know how to square a number. *To **square** a number, multiply it by itself.* For example, to square 11, multiply 11 x 11 to get 121.

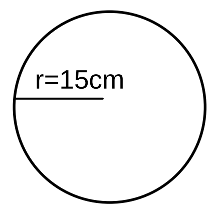
If you square a number that has units attached, the units are also squared. For example, the square of 5 meters (5m x 5m) is 25 meters squared. To show that meters, or any units, have been squared, we write a small two after the unit and slightly above it, like this: m^2. (The small raised 2 is called an exponent.) 5 m squared is equal to 25 m^2.

To square a number and its unit of measurement, we put brackets around *both of them* and write the exponent ^{2} after the brackets. So $(9 \text{ m})^2$ means you square 9 m by multiplying 9 m by 9 m.

$$(9 \text{ m})^2 = 9 \text{ m} \times 9 \text{ m} = 81 \text{ m}^2$$

Now let's use the formula $A = \pi r^2$ to find the area for the following circle.

Example: Find the area of a circle with a radius of 15 cm:



Known
r =15 cm
π = 3.14

We need to know:
- the area of the circle

We use the formula $A = \pi r^2$

$A = \pi r^2$	write the formula
$A = 3.14 \times (15 \text{ cm})^2$	substitute the values for known quantities
$A = 3.14 \times 225 \text{ cm}^2$	solve for A
$A = 706.5 \text{ cm}^2$	

Note that the units are squared in the answer. Whenever you find the area of a circle, a square, a rectangle etc, the units are squared.

Example: Find the area of a circle with a diameter of 10 m.

Known
radius = diameter ÷ 2
r = 10 m ÷ 2
= 5 m

Find the area:

$$A = \pi r^2$$
$$= 3.14 \times (5 \text{ m})^2$$
$$= 3.14 \times 25 \text{ m}^2$$
$$= 78.5 \text{ m}^2$$

Using $A = \pi r^2$ to find the radius: If you know the area of a circle, you can find its radius by manipulating or changing around the position of the variables (the letters and symbols) in the formula. Rewrite the formula so that r^2 is by itself on the left. The letters in equations can be moved around just like numbers as long as you follow the rules of basic algebra.

Example: Find the radius of a circle with an area of 78.5 cm^2. First change the formula so that r^2 is by itself on the left:

Known
A= 78.5 cm^2
π = 3.14

Find the radius

Area = πr^2	reverse the equation
$\pi r^2 = A$	divide by π to isolate r^2
r^2 = 78.5 cm^2 ÷ 3.14	
r^2 = 25 cm^2	
r = 5 cm	Find the square root of r^2 and 25 cm^2

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Answer the following questions about the circle. Answers are on the last page.

1. Find the radius of a circle with a diameter of 24 inches.

2. Find the diameter of a circle with a radius of 3 feet.

3. Square the following numbers:
   a) 7 b) 31 c) 4 km d) 15 in e) 2 cm

4. Find the circumference of a circle that has a diameter of 12 cm.

5. A sheet metal worker must solder the seam at the bottom of a round tank with a diameter of 1.5 m. What is the length of the soldered seam?

6. What is the cost of spraying insulating foam around a circular window with a radius of 24 inches if the cost of the foam works out to $.50 a foot? (Change the inches to feet first.)

7. A ball on a string that is 4 meters long is swung around in a circle two and a half times. How far did the ball travel? (Find the circumference and multiply it by 2 1/2)

8. Find the area of a circle with a radius of 28 cm.

9. Find the area of a circle that has a diameter of 14 m.

10. A welder must improve his skill by filling in a circular hole with an area of 28.26 cm^2 . Find the radius of the hole using 3.14 as π. (After dividing by π, find the square root of the answer. Use a calculator if you have one.)

11. A sheet metal worker cuts a circular hole with a radius of 20 cm is cut out of a piece of steel measuring 100 cm by 200 cm, how much metal is left?

12. If a circle with a diameter of 10 cm is cut in half, what is the area of each half.

13. You must cover a circular area that has a diameter of 1.2 m with r20 insulation. If a batt of insulation covers 2 m^2, how many batts will you need? (First calculate the area of the circle.)

14. What is the cost of making a metal cover for a barrel that is 2 yards in diameter if the cost of the metal is $5.00 a square yard? The waste metal can be reused, so only the amount used for the cover needs to be calculated.

ANSWER PAGE

1. $r = 12$ in.

2. $d = 6$ ft.

3. a) 49 b) 961 c) 16 km^2 d) 225 in^2 e) 4 cm^2

4. $C = \pi d = 37.68$ cm

5. $C = 4.71$ m — Soldered seam length = circumference of circle

6. $R = 24 \text{ in} \div 12 = 2$ ft
$C = 12.56$ ft
Cost = 12.56 x \$.50 = \$6.28

7. $r = 4$ m — Don't forget that the ball went around not once, but 2.5 times
$C = 2 \times 3.14 \times 4$ m — So, we have to multiply the circumference of 25.12 m by 2.5
$= 25.12$ m
25.12 m x 2.5 = 62.8 m.
The ball traveled 62.8 m.

8. $A = \pi r^2$
$= 3.14 \times (28 \text{ cm})^2$
$= 3.14 \times 784 \text{ cm}^2$
$= 2461.76 \text{ cm}^2$

9. Radius = diameter ÷ 2
= 7 m
Area $= 3.14 \times 7^2$
$= 153.86 \text{ m}^2$

10. r^2 = area ÷ π
$= 28.26 \text{ cm}^2 \div 3.14$
$= 9 \text{ m}^2$
$r = \sqrt{9m^2}$
$r = 3m$

11. Area of circular hole = 1256 cm^2
Original area of the sheet = 20,000 cm^2
Area of metal left = 20,000 cm^2 - 1256 cm = 18,744 cm^2

12. Area = 78.5 cm^2
½ of area = 39.25 cm^2

13. A = 3.14 x $(.6 \text{ m})^2$
    = 1.13 m^2
    One batt covers 2 m^2. This is more than the area to be covered (1.11 m^2), so 1 batt is enough.

14. A = 3.14 sq. yd.
    Cost of one square yard = $5.00
    Cost of 3.14 sq. yd. = 3.14 x $5.00 = $15.70