# EVALUATING
# ACADEMIC READINESS
# FOR APPRENTICESHIP TRAINING

**Revised for**

# ACCESS TO APPRENTICESHIP

## MATHEMATICS SKILLS
## CALCULATION OF PERIMETER, AREA & VOLUME OF GEOMETRIC FIGURES

## AN ACADEMIC SKILLS MANUAL
## for
## The Metal Work Trades

This trade group includes the following trades:
Heat & Frost Insulator, Iron Worker,
Precision Metal Fabricator, Sheet Metal Worker, and
Welder & Fitter

***Workplace Support Services Branch***
***Ontario Ministry of Training, Colleges and Universities***

***Revised 2011***

In preparing these Academic Skills Manuals we have used passages, diagrams and questions similar to those an apprentice might find in a text, guide or trade manual.

**This trade related material is not intended to instruct you in your trade. It is used only to demonstrate how understanding an academic skill will help you find and use the information you need.**

# MATHEMATICS SKILLS: CALCULATION OF PERIMETER, AREA & VOLUME OF GEOMETRIC FIGURES

***An academic skill required for the study of the Metal Work Trades***

### *INTRODUCTION*

Drills, shafts, saws, bearings -- most objects used on the job every day are three dimensional objects. To ensure that you have the right-sized object for the job, you might have to measure a saw blade or a drill bit or find the area of a two dimensional object like a piece of sheet metal. In this skills manual, we will look at finding the perimeter and area of both two and three dimensional geometric figures and calculating the volume of three dimensional figures.

The skills manual will cover:

- ♦ Two dimensional figures
- ♦ Finding the perimeter
- ♦ Finding the area
- ♦ Calculating the cost of covering an area
- ♦ Three dimensional figures
- ♦ Finding the surface area
- ♦ Finding the volume

Questions at the end of the skills manual let you test your knowledge of two and three dimensional figures.

### *TWO DIMENSIONAL GEOMETRIC FIGURES*

A simple, closed, two dimensional (flat) figure with three or more straight sides is called a **polygon**.

- Triangles, squares, rectangles, and parallelograms (figures with 2 pair of opposite sides parallel) are all examples of polygons.

A **circle** is also a flat, closed figure but it is a curve, consisting of points that are all the same distance from the center.

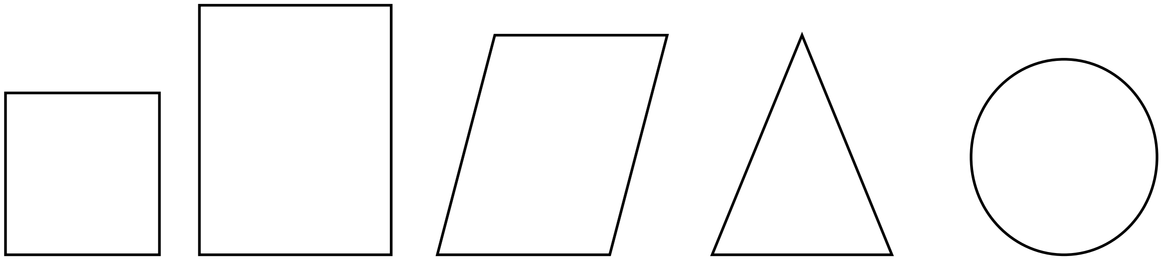



**FIGURE 1:** Some simple geometric shapes

These figures can be measured in different ways.

- Whenever we use measurements to make calculations with geometric figures, all measurements must be in the same linear units.
    - The units might be meters or centimeters, but they can't be a mix of meters and centimeters.

## *FINDING THE PERIMETER*

*The **perimeter** (P) of any polygon is the distance around its boundary*. Perimeter is found by adding together the lengths of the sides.

### *Perimeter of a Rectangle*

*A **rectangle** is a polygon with four 90°angles (right angles) and with each pair of parallel sides the same length* (see Figure 1).

- This means that we can find the perimeter of a rectangle by adding the lengths of the two long side to the lengths of the two shorter side.

**The perimeter of a rectangle equals twice the length (*l*) added to twice the width (*w*).** The formula is written in two forms:

$P = 2l + 2w$ where P is the perimeter, *l* is the length and *w* is the width of the rectangle.

or $P = 2(l + w)$

**Example**: Find the perimeter of a generator base that is 6 m long and 3 m wide.

$P = 2l + 2w$
$= 2(6 \text{ m}) + 2(3 \text{ m})$
$= 12 \text{ m} + 6 \text{ m}$
$= 18 \text{ m}$

The perimeter is 18 m.

**Example:** Find the amount of iron fencing required to close in a field that is 400 yd wide and 1500 ft long.

Known:
l = 1500 ft
w = 400 yd = 1200 ft 400 yd x3 = 1200 ft

Find perimeter (P)

P = 2(*l* + *w*)
= 2 (1500 ft + 1200 ft)
= 2(2700 ft)
= 5400 ft

The space will require 5400 ft of fencing.

***Perimeter of a Square***
*A square is a rectangle with all four sides the same length.*

**To find the perimeter of a square, multiply the length by 4.**

Perimeter of a square = 4*l*

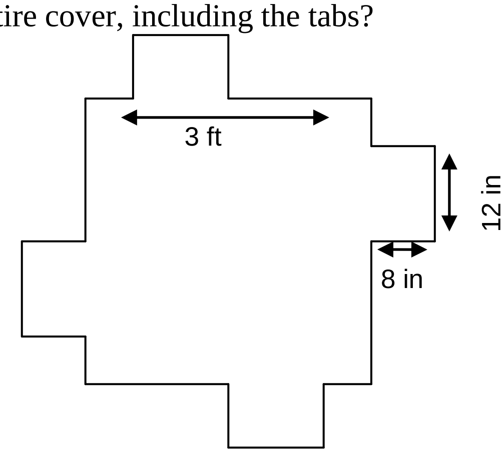
**Example:** How much railing is required for a lift that is 12 ft square?
(If a room is 12 ft square, it measures 12 ft by 12 ft.)

P = 4*l*
= 4(12)
= 48 ft

48 ft of trim is required.

Often a fabrication is more complicated than a simple square. A diagram can help you with these calculations. When you have to find the perimeter or area and there is no diagram, it is helpful to draw one.

**Example:** If a 3 foot square box cover has four tabs, each measuring 12 by 8 inches, how much rolled edging will be required to go around the entire cover, including the tabs?




Use the diagram.
You can see from the diagram that the 12 inch length of each tab is included in the 3 foot length of each side.
You can also see that each tab has two 8 inch edges (their widths).

All of this will be included in the calculation.

Known:
Sides are 3 ft including the length of the tabs
Each of the four tabs has two 8 in sides. 4 tabs x 2(8 in)

Find:
The perimeter of the entire cover
Add the sides of the tabs to the perimeter of the square

First find the perimeter of the square
P= 4l
P= 4×3 ft
P= 12 ft

Now find the total length of the widths of the four tabs and add them to the perimeter.

| | |
|---|---|
| 2 × 8 in = 16 in | Each tab has two widths of 8 inches. |
| 4×16 in = 64 in | There are 4 tabs. |
| 64 in ÷ 12 = 5 ft 4 in | Convert the inches to feet. There are 4 inches remainder. |
| 12 ft + 5 ft 4 in = 17 ft 4 in | Add the perimeter of the square and the widths of the tabs. |

17 ft 4 in of rolled edging is required.

### *Finding the Length of an Unknown Side When the Perimeter Is Known*

If you know the perimeter of a rectangle and the length of one side, you can find the other side.

1. Manipulate (or rearrange) the variables in the formula for perimeter so the letter for length or width is by itself on the left side.
2. Solve to find the unknown side.
3. *Remember, whatever you do to one side of the formula, you need to do to the numbers and letters on the other side.*

**Example:** The external perimeter of a building is 70 m. The width of the building is 10 m. What is the length?

Known:
P = 70 m
w = 10 m

Find l

| | |
|---|---|
| P = 2(*l* + *w*) | |
| 70 = 2(*l* +10) | divide both sides by 2 |
| 70/2 = 2/2(*l* + 10) | |
| 35 = *l* + 10 | subtract 10 from both sides |
| 35 – 10 = *l* + 10 – 10 | |
| 25 = *l* | reverse the equation |
| *l* = 25 m | write in the units, meters |

The length is 25 m.

## *FINDING THE AREA*

*The* ***area*** *of a polygon is the measure of the surface inside the boundary*. The units of area are squared units.

### *Area of a Rectangle*

The area of a rectangle is the amount of surface enclosed within its boundaries of ***length*** and ***width***.

**Example:** The area of a room is the amount of floor space it has.

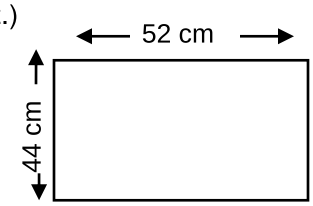
**Area is calculated by multiplying the length of the rectangle times its width.**
The formula for area is:

$A = lw$

***Remember:*** *When finding the area of a rectangle, the units used to measure the length and the width must be the same. If the length is in meters, the width must also be in meters. If the units are different, one must be converted to the other before you can multiply.*

**Example**: Find the area of a rectangular piece of sheet metal 52 cm long and 44 cm wide.
(The units are the same so we don't have to convert.)

Draw the rectangle




Known:
l = 52 cm
w= cm

Find:
Area
Use A = lw

$A = lw$
$= 52 \text{ cm} \times 44 \text{ cm}$
$= 2288 \text{ cm}^2$

**Note:** When two of the same units are multiplied together, such as the centimeters in our example, they become square units. Instead of writing square centimeters, you can use the short form of $cm^2$ or sq cm. (Sq is the short form for square.) Four square feet is written 4 sq ft or 4 $ft^2$.

**Example**: Find the area of a space with length 5 m and width 142 cm.

We must convert one of the units so both are the same.

*Known:*
l = 5 m
w = 142 cm or 1.42 m

*Find:*
area
Use A = lw

A = *lw*
A = 5 m x 1.42 m
A = 7.1 $m^2$

**Example:** A heat resistant panel is to be fabricated for an 8 ft by 15 ft wall. What is the area of the wall?

Known
l = 15 ft
w = 8 ft

Find
Area
Use A= lw

A = *lw*
= 8 x 15
= 120 sq ft

**Example:** Find area of a foundation of a structural steel building that is 15 m by 10 m.

A = *lw*
= 15 x 10
= 150 $m^2$

### *Area of a Square*

The four sides of a square are all the same length. To find the area of a square, square the length. (To square a number, multiply it by itself. Three squared is 3 x 3 = 9.)

**Example:** Find the area of a square with sides 15 ft long.

Known: l = 15 ft
w = 15 ft

Find A

A = $l\,w$ or $l^2$
A = 15 ft x 15 ft
A = 225 sq ft

***Finding the Length of an Unknown Side When the Area Is Known***

If you know the area of a rectangle and the width of one side, you can find the length of the other side.

1. Manipulate the variables in the formula for area so the letter for length ends up by itself on the left side.
2. Substitute the known measurements for area and width.
3. Solve to find the unknown side.

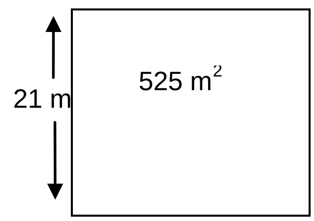
You can substitute the given measurements either before or after you manipulate the formula.

**Example**: If the area of a rectangle is 525 $m^2$ and the width is 21 m, what is the length?

Known: Find l

w = 21 m

A = 525 $m^2$




A = $lw$

$525m^2 = l \times 21m$ Fill in the given quantities.

$l \times 21\text{ m} = 525m^2$ Reverse the equation.

$l \times \frac{\overset{1}{\cancel{21m}}}{\underset{1}{\cancel{21m}}} = \frac{\overset{25}{\cancel{525m^2}}}{\underset{1}{\cancel{21m}}}$ Divide both sides by 21 m to isolate $l$ on the left.

When you divide a squared unit by a linear unit, such as square meters by meters, the meters on the bottom cancel one of the units on the top, leaving meters in the answer.

$l = 25\text{ m}$

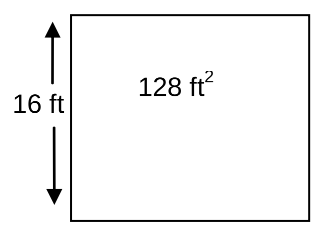
The length is 25 meters.

**Example:** Find the height of a wall that has an area of 128 sq ft and a length of 16 ft.

Known:

w = 16 ft

A = 128 $ft^2$




Find l

A = $lw$

128 sq ft = 16 ft($wl$) divide by 16 ft

$\frac{^{8}\cancel{128\text{ sq}}\text{ ft}}{\cancel{16\text{ ft}}_1} = \frac{\cancel{16\text{ ft}}^1 (w)}{_1\cancel{16\text{ ft}}}$ cancel the units and reduce where possible

8 ft = $w$ reverse the equation

$wl$ = 8 ft

The width is 8 ft.

## *CALCULATING THE COST OF COVERING AN AREA*

To find the cost of covering an area like a wall with a material such as stainless steel, first find the area and then multiply it by the cost per unit area. The cost of material is usually given as a rate, such as $16.95 per sq m.

If your area is calculated in square meters, you multiply the number of square meters by $16.95. The units sq m cancel out, leaving only the dollar unit. If the area is calculated in square centimeters or in square feet, you have to convert to square meters before multiplying.

The cost of material can be listed as a rate per given area, such as $142.08 per 4 ft × 8ft sheet of 24 gage stainless covers 24 sq ft.

**To find the cost of covering an area:**

1. First calculate the area to be covered.
2. Then multiply the area by the cost per unit area.

**Example:** Find the cost of installing a ½ in thick A 36 steel plate floor in a metal storage bin 16 ft long and 11 ft wide. The flooring costs $449.28 per 4 ft x 8 ft sheet.

Known:
L = 16 ft
W = 11 ft
Flooring cost 449.28 per 4ft x 8ft sheet
Each sheet covers 4ft x8ft = 32 $ft^2$

Find
1. Area of the floor
2. Cost to cover 1 square foot
3. Cost to cover the entire floor

Area of the floor.

A = *lw*
= 16 ft x 11 ft
= 176 sq ft

Cost to cover 1 square foot:

4ft × 8 ft = 32 sq ft — each sheet covers 32 sq ft

$449.28 ÷ 32 sq ft = $14.04/sq ft — divide the cost per sheet by the area it will cover

Cost to cover the entire floor
Multiply the ***number of square feet*** (the area) times the ***cost per square foot*** to find the total cost:

176 sq ft x $14.04/sq ft = $2471.04 — (Notice that the square feet cancel.)

The cost to cover the floor is $2471.04.

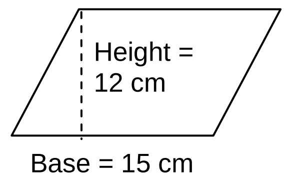
### *Finding the Area of a Parallelogram*

The area of a parallelogram is equal to the altitude or height times the base. The formula is:

A = *ab* or *bh*

**Example:** Find the area of a parallelogram with a height of 12 cm and a base of 15 cm.

Draw and label a parallelogram




Known: b = 15 cm
h = 12 cm

Find A
A= *bh*
= 15 cm x 12 cm
= 180 $cm^2$

## *THREE DIMENSIONAL FIGURES*

A closed, solid geometric figure has three dimensions. It has length, width and height or depth. Some solid figures are the rectangular solid, the cube, the cylinder and the sphere.

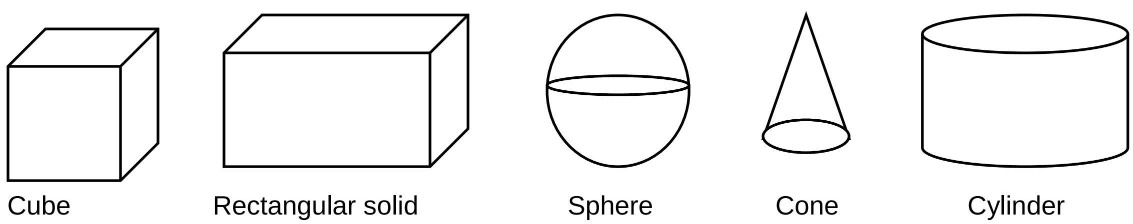



**FIGURE2:** Solid geometric figures

## FINDING THE SURFACE AREA OF SOLID FIGURES

### *Surface Area of Three Dimensional Figures*

Surface area of a three dimensional figure is the combined areas of all the outside surfaces or faces of the figure. When finding the surface area, all measurements must be in the same linear units. The answer will be in square units.

### *Finding the surface area of a rectangular solid*

To find the total area of the outside surface of a rectangular solid, we have to find the areas of each face of the figure.

1. First find the area of the front surface by multiplying the length times the height.
   - The back surface is the same area, so multiply that answer by 2.
2. Next find the area of one side by multiplying the width times the height.
   - Since the opposite side is the same, multiply the answer by 2.
3. Now find the base by multiplying the length times the width.
   - The top is the same as the base, so multiply that answer by 2 also.

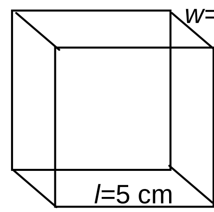
The formula is:

A = $2lh + 2wh + 2lw$
or A = $2(lh + wh + lw)$

or A = $2(lh + wh + lw)$

**Example:** Find the total area of the outside surface of a rectangular solid 5 cm long, 3 cm wide and 6 cm high.

Draw and label the solid




Known:
l = 5 cm
w = 3 cm
h = 6 cm

Find:
Outside surface area of the solid A = 2(lh + wh + lw)

A = 2 ($lh + wh +$ $lw$)
= 2(5cm x 6cm + 3cm x 6cm + 5cm x 3cm)
= 2(30 $cm^2$ + 18 $cm^2$ + 15 $cm^2$)
= 2(63 $cm^2$ )
= 126 $cm^2$

***Finding the surface area of a cube***
A cube is made of six identical squares. Each edge is the same length, each side has the same area.

**To find the area of a cube:**

1. Find the area of one side ($l^2$) and multiply it by 6.

The formula is:

A = 6 ($l^2$)

**Example:** Find the total surface area of a cube whose edges measure 10 in.

Known
Edges of cube = 10 in

Find;
Surface area of cube A = 6($l^2$)

A = 6 ($l^2$)
= 6($10^2$)
= 6(100)
= 600 sq in.

**The surface area of a cylinder**

A tin can, a snare drum and an oil drum are all cylinders. The surface area of a cylinder consists of the outside curved surface, which is actually a rectangle if it is straightened, and the circular areas at the top and bottom.

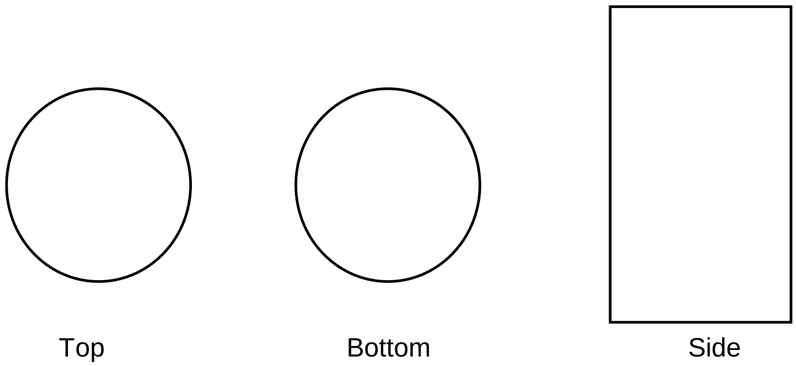



**FIGURE 3:** Finding the surface area of a cylinder

**To find the surface area of a cylinder:**

1. Find the area of each of the top and bottom circles.
2. Find the area of the rectangular side:
3. Add the areas together.

**1. To find the area of the top and bottom:** Use the formula $A = \pi r^2$. A cylinder has two circles (the top and the bottom), so we need to find the two areas, $2\pi r^2$.

***Remember:*** *$\pi = 3.14$*

**2. To find the area of the side of the cylinder (a rectangle):** Multiply the length times the width.

The formula is: $A = lw$.

This rectangle has a width equal to the height of the cylinder so substitute height (*h*) for the width.

The formula is now: $A = 2\ lh$.

The length of the rectangle is the same as the perimeters of the circles at the top and bottom. We find the perimeter of a circle using the formula $P = 2\pi r$. Substitute this formula for the length of the rectangle.

The formula becomes $A = 2\pi rh$.

**3. To find the area of the cylinder** add the areas of the top and bottom ($2\pi r^2$) to the area of the rectangle ($2\pi rh$).

$A = 2\pi r^2 + 2\pi rh$.

4. **The formula is rearranged** to become:

$$A = 2\pi r(r + h)$$

**Example:** Find the total area of a cylinder when its radius is 8 ft and its height is 20 ft.

Known:
r of cylinder = 8 ft
h of cylinder = 20 ft

Find the surface area of the cylinder

$$\begin{aligned} A &= 2\pi r(r + h) \\ &= (2 \times 3.14 \times 8)(8 + 20) \\ &= (50.24)(28) \\ &= 1406.72 \text{ sq ft} \end{aligned}$$

### *Finding the surface area of a sphere*

A sphere is a ball. The surface area of a sphere is equal to 4 times π times the radius squared. The formula is:

$$A = 4\pi r^2$$

**Example:** Find the surface area of a sphere with a radius of 5 cm.

Known
r = 5 cm

Find the surface area of the sphere

$$\begin{aligned} A &= 4\pi r^2 \\ &= 4 \times 3.14 \times 5^2 \\ &= 314 \text{ cm}^2 \end{aligned}$$

### *THE COST TO COVER THE OUTSIDE SURFACE OF A THREE DIMENSIONAL OBJECT*

To find the cost of covering the outside surface of an object:

1. Find the surface area, and
2. Multiply it by the cost per unit area.

**Example**: It takes 3 cans of spray paint to cover the outside of a metal box. Each can covers about 10 sq ft. What is the approximate surface area of the box? What will the job cost if each can costs $4.99.

One can covers 10 sq ft; three cans will cover
3 x 10 sq ft = 30 sq ft

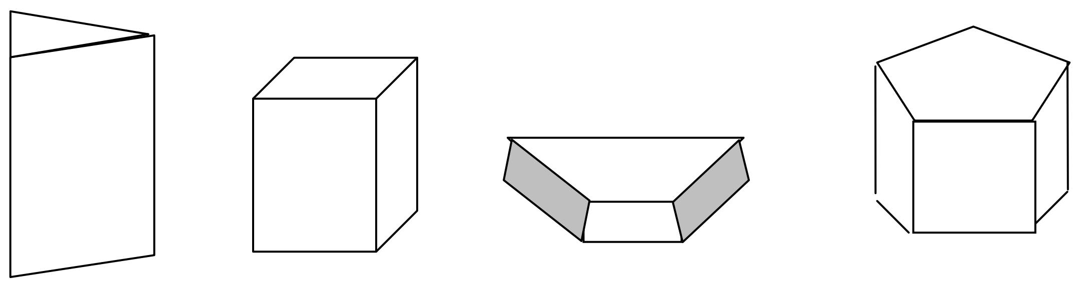
The surface area of the box is approximately 30 sq ft.

If one can of paint costs $4.99, three cans will cost
3 x $4.99 = $14.97

## *FINDING THE VOLUME OF THREE DIMENSIONAL GEOMETRIC FIGURES*

The ***volume*** *or capacity* of a solid figure is the amount of space contained within its boundaries.

**Regular solids**
A regular solid is a three dimensional object with straight sides. A solid is named for the shape of its base.



Triangular solid Rectangular solid Trapezoidal solid Pentagonal solid

**To calculate volume of a solid,** multiply the area of its base by its height.

Since each linear measurement has a unit, the units in the answer become cubic units. Area (with units meters x meters) x height (with units meters) equal cubic meters. The short form for cubic units such as cubic inches is $in^3$ or cu in.

***Volume of a Rectangular Solid***
The volume of a rectangular solid equals the length times the width times the height. The formula is:

$V = lwh.$

**Example:** Find the volume of a rectangular solid 9 m long, 4 m wide and 3 m high.

$V = lwh$
$= 9 \times 4 \times 3$
$= 108\ m^3$

**Volume of a cube**
The volume of a cube equals the length of one edge cubed. The formula is:

$V = l^3$

**Example:** Find the volume of a cube whose length measures 2 m.

$V = l^3$
$= 2^3$ (2 x 2 x 2)
$= 8\ m^3$

***Volume of a Cylinder***

The volume of a cylinder equals π times the square of the radius of the base times the height. The formula is:

$$V = \pi r^2 h$$

**Example:** Find the volume of a cylinder with a radius of 12 ft and a height of 72 in.

72 in ÷ 12 = 6 ft    Change the units of height to feet by dividing by 12.

Now use the formula.

$V = \pi r^2 h$

$= 3.14\ (12^2)\ (6)$

$= 2713$ cu ft

***Note:*** *A volume of 1000 cm*$^3$ *of liquid is equivalent to 1 liter of the liquid. If you know the volume, you can calculate the number of liters of liquid.*

***Volume of a Sphere***

The volume of a sphere equals 4/3 times *π* times the cube of the radius. The formula is:

$$V = \frac{4\pi r^3}{3}$$

**Example:** Find the volume of a sphere with a radius of 10 inches.

$$V = \frac{4\pi r^3}{3}$$

$$V = \frac{4(3.14)(10^3)}{3}$$

$= 4186.67$ cu in

Answer the following questions about geometric figures. **(Answers are on the last page of the skills manual.)**

1. Find the number of centimeters of split pipe needed to go around a metal sink that is 60 cm by 75 cm.

2. Find the area of a rectangle that is 18 meters long and 400 centimeters wide.

3. Find the area of a parallelogram with a base of 7.2 m and a height of 4.7 m.

4. Find the area of a square with sides that are 16 yards long.

5. Find the amount of steel fencing needed to fence in a garden that is 30 feet by 10 yards.

6. Find the cost of sheeting an external wall section that measures 18.4 m long by 10.1 m high. The sealant coating costs $8.42 per square meter.

7. Find the cost of covering a ceiling of a warehouse 30 ft by 50 ft with a layer of radiant insulating foil if a roll of the foil covers 500 sq ft and each roll costs $89.95.

8. If a sheet of metal measures 4 ft by 8 ft, find the amount required to make a rectangular metal box that has a height of 48 in, a width of 48 in and a length of 60 in. (First change the inches to feet.)

9. Find the volume of flotation foam needed to fill a space that measures 14 ft by 8 ft by .5 ft.

10. Find the volume of a metal casting mold whose sides are all equal and measure 8 cm.

11. What is the surface area of a cylinder with a radius of 100 cm and a height of 150 cm?

**ANSWER PAGE**

1. P = 2l + 2w
 = 2(60 cm) + (75 cm)
 = 120 cm + 150 cm = 270 cm

2. Change cm to m. 400 cm = 4 m
 18 x 4 = 72 $m^2$

3. A = 7.2 m x 4.7 m
 A = 33.84 $m^2$

4. 256 sq yd

5. Change feet to yards. 30 ft = 10 yd
 P = 2(10 yd.) + 2(10 yd.)
 = 40 yd

6. A = 18.4 m x 10.1 m
 = 185.84 $m^2$
 Cost = 185.84 $m^2$ x \$8.42/ $m^2$
 = \$1564.7728

 Round off to \$1564.77

7. A = 30 ft x 50 ft
 = 1500 sq ft
 One roll of foil covers 500 $m^2$.
 Number of rolls needed = 1500 $m^2$ ÷ 500 $m^2$ = 3 rolls
 One roll costs \$89.95. Three rolls cost 3 x \$89.95 = \$269.85

8. First change inches to feet. 48 in = 4 ft, 60 in = 5 ft
 Area of top and bottom = 2 x 4 x 5 = 40 sq ft
 Area of long sides = 2 x 4 x 5 = 40 sq ft
 Area of short sides = 2 x 4 x 4 = 32 sq ft
 Find the total area and divide that by the area of 1 sheet of metal.
 Total surface area of the box = 40 + 40 + 32 = 112 sq ft
 Area of a sheet of metal = 4 x 8 = 32 sq ft
 Number of sheets needed = 112 ÷ 32 = 3.5 sheets

9. V = lwh
 = 14 ft x 8 ft x .5 ft
 = 56 cu ft

10. V = $l^3$
 = $8^3$
 = 8 x 8x 8
 = 512 cu cm

11. A = 2πr (r + h)
 =( 2 x 3.14 x 100)(100 + 150)
 = (628)(250)
 = 157 000 cc (cubic centimeters)