

**EVALUATING
ACADEMIC READINESS
FOR APPRENTICESHIP TRAINING**
Revised for
ACCESS TO APPRENTICESHIP

**MATHEMATICS SKILLS
PERCENT, DECIMAL, FRACTION CONVERSION
AND COMPARISON**

AN ACADEMIC SKILLS MANUAL
for
The Construction Trades: Mechanical Systems

This trade group includes the following trades:
Electrician (Construction, Maintenance & Industrial),
Network Cabling Specialist,
Plumber, Refrigeration & Air Conditioning Mechanic,
Sprinkler & Fire Protection, and Steamfitter,

*Workplace Support Services Branch
Ontario Ministry of Training, Colleges and Universities*

Revised 2011

In preparing these Academic Skills Manuals we have used passages, diagrams and questions similar to those an apprentice might find in a text, guide or trade manual.

This trade related material is not intended to instruct you in your trade. It is used only to demonstrate how understanding an academic skill will help you find and use the information you need.

MATHEMATICS SKILLS

PERCENT, DECIMAL, FRACTION CONVERSION AND COMPARISON

*An academic skill required for the study of the
Construction Trades: Mechanical Systems*

INTRODUCTION

Fractions, decimals, and percent are closely related. Each is a different way of expressing a partial amount. For example, twenty-seven parts out of a total of one hundred parts can be written as the fraction $27/100$, as the decimal $.27$ or as the percent 27% . Each of these expressions indicates the same partial amount which is 27 parts out of a total of 100 parts. Because of the close relationship between fractions, decimals and percent, one form can be easily converted to either of the other two forms.

It can be useful to know how to change from one form to another.

Example: Your code book states that an opening must be 2.5 inches wide. You prefer to work with a fractional measurement, changing the decimal measurement of 2.5 inches to $2\frac{1}{2}$ inches.

Example: Or you might want to know that a 10% increase in your forty hour week means that you will have to work four hours of overtime this week.

In this skill manual, we will review how to write numbers as fractions, decimals and percents. Then we will examine how to convert from one form to another. The following topics are covered:

- ◆ Writing partial amounts as fractions, decimals or percent
- ◆ Steps in converting one form to either of the other forms
- ◆ Chart of equivalent forms
- ◆ Mixed forms
- ◆ Comparing sizes of two percents, two fractions, or two decimals
- ◆ Comparing fractions, decimals and percents

EXPRESSING PARTIAL AMOUNTS

Writing Fractions

A **fraction** is one way to express a partial amount. A fraction tells you how many equal parts you have and how many equal parts make up one whole. Two parts out of a total of three is written as the fraction $2/3$ or $\frac{2}{3}$. We say it as two thirds.

Parts of a fraction:

- The top part of a fraction is called the **numerator**
- The bottom part is called the **denominator**.
- The dividing line is called the fraction line.
- A **mixed number** such as $7\frac{1}{2}$ consists of a whole number and a fractional amount.

Fractions are usually reduced to **lowest terms**. To do this we do the following:

1. Examine both the numerator and the denominator to see if any number will divide *evenly* into both of them.
 - Any such number is called a **common factor**.
2. If there is a common factor, it is divided into both the numerator and the denominator.
3. The final division answer is written as the fraction reduced to lowest terms.

Example: Reduce $\frac{12}{72}$ to lowest terms.

$$\frac{12 \div 12}{72 \div 12} \quad 12 \text{ and } 72 \text{ have } 12 \text{ as a common factor.}$$

$\frac{12}{72}$ reduced to lowest terms is $\frac{1}{6}$.

Writing Decimals

A **decimal number** such as .86 is another way of indicating a partial amount. A decimal number is written with a decimal point followed by the partial amount.

- A decimal amount is actually a fraction which has a denominator that is a power of ten, such as 10, 100, 1000 or 10,000.
- Only the numerator is written in a decimal number; the fraction line and the denominator are replaced by the decimal point.

Example: The decimal number .86 is a fraction that has the numerator 86 and the denominator 100.

- The numerator 86 is written to the right of the decimal point.
- The decimal point takes the place of both the fraction line and the denominator 100, neither of which is shown.

A decimal number can be a partial amount such as .594. Or it can have a whole number component such as 45.594, which indicates an amount consisting of the whole number 45 and a part of a whole number .594.

Writing Percent

A **percent** is still another way of expressing a partial amount. A percent is a fraction in which the denominator is always 100.

- Only the numerator of the fraction is written.
- The % sign takes the place of the fraction line and the denominator.

Example: Fifteen out of a hundred written as a percent is 15%. So 15% is also the fraction $\frac{15}{100}$, with the percent sign taking the place of the fraction line and the denominator 100.

STEPS IN CONVERTING

Because of the close connection between fractions, decimals and percents, converting from one form to another involves only a few basic steps. Six different conversions can be carried out among the three forms. The six possible conversions and the steps involved are each described in turn.

To change a fraction to a decimal:

1. Divide the numerator by the denominator.
2. Write the answer as a decimal number.

To change a mixed number to a decimal:

1. Write the whole number.
2. Place a decimal point to its right.
3. Change the fraction to a decimal as above and write it to the right of the decimal point.

Example: Change $\frac{2}{5}$ to a decimal.

$$\begin{array}{r} .4 \\ 5 \overline{) 2.0} \\ \underline{2.0} \end{array} \quad \text{divide the numerator 2 by the denominator 5}$$

$\frac{2}{5}$ written as a decimal number is .4

Example: Change $6\frac{3}{4}$ to a decimal number.

1. Write the whole number, 6.
2. To change the fraction $\frac{3}{4}$ to a decimal, divide the numerator 3 by the denominator 4.

$$3 \div 4 = .75 \quad \text{Write the .75 after the 6.}$$

$6\frac{3}{4}$ written as a decimal number is 6.75

Rounding off: Sometimes when you divide a denominator into a numerator, the answer doesn't come out evenly. Usually you round off a division answer to two places past the decimal point (called rounding off to the nearest hundredths).

To round off to two places, continue division to three places. If that third digit to the right of the decimal point is five or more, drop it. Round up the second digit to the right of the decimal point to make it one digit higher. If the third digit is less than 5, drop it and leave the second digit as it is.

Example: Round off .143 to two places.

Look at the third digit past the decimal point, which is 3. Since it is less than five, drop it and leave the second digit as it is.

.143... rounded to two places is .14

Example: Round off .6666 to two places.

The third digit past the decimal place is 6, which is greater than 5. The second digit, also a 6, becomes one digit higher, 7. Then drop the third digit.

.6666... rounded to two places is .67

Example: Round off .6995 to two places.

The third digit past the decimal point is greater than 5, so the second digit 9, becomes one larger, which makes it 10. The zero is written in the place where the 9 was. The 1 is added to the 6 in the next place to the left, changing it into a 7.

.6995 rounded off to two places becomes .70

Example: Change $5 \frac{1}{12}$ to a decimal. Round off to two places.

$$1 \div 12 = .0833\dots$$

Change the fraction part to a decimal.

Round off the decimal to two places.

The third digit to the right is less than 5, so leave the second digit as it is and drop all the digits past it.

.0833...becomes .08

Write the whole number with the decimal part.

$5 \frac{1}{12}$ written as a decimal rounded off to two places is 5.08.

To change a decimal to a fraction:

1. The digits to the right of or after the decimal point are written as the numerator.
2. Count the number of digits to the right of or after the decimal point.
3. The denominator is written as a 1 with as many zeros after it as there are digits after the decimal point.
4. If there are any digits to the left of the decimal point, write them as whole numbers with the fraction.
5. Reduce to lowest terms by dividing the numerator and denominator by any common factors.

Example: Change .25 to a fraction.

The digits that appear after the decimal point become the numerator, 25.

There are two digits after the decimal point, so the denominator becomes a 1 followed by two zeros, 100.

The fraction is written as $\frac{25}{100}$.

$\frac{25}{100}$ is reduced to $\frac{1}{4}$.

Example: Change 16.5 to a fraction.

The 5 becomes the numerator.

The denominator is 10.

5/10 is reduced to 1/2.

The whole number 16 is placed in front of the fraction to make a mixed number.

$$16.5 = 16 \frac{1}{2}.$$

To change a decimal to a percent:

1. Multiply the decimal number by 100. (To multiply by 100 quickly, move the decimal point two places to the right.)
2. Write the percent sign with the answer.

Example: Change .75 to a percent.

$$.75 \times 100 = 75 \quad \text{Multiply by 100. Write \% sign.}$$

$$.75 = 75\%$$

Example: Change 1.05 to a percent.

$$1.05 \times 100 = 105\%$$

To change a percent to a decimal:

1. Remove the percent sign.
2. Divide the percent by 100. (To divide by 100, move the decimal point two places to the left.)

Example: Change 33% to a decimal.

$$33 \div 100 = .33 \quad \text{Remove \% sign, divide by 100.}$$

Example: Change 2% to a decimal.

$$2 \div 100 = .02$$

To change a fraction to a percent:

1. Change the fraction to a decimal number by dividing the numerator by the denominator.
2. Change the decimal number to a percent by multiplying by 100.
3. Write the percent sign.
4. If the fraction is a mixed number, change the fraction part to a decimal. Write the whole number with the decimal part after it. Multiply by 100 and write %.

Example: Change 3/8 to a percent.

$$3 \div 8 = .375$$

$$375 \times 100 = 37.5$$

$$3/8 = 37.5\%$$

Divide the numerator by the denominator.

Multiply the decimal number by 10.

Write the % sign.

We can express a partial amount within a percent as a fraction instead of as a decimal. The percent above can be written as $37\frac{1}{2}\%$. One-third expressed as a percent is written 33.33% or $33\frac{1}{3}\%$. This form is often used in advertisements to indicate that the price is one-third off. However, when doing calculations with percent, it is usually easier if any partial amount is expressed as a decimal instead of a fraction.

Example: Change $2\frac{4}{5}$ to a percent.

$4 \div 5 = .8$	Change the fraction to a decimal.
2.8	Write the whole number with the decimal part.
$2.8 \times 100 = 280\%$	Multiply by 100 and write the % sign.
$2\frac{4}{5} = 280\%$	

Sometimes, the division answer does not come out evenly. In this case, the answer is usually rounded off to four places. When it is multiplied by 100, there will be two decimal places in the final answer.

Example: Change $\frac{5}{12}$ to a percent.

$$5 \div 12 = .41666\dots$$
$$.4167 = 41.67\%$$

Since .6666 is the same as the fraction $\frac{2}{3}$, you could write the percent as $41\frac{2}{3}\%$.

Example: Change $7\frac{4}{9}$ to a percent

$$4 \div 9 = .4444\dots$$
$$= .4444$$
$$7.4444 = 744.44\%$$

Example: Change $\frac{1}{3}$ to a percent.

$1 \div 3 = .33333\dots$	Divide and round off to four places
$1 \div 3 = .3333$	
$.33333\dots = 33.33$	Multiply by 100.
$.3333 = 33.33\%$	Add the % sign.
$\frac{1}{3} = 33.33\%$	

Example: Change $\frac{2}{3}$ to a percent.

$2 \div 3 = .6667\dots$	Divide and round off to four places
$.6667 \times 100 = 66.67$	Multiply by 100.
$.6667 = 66.67\%$	Add the % sign.
$\frac{2}{3} = 66.67\%$	

$\frac{1}{3}$ and $\frac{2}{3}$ are the two fractions most commonly written with the partial amount as a fraction instead of a rounded off decimal. We noted earlier that $\frac{1}{3}$ expressed as a percent is written as 33.33% or $33\frac{1}{3}\%$. $\frac{2}{3}$ expressed as a percent is written as 66.67% or as $66\frac{2}{3}\%$.

To change a percent to a fraction:

1. remove the percent sign
2. write the given percent as the numerator over 100 as the denominator
3. reduce to lowest terms

Example: Change 15% to a fraction.

15% = 15/100 Remove the % sign. Write 15 as the numerator over 100 to make the fraction.
15/100 = 3/20 Reduce the fraction.
15% = 3/20

Example: Change 330% to a fraction.

330% = 330/100
330/100 = 3 30/100
330% = 3 3/10

CHART OF EQUIVALENT FORMS

The following chart shows equivalent forms of the most commonly used percents, decimals and fractions. It can be useful to memorize the most often used equivalents such as 50%, .5 and 1/2.

<i>Percent</i>	<i>Decimal</i>	<i>Fraction</i>
5%	.05	1/20
6 1/4 %	.0625	1/16
8 1/3%	.083	1/12
10%	.01	1/10
12.5 %	.125	1/8
20%	.2	1/5
25%	.25	1/4
33 1/3%	.33	1/3
50%	.5	1/2
66 2/3%	.667	2/3
75%	.75	3/4
100%	1.0	1/1 or 1

MIXED FORMS

Sometimes amounts will be expressed in more than one form.

Example: Your regular hourly wage is \$12.50 and overtime is paid at a rate of time and a half ($1\frac{1}{2}$). To calculate your overtime rate, multiply the decimal number \$12.50 by the fraction $1\frac{1}{2}$

This calculation is much easier if you first change the fraction to a decimal and then multiply.

$$\begin{aligned} 1/2 &= 1 \div 2 = .5 \\ 1\ 1/2 &= 1.5 \\ \$12.50 \times 1.5 &= \$18.75 \end{aligned}$$

Example: You have to pay a minimum of 20% of your credit card debt every month. Your statement shows you owe \$3675.15. How much do you need to pay this month?

Multiply $\$3675.15 \times 20\%$.

$$20\% = .2 \quad \text{Change the percent to a decimal.}$$

$$\$3675.15 \times .2 \quad \text{Multiply.}$$

$$= \$735.03$$

You have to pay \$735.03 this month.

COMPARING RELATIVE SIZES OF PERCENTS, FRACTIONS AND DECIMALS

Some questions ask you to compare the values, or to find equivalents of percents, fractions and decimals. As a first step you need an understanding of the relative sizes of different percents, fractions and decimals.

1. Relative Sizes of Percents

It is easiest to identify the relative sizes of percents. You know that as you start counting at 1, the numbers get larger.

- So you know 35% is larger than 12% because 35 is larger than 12.
- In the same way, you know that 67% is smaller than 75% because 67 is smaller than 75.

2. Relative Sizes of Fractions

To compare the relative sizes of fractions such as $\frac{2}{3}$ and $\frac{4}{5}$, you have to change them into fractions with a common (the same) denominator. Then look at the numerators to see which is larger or smaller.

To find a common denominator for two fractions, the easiest method (there are several methods) is to multiply the denominators together. The answer becomes the new common denominator.

Example: Compare the fractions $\frac{2}{3}$ and $\frac{4}{5}$.

First, find the common denominator of $\frac{2}{3}$ and $\frac{4}{5}$.

$\frac{2}{3}$ and $\frac{4}{5}$ The denominators are 3 and 5. Multiply $3 \times 5 = 15$
15 is the common denominator.

$\frac{\quad}{15}$ and $\frac{\quad}{15}$

Second, change the fractions into equivalent fractions with a common denominator.

Start with $\frac{2}{3}$.

$15 \div 3 = 5$ Divide the common denominator 15 by the denominator 3 in $\frac{2}{3}$.

$\frac{2 \times 5 = 10}{3 \times 5 = 15}$ Multiply both the numerator and the denominator
by the division answer 5.

$\frac{2}{3} = \frac{10}{15}$ The multiplication answer becomes the equivalent fraction.

Now, change $\frac{4}{5}$ into an equivalent fraction with the denominator 15.

$15 \div 5 = 3$ Divide the common denominator 15 by the denominator 5.

$\frac{4 \times 3 = 12}{5 \times 3 = 15}$ Multiply the numerator and the denominator by the division answer.

$\frac{4}{5} = \frac{12}{15}$ The multiplication answer becomes the equivalent fraction.

Third, compare the *numerators* of the fractions once they have common denominators.

$\frac{2}{3} = \frac{10}{15}$ and $\frac{4}{5} = \frac{12}{15}$

Fourth, note that 12 is larger than 10, so $\frac{12}{15}$ ($\frac{4}{5}$) is larger than $\frac{10}{15}$ ($\frac{2}{3}$).

3. Relative Sizes of Decimals

To compare decimals:

1. Look at the digits in the tenths place (the first place after the decimal point) in each decimal number.
2. The number with the largest digit in the tenths place is the largest.

Example: Compare the numbers .5496 and .687 to find the largest.

Look at the digits in the tenths place, to the right of the decimal point.

.5496 5 is the digit in the tenths place
.687 6 is the digit in the tenths place

6 is larger than 5; therefore .687 is larger than .5496.

When you compare decimal numbers

1. If the digits in the tenths place are the same, such as in .381 and .349, look at the second, or hundredths, place.
 - The digit 8 in the second decimal place is larger than the digit 4, so .381 is larger than .349.
2. If the digits in the hundredths place are the same, look at the third decimal place, etc.
3. If all the digits are the same, but one number has more decimal places (such as in .213 and .2131), the number with the greater number of decimal places is the larger number.

COMPARING FRACTIONS, DECIMALS AND PERCENTS

To compare fractions, decimals and percents to each other, change them all to one form. Then compare their relative sizes.

Example: Which fraction is equivalent to .25?

- a) $1/5$
- b) $1/3$
- c) $1/4$

$1/5 = .2$ Change the fractions to decimals and examine them.
 $1/3 = .33$
 $1/4 = .25$

c) is the correct answer because $1/4$ is equivalent to .25.

Example: Which of the following is equivalent to 50%?

- a) .75
- b) $1/2$
- c) .55
- d) $2/3$

$.75 = 75\%$ Change the numbers to percent.
 $1/2 = 50\%$
 $.55 = 55\%$
 $2/3 = 66.67\%$

b) is the correct answer because $1/2$ is equivalent to 50%.

Example: Find the number with the largest value.

- a) 75%
- b) $\frac{5}{8}$
- c) .88
- d) $\frac{4}{5}$

Change them all to decimals.

$$75\% = .75$$

$$\frac{5}{8} = .625$$

$$.88 \text{ doesn't have to be changed}$$

$$\frac{4}{5} = .8$$

The answer is c). When you compare the four decimal numbers, .88 is the largest.

Answer these questions on converting between fractions, decimals and percents. **Answers are on the last page of the skills manual.**

Round off your answers to two decimal places if they do not come out evenly.

1. Change the following fractions to decimal numbers.

- a) $\frac{1}{2}$ b) $\frac{3}{4}$ c) $\frac{1}{4}$ d) $7\frac{3}{5}$ e) $\frac{3}{8}$ f) $4\frac{7}{10}$ g) $\frac{1}{3}$

2. Change the following decimal numbers to fractions.

- a) .5 b) .3 c) .75 d) .625 e) .125 f) 5.25 g) 43.1

3. Change the following decimal numbers to percent.

- a) .5 b) .3333 c) .25 d) .99 e) 1.75 f) 4.40 g) 1.1

4. Change the following percents to decimal numbers.

- a) 25% b) 100% c) 50% d) 10% e) 89% f) 175% g) 550%

5. Change the following fractions to percent.

- a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) $\frac{7}{12}$ e) $\frac{3}{8}$ f) $4\frac{3}{4}$ g) $\frac{2}{3}$

6. Change the following percents to fractions.

- a) 75% b) 50% c) 55% d) 12% e) 110% f) 525% g) 99%

7. Which of the following is equivalent to .625 ?

- a) $\frac{3}{4}$
- b) $\frac{4}{5}$
- c) $\frac{5}{8}$
- d) $\frac{7}{12}$

8. Which of the following is equivalent to 34.55% ?

- a) 3.455
- b) $34 \frac{1}{5}$
- c) .03455
- d) 455

9. Which of the following has the largest value?

- a) 48%
- b) .489
- c) $\frac{3}{7}$
- d) .45

ANSWER PAGE

1. a) .5 b) .75 c) .25 d) 7.6 e) .375
f) 4.7 g) .33
2. a) $\frac{1}{2}$ b) $\frac{3}{10}$ c) $\frac{3}{4}$ d) $\frac{5}{8}$ e) $\frac{1}{8}$
f) $5\frac{1}{4}$ g) $43\frac{1}{10}$
3. a) 50% b) 33.33% c) 25% d) 99% e) 175%
f) 440% g) 110%
4. a) .25 b) 1 c) .5 d) .1 e) .89
f) 1.75 g) 5.5
5. a) 25% b) 50% c) 33.33% d) 58.33% e) 37.5%
f) 475% g) 66.67%
6. a) $\frac{3}{4}$ b) $\frac{1}{2}$ c) $\frac{11}{20}$ d) $\frac{3}{25}$ e) $1\frac{1}{10}$
f) $5\frac{1}{4}$ g) $\frac{99}{100}$
7. c) $\frac{5}{8}$
8. d) .3455
9. b) .489