EVALUATING
ACADEMIC READINESS
FOR APPRENTICESHIP TRAINING

Revised for

ACCESS TO APPRENTICESHIP

MATHEMATICS SKILLS
RATIO AND PROPORTION

AN ACADEMIC SKILLS MANUAL
for
The Construction Trades: Mechanical Systems

This trade group includes the following trades:
Electrician (Construction, Maintenance & Industrial),
Network Cabling Specialist,
Plumber, Refrigeration & Air Conditioning Mechanic,
Sprinkler &Fire Protection, and Steamfitter,

Workplace Support Services Branch
Ontario Ministry of Training, Colleges and Universities

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In preparing these Academic Skills Manuals we have used passages, diagrams and questions similar to those an apprentice might find in a text, guide or trade manual.

This trade related material is not intended to instruct you in your trade. It is used only to demonstrate how understanding an academic skill will help you find and use the information you need.

MATHEMATICS SKILLS
RATIO AND PROPORTION

An academic skill required for the study of the Construction Trades: Mechanical Systems

INTRODUCTION

Comparing Numbers
Numbers are compared in a variety of ways. One way to compare numbers is to note their difference.

> **Example:** If one car sells for \$36 000 and another sells for \$18 000, the first car costs \$18 000 more than the second. In this comparison, we subtracted the cost of the less expensive car from the more expensive one to find the difference.

We can also compare the cost of the two cars by division.

> **Example:** If we divide the cost of the first car by the cost of the second (\$36 000 ÷ \$18 000 = 2), we find that the first car costs twice as much as the second.

We can compare by using a ***ratio***. A ratio also compares numbers in a form that indicates division. Usually the numbers in a ratio are reduced to lowest terms but not actually divided.

Ratios give useful information about the relationship between numbers. If a certain length of PVC pipe weighs 10 kilograms and the same length of steel pipe weighs 50 kilograms, you can make a ratio and say the PVC is 5 times lighter than steel. Without knowing the actual weights of the same lengths of PVC and steel pipe, you know that the PVC is much lighter. We also use ratio to solve problems by proportion and to read blueprints that are drawn to scale.

This skill manual looks at the following topics concerning ***ratio, proportion, and scale***:

- Ratio, including
  - finding ratios from given information
  - rates
- Proportions, including
  - solving a proportion when 3 out of 4 terms are known
  - direct and inverse proportions
- Scale

RATIO

Comparing two numbers by writing a ratio: If one panel is 3 feet long and another is 4 feet long, you can compare the two lengths by writing them as a ratio.

There are several ways to indicate this ratio:

- **By comparing one amount to another**, as when we say 6 out of 10.
- **By putting a colon between the numbers.** The ratio is written 6 : 10. We read this as "*the ratio of six to ten*".
- **By writing the ratio as a fraction.** The first number being compared becomes the numerator, which is placed over the second number, the denominator. The fraction is usually written in lowest terms. So 6 out 10 becomes 6/10 and can be reduced to 3/5.

When you write a ratio, you don't actually do the division unless you want one of the terms of the ratio to be 1.

Lowest terms: The ratio 3:4 is already in lowest terms. The ratio 8 to 32 is not in lowest terms. When this ratio is reduced to lowest terms, it is written as 1 to 4. A ratio, like a fraction, is usually, but not always, written in lowest terms.

To reduce a fraction or a ratio to lowest terms:

1. Look for a number (a common factor) that will divide evenly into the numerator and denominator of the fraction or the terms of the ratio.
2. Divide the common factor into the numerator and the denominator or into each term.
3. Continue dividing until there are no more common factors.
4. The last division answers form the fraction or ratio in lowest terms.

Example: The bend radius of a conduit can be expressed as a ratio. If the conduit has an internal diameter of 5.1 centimeters or less, the bend radius must be 6 times the internal conduit diameter.

If the conduit diameter is 3 centimeters, the bend must be 3 times 6 = 18 centimeters.

This gives a ratio of 3 to 18.
The common factor is 3.
Reduced to lowest terms, the ratio is 1 to 6.

If the conduit has a diameter of more than 5.1 cm, the bend radius must be 10 times the internal conduit diameter.

If the diameter is 6 centimeters, the bend must be 6 times 10 cm = 60 cm.
The common factor is 6.
Reduced to lowest terms, the ratio is 1:10.

Notice that there are no units in these ratios. Because we are comparing centimeters to centimeters, the units cancel out. When the numbers being compared have the same unit of measurement, there are no units in the ratio.

Ratios with 1: The ratio 2:1 has the number 1 as one of its terms. The ratio 3:4 does not. Sometimes a ratio like 3:4 is more useful if one of the terms is 1. You could divide both terms by 4 and then express the ratio as .75 to 1, or you could divide both terms by 3 and express the ratio as 1 to 1.33.

Equivalent ratios: Reducing a fraction to lowest terms does not change the value of the fraction, nor will it change the value of a ratio. The fractions 2/8 and 4/16 can each be reduced to 1/4. 1/4, 2/8, and 4/16 are ***equivalent fractions***. They each represent the same amount.

In the same way, ratios representing the same amount are called ***equivalent ratios***. The ratio 3 to 4 and the ratio .75 to 1 represent the same comparison of lengths and are equivalent ratios.

Finding Ratios from Given Information

Before using ratios to solve problems or to read blueprints, we will look at setting up ratios from given information.

Questions that ask you to set up ratios are generally worded in one of two ways.

1. You might need to compare part of an amount to the total amount; or
2. You might be asked to compare two parts to each other.

Situation one: You are asked to compare part of the amount to the total amount. If the total amount isn't given, you first have to find it.

Example: A class of apprentices consisted of 6 women and 24 men. What is the ratio of women to the whole class and the ratio of men to the whole class?

First you have to find the total number of students.
Adding 6 + 24 gives a total of 30 apprentices in the class.

Now find the ratios:

a) Ratio of women to the whole class is 6 out of 30, reduced to 1 out of 5, 1/5 or 1:5.
b) Ratio of men to the whole class is 24 out of 30, reduced to 4 out of 5, 4/5 or 4:5.

Situation two: The question asks you to compare one amount to another. This time you don't need to know the total.

Example: Using the class of 6 women and 24 men, what is the ratio of women to men and men to women?

Ratio of women to men is 6 to 24, reduced to 1 to 4, 1/4 or 1:4.

Ratio of men to women is 24 to 6, reduced to 4 to 1, 4/1 (or 4:1.

Note: if the denominator is 1 when writing a ratio, you must show it)

General Rules For Reading And Writing Ratios

Rule 1: *When you read or write ratios, compare the parts in the same order in language and numerically, unless they are part of a table or formula*.

To compare the number of women to the class total, the number of women is stated before the class total.

Ratio of women to class = 6:30
This is reduced to 1:5.

To compare the number of men to women, the number of men is written before the number of women.

Ratio of men to women = 24:6
This is reduced to 4:1.

Rule 2: *If the units in each term of the ratio are the same, they will cancel each other out. If the units cancel out, you don't need to include them in the ratio.* (There are situations, however, where you want to keep the units in the ratio. We will look at them later.)

The ratio of 25 centimeters to 1 meter is not 25:1. The ratio has to be written as 25 cm to 1 m or 25cm:1m.

Usually it is easier to work with ratios if there are no units, so make the units the same. If you convert 1 meter to 100 centimeters, the units will be the same. You can then cancel them out. The ratio is then written as 25:100 without any units.

If you can't write the ratio with the same unit for all terms, the units must remain in the ratio.

Rule 3: *Ratios without units are usually expressed in lowest terms.*

Example: Write the ratio of part time to full time electricians in a company with 25 part time and 15 full time workers.

Example: Write the ratio of part time to full time employees in a shop with 25 part time and 15 full time workers.

Answer: The units, which are employees, are the same. Since the question lists part time before full time, that is how the numbers are listed. The ratio is 25:15

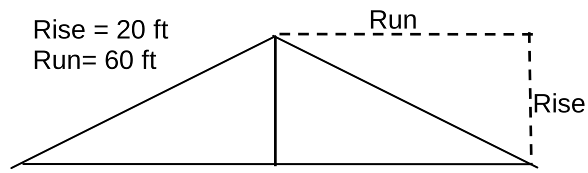
Reduce the ratio to lowest terms. Five is a common factor that divides into 25 and 15, giving the answers 5 and 3. The ratio 25:15 reduced to lowest terms is 5:3. The ratio of part time to full time workers is 5:3.

Example: What is the roof slope or pitch of a building if the rise (the vertical distance) is 20 feet and the run or span (the horizontal distance) is 60 feet. Use the ratio:

$$\text{pitch} = \frac{\text{rise}}{\text{run}}$$

$$\text{Pitch} = \frac{20}{60}$$ reduce to lowest terms

$$= \frac{1}{3}$$



Slope of roof is 1 to 3.

Rates

When the units of the quantities in a ratio are the same, they cancel out and so are not shown. When the units in a ratio are different, or there is only one unit, the units must be included in the ratio.

Ratios can be used to compare quantities of different types, such as kilometers per hour or cost per kilowatt-hour. These comparisons are called **rates**.

*A **rate** is a quantity or amount of something measured per unit of something else.* A rate includes the word "per" which is indicated by the fraction line.

Usually a ratio is divided so the amount of the unit following the word "per" is 1. If a rate involves two different kinds of units, they must be included in the ratio.

Driving speed is a rate. Say you drove 300 km in 3 hours. The ratio 300 km/3 hr is reduced to 100 km/1 hr or 100 km/hr. Your rate of speed is 100 km per hr.

Rates that involve a cost per unit, such as the rate you pay for electricity, include the dollar sign. Your electrical bill might say that your electrical rate is $.30/kw-hr. For every kilowatt-hour of electricity you use, you pay $.30.

Answer the following questions on ratios. Answers are on the last page.

1. Write as ratios using a colon between the two quantities. Convert quantities to the same unit where possible (that is, if the units are cm and m, convert so both quantities are either cm or m). Reduce to lowest terms.

   a) 1 to 4   b) 5 to 8   c) 5 in to 1 ft   d) 2 kg to 125 g

   e) 1 m to 50 cm   f) 15 min to 1 hr   g) 5 ft to 6 ft 6 in   h) one nickel to a quarter

2. Write as ratios using the fraction form. Reduce to lowest terms.

   a) 6 m to 3 m   b) 20 in to 45 in   c) 15 L to 9 L   d) 3 m to 90 cm

3. What is your rate of speed if you travel 400 km in 4 hr?

4. What is the cost of gas per liter if you pay $5.90 for 10 L?

5. If a pipe falls 3 inches over a distance of 12 feet, what is the fall per foot? In other words, express the two numbers as a rate. (Divide both numbers by 12. The first number in the ratio will be a fraction.)

PROPORTIONS

Two equivalent ratios express the same relationship but are written using different but related terms or numbers. For example, 1/4 and 2/8 are equivalent ratios and they represent the same amount. We can say that 1/4 equals 2/8. We can write this statement as:

1/4 = 2/8

Equivalent ratios written in fraction form with an equal sign between them form a ***proportion***.

- A proportion has four *terms*, or parts.
  - The terms of the proportion above are 1, 4, 2 and 8.
  - When we read the proportion, we name all four terms. 1/4 = 2/8 is read as "1 to 4 equals 2 to 8."

Direct and Indirect (or Inverse) Proportions

There are two basic types of proportions: direct proportions and indirect (sometimes called inverse) proportions.

In a ***direct proportion****, as one quantity increases, the corresponding quantity also increases.* Similarly, as one quantity decreases, the other one also decreases.

Example: The relationship between the size of drill bit you choose and the size of hole you drill is a direct proportion.

- The larger the bit, the larger the hole.
- The smaller the bit, the smaller the hole.

This is a direct proportion because as the bit changes in size, the hole changes in size *in the same way*.

In an ***inverse, or indirect, proportion****, as one quantity increases, the corresponding quantity decreases.* As one quantity decreases, the other one increases.

Example: The relationship between the efficiency of your car engine and fuel consumption is in inverse proportion.

- The more efficiently the car runs, the less fuel you use.
- The less efficient the engine is, the more fuel you use.

Solving a Proportion When Three of Four Terms Are Known

Proportions such as 1/4 = 2/8 in the example above don't tell us much. We already know that two ratios or fractions that represent the same amount equal each other. However, if we only know three of the four terms, a proportion can be used to find the fourth term.

The following are the general steps of finding an unknown amount using a proportion. Here are the steps to find the fourth, unknown term:

Here are the steps to find the fourth, unknown term:

1. **Set up a proportion using a letter to represent the unknown amount** in one of the ratios. The letter can be manipulated (moved around) in an equation just like a number. Write the ratios with an equal sign between them, forming an equation.

   Example: Write the proportion using the two ratios n:10 and 8:20.

   $$\frac{n}{10} = \frac{8}{20}$$

2. ***Cross-multiply to get rid of the denominators on both sides.*** To cross-multiply, multiply the diagonal numbers across the equal sign. In other words, multiply the numerator of one ratio by the denominator of the other ratio.

   If an unknown term is represented by a letter, cross multiply in the same way.

   Example: Cross-multiply in the equation below to get rid of the denominators.

   $$\frac{n}{10} = \frac{8}{20}$$

   Notice that n represents the unknown term.

   Multiply n by 20 and 10 by 8. Keep the equal sign.

   $20n = 10(8)$

   $20n = 80$

3. ***Isolate the unknown term*** (get it alone on one side of the equal sign). To do this divide both sides by the number in front of the unknown term.

   Example: Isolate n in the following equation.

   $20n = 80$

   $$\frac{20n}{20} = \frac{80}{20}$$

   Divide both sides by 20.

   $n = 4$

Here are some other manipulations that can help isolate the letter representing the unknown term.

A. If the letter representing the unknown term is on the right side, reverse the equation before dividing. You can reverse an equation without changing its value.

Example: You can reverse:

$3(15) = 5n$ to $5n = 3(15)$.

Both equations have the same value.

B. You can invert (turn all of the terms upside down) both sides of the equation without changing its value.

Example: You can invert

4/s = 5/6 to s/4 = 6/5.

Both equations have the same value.

Note: *If you invert one side of an equation, you must invert the other side to keep the equation equal.*

Now let's look at some examples of finding an unknown term in a proportion using these steps.

Example: Solve for n in the following proportion.

$\frac{4}{5} = \frac{n}{15}$ Set up the proportion

4(15) = 5n cross-multiply

60 = 5n

5n = 60 Reverse the equation so that n is on the left side of the equal sign.

5n ÷ 5 = 60 ÷ 5 Divide both sides of the equation by the number in front of the unknown term.

n = 12 The letter is isolated on the left hand side of the equation.
The answer is on the right hand side

Substitute 12 for n to write the complete proportion.

$\frac{4}{5} = \frac{12}{15}$

Example: Find the value of n when:

$$\frac{n}{12} = \frac{5}{15}$$

15n = 5(12) cross multiply

5n = 60 divide by 15 to isolate n

$$\frac{15n}{15} = \frac{60}{15}$$

n = 4

4/12 = 5/15 Substitute 4 for n to write the complete proportion.

Example: Find the value of n when:

$$\frac{n}{8} = \frac{10}{16}$$

16n = 10(80) cross multiply

16n = 80 divide both sides by 16

n = 5

Example: Find the value of s.

$$\frac{3}{4} = \frac{9}{s}$$

3s = 9(4) cross multiply
3s = 36
3s ÷ 3 = 36 ÷ 3 divide by 3
s = 12

Solving Problems Using Proportions

Proportions can be used to solve problems. You have to figure out what goes with what and then set up your proportion to find the unknown quantity. Notice that when you first set up your ratios, you do not usually reduce to lowest terms.

Method 1: These suggestions are one method to set up a proportion.

a) Set up the ratios (or fractions) so the same units are over each other.
    a. Set up minutes over minutes, kilometers over kilometers, or meters over meters.
b) The units of the two given quantities that form one fraction will cancel out.
    a. The unit of the third known quantity will be the unit of the unknown quantity
c) Set up the smaller unit over the larger unit. The proportion will look like this:

$$\frac{\text{small}}{\text{large}} = \frac{\text{small}}{\text{large}}$$

Example: A plumbing contractor can install an average of 25 low flush toilets in an apartment building a day when 5 apprentices are working. What is the average number of toilets that can be installed when 10 apprentices are working?

Set up your proportion.

- Put number the of apprentices over the number apprentices.
- Let x equal the unknown number of hours.

The proportion looks like this:

$$\frac{5 \text{ apprentices}}{10 \text{ apprentices}} = \frac{25 \text{ toilets}}{x} \qquad \frac{\text{small}}{\text{large}} = \frac{\text{small}}{\text{large}}$$

This looks like the proportions we already know how to solve.
Find the answer by solving for x:

$$\frac{5 \text{ ~~apprentices~~}}{10 \text{ ~~apprentices~~}} = \frac{25 \text{ toilets}}{x}$$ apprentices cancel out

$$\frac{5}{10} = \frac{25}{x}$$ cross multiply

$5x = 25 \times 10$ divide both sides by 10

$x = 50$

10 apprentices can install 50 toilets in one day.

Method 2: You can also set up the two ratios so each is given as a rate. When the ratios are set up as rates, in each ratio, one unit is over the other, different, unit. The following example shows how to set up the proportion.

Example: You travel 25 km in 50 minutes. How long will it take to travel 75 km at that speed?

The first ratio or rate is 50 min/25 km.
The second ratio is *unknown minutes*/75 km.
Set up the proportion by writing the two ratios.
Let m represent the unknown time.

$$\frac{50\text{ min}}{25\ \cancel{\text{km}}} = \frac{m}{75\ \cancel{\text{km}}}$$

km cancel
you can leave out the other unit, minutes, until the end

50(75) = 25m   cross-multiply

25m = 3750 min   reverse the equation
m = 150 min   divide both sides by 25 and put in the unit min

It will take 150 minutes to travel 75 km.

Example: If a network cabling specialist takes 4 hours to wire three rooms, how many rooms can she wire in an 8 hour day?

We will use the second method, although either method will get the same answer.
The first ratio is 4 hours/3 panels.
The second ratio is 8 hours/unknown number of panels.
Let t represent the unknown number of panels.
Set up the proportion.

$$\frac{4\text{ hrs}}{3} = \frac{8\text{ hrs}}{t}$$

cross multiply

4t hrs = 24 hrs   divide both sides by 4 hrs

t = 6 rooms   put in the unit houses

An apprentice can wire 6 rooms in an 8 hour day.

Here are some questions on proportions. Answers are on the last page.

6. Solve for the unknown quantity.

a) $\frac{n}{24} = \frac{1}{2}$ b) $\frac{2}{x} = \frac{10}{40}$ c) $\frac{16}{2} = \frac{s}{3}$

d) $\frac{5}{10} = \frac{12}{n}$ e) $\frac{n}{7} = \frac{3}{21}$ f) $\frac{2}{6} = \frac{2.45}{7.35}$

7. If it takes 70 minutes to travel 35 km, how long will it take to travel 85 km at the same speed?

8. Connectors costs $32.50 for 100. How much would 360 connectors cost?

9. If 65 feet of cable weighs 12.5 pounds, how much will 125 feet weigh?

SCALE

You have to know how to read blueprints in order to install the appropriate electrical, plumbing, heating, cooling or cable equipment. A blueprint is a drawing of a building or part of a building. The dimensions on a blueprint are scaled down representations of the actual dimensions. It would be impossible to manage the drawing of a large building if the actual dimensions were used.

The ***scale*** of a blueprint expresses the relationship between the dimensions of the blueprint and the actual dimensions of the building shown in the diagram. *The **scale** is the ratio of the drawing size to the actual size.* An equal sign (=) is used instead of a colon when indicating a scale.

Scale uses a smaller unit of measure to represent an actual, larger unit of measure. Scale can be considered as similar to rate, where the distance on the blueprint is compared to a standard unit, usually 1 foot.

> **Example:** A 1/4 inch line on a drawing can represent an actual length of 1 foot. The scale of the drawing is 1/4 in = 1 ft. In this case, a 5 inch pipe on the drawing represents a pipe that is actually 20 feet long.

The word scale is used to indicate the relationship between the drawing and the actual dimensions, as described above.

- In equations used to convert drawing dimensions to actual dimensions, scale indicates the number with the first, smaller unit.
- The unit of the second number is called the *standard unit*.
- So in the example above, the scale is 1/4.

To convert a length on a drawing to an actual measurement, follow these steps:

1. Divide the length on the diagram by the scale, the first number listed.
2. Use the unit of the standard unit (the larger unit) in the answer.

Example: If the length on a blueprint of a pipe is 2 1/2 inches and the scale is 1/4 inch = 1 feet, what is the actual length of the pipe?

length on diagram ÷ scale = actual length

The scale is 1/4.

2 1/2 ÷ 1/4 = 10 ft use the standard unit ft in the answer

2 1/2 inches represent a 10 foot pipe.

Example: A blueprint is drawn to the scale of 1/4 inch = 1 inch. If the dimensions of the space for a humidifier in a forced air system are drawn as 6 inches by 7 1/2 inches, what are the actual dimensions?

length on diagram ÷ scale = actual length

The scale is 1/4. The standard unit is inches.

6 ÷ 1/4 = 24 inches
7 1/2 ÷ 1/4 = 30 inches

The actual dimensions are 24" by 30".

In metric dimensions, the scale is usually a decimal or whole number, not a fraction.

Example: Find the actual length of a conduit if it is 4 centimeters long on a diagram. The scale is 1 centimeter = 1 meter.

length on diagram ÷ scale = actual length

The scale is 1. The standard unit is meters.

4 ÷ 1 = 4 m

The conduit is 4 meters long.

If you know a distance on a diagram and the actual distance, you can find the scale by following these steps:

- Divide the scale length by the actual length.
- If the scale is imperial, write the answer as a fraction. If the scale is metric, write it as a decimal.

Example: Find the scale of a blueprint if 10 centimeters on the diagram represents 20 meters.

scale distance ÷ actual distance = 10 m ÷ 20
= .5

The scale is .5. To express this as a ratio, put the unit of the blueprint length with the scale. The standard unit is 1 followed by the unit of the actual object. The two units are separated by an equal sign.

.5 cm = 1 m

Answer the following questions about scale. Answers are on the last page.

11. If the scale of a blueprint is 1/5 in = 1 ft, what is the actual length of an object that is 3 inches on the diagram?

12. What are the actual dimensions of a shed for a pool pump that measures 20 centimeters by 24 centimeters on a blueprint with a scale of 4 cm = 1 m?

13. If 2 inches on a blueprint represents 6 feet, what is the scale? Express the scale as a ratio.

14. What is the actual length and width of a furnace room if it is shown as 4 inches by 3 inches on the blueprint. The scale is 1/2 in = 1 ft?

15. A blueprint has a scale of 10 cm = 1 m. What is the actual diameter of a circle if the diameter measures 20 cm on the blueprint?

ANSWER PAGE

RATIOS Page 5

1. a) 1 : 4
   2. 5 : 8
   c) 5 : 12 (change 1 ft to 12 in)
   d) 16 : 1 (change kg to g)
   e) 2 : 1 (change m to cm)
   f) 1 : 4 (change hr to min)
   g) 10 : 13 (change 5' to 60" and 6' 6" to 78")
   l) 1 : 5 (change nickels and quarters to cents)

2. a) 6/3 = 2/1
   b) 20/45 = 4/9
   c) 15/9 = 5/3
   d) 200/90 = 10/3 (change m to cm)

3. 100 km/hr

4. $.59/L

5. 3/12 inches per 12/12 ft Reduce the fraction.
   1/4 inch per 1 foot
   1/4 inch fall per foot

PROPORTIONS Page 10

6. a) 12
   b) 8
   c) 24
   d) 24
   e) 1
   f) 2.45

7. $\frac{70\text{ min}}{35km} = \frac{m}{85km}$

   70(85) = 35m
   35m = 5950
   m = 170 min

8. $\frac{\$32.50}{100} = \frac{k}{360}$

100k = $32.50 x 360
k = $117.00

9. $\frac{65\,ft}{12.5lb} = \frac{125\,ft}{n}$

65n = 12.5 x 125
65n = 1562.5
n = 24 lb

SCALE Page 13

11. 3 ÷ 1/5
= 15 ft

12. 20 ÷ 4
= 5
24 ÷ 4
= 6
Dimensions are 5 m by 6 m

13. 2 ÷ 6 = 2/6 = 1/3
Scale is 1/3 inch = 1 ft.

14. 4 ÷ 1/2
= 8
3 ÷ 1/2
= 6
Length of the room is 8 ft.
Width of the room is 6 ft.

15. 20 ÷ 10
= 2
Diameter is 2 meters.