EVALUATING ACADEMIC READINESS FOR APPRENTICESHIP TRAINING

Revised for

ACCESS TO APPRENTICESHIP

MATHEMATICS SKILLS
CALCULATION OF AREAS OF GEOMETRIC FIGURES

AN ACADEMIC SKILLS MANUAL
for
The Construction Trades: Mechanical Systems

This trade group includes the following trades:
Electrician (Construction, Maintenance & Industrial),
Network Cabling Specialist,
Plumber, Refrigeration & Air Conditioning Mechanic,
Sprinkler &Fire Protection, and Steamfitter,

Workplace Support Services Branch
Ontario Ministry of Training, Colleges and Universities

Revised 2011

In preparing these Academic Skills Manuals we have used passages, diagrams and questions similar to those an apprentice might find in a text, guide or trade manual.

This trade related material is not intended to instruct you in your trade. It is used only to demonstrate how understanding an academic skill will help you find and use the information you need.

MATHEMATICS SKILLS
CALCULATION OF AREAS OF GEOMETRIC FIGURES

An academic skill required for the study of the Construction Trades: Mechanical Systems

INTRODUCTION

To successfully install any electrical, plumbing or cable system for a new office building, you need to correctly measure and then make accurate calculations involving those measurements. You might need to measure the perimeter of a room to figure out how much wire you will need to run and to determine the necessary equipment and supplies, or you may have to calculate the area of rooms in order to install the correct heating or air conditioning system.

Part I of this skill manual looks at calculating the perimeter and area of a rectangular space such as a room. The topics covered include:

- ♦ Two dimensional geometric figures
- ♦ Finding the perimeter of a rectangle
- ♦ Finding the area of a rectangle
- ♦ Calculating cost

Part II of this skills sheet looks at calculating the area of circles. The topics covered include:

- ♦ Terms used to describe a circle, including
  - degrees in a circle
  - circumference of a circle
  - diameter and radius
  - meaning of pi
- ♦ Finding the circumference of a circle
- ♦ Finding the area of a circle

PART I
CALCULATING PERIMETER AND AREA OF RECTANGULAR SPACE

TWO DIMENSIONAL GEOMETRIC FIGURES

A simple, closed, two dimensional (flat) figure with three or more straight sides is called a **polygon**.

- Triangles, squares, rectangles, and parallelograms (figures with 2 pair of opposite sides parallel) are all examples of polygons.

A **circle** is also a flat, closed figure but it is a curve, consisting of points that are all the same distance from the center.

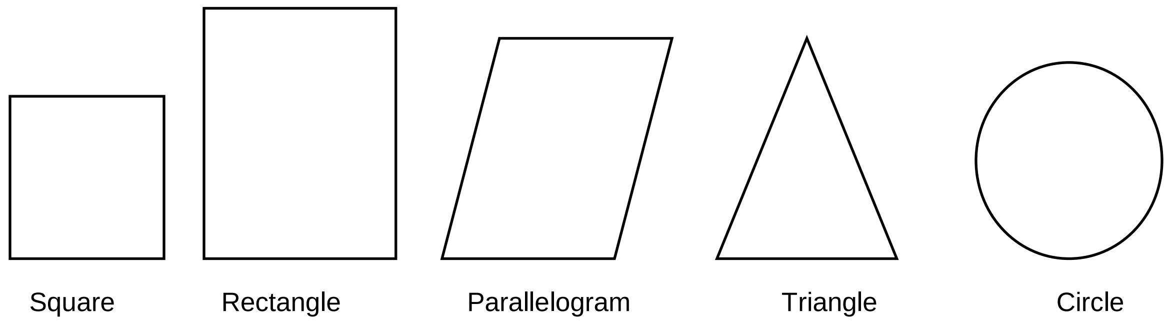


FIGURE 1: Some simple geometric shapes

These figures can be measured in different ways.

- ♦ Whenever we use measurements to make calculations with geometric figures, all measurements must be in the same linear units.
  - The units might be meters or centimeters, but they can't be a mix of meters and centimeters.

FINDING THE PERIMETER

The ***perimeter*** *of a polygon is the distance around its boundary*. Perimeter (p) is found by adding together the lengths of the sides.

Perimeter of a Rectangle

A ***rectangle*** *is a polygon with four 90 ° angles (right angles) and with each pair of parallel sides the same length.* (see Figure 1). Most rooms and offices have a rectangular shape.

- This means that we can find the perimeter of a rectangle by adding the lengths of the two long side to the lengths of the two shorter side.

The perimeter of a rectangle equals twice the length (*l*) added to twice the width (*w*). The formula is written in two forms:

$P = 2l + 2w$ where P is the perimeter, l is the length and w is the width of the rectangle.

or $P = 2(l + w)$

Remember: *When finding perimeter, all units must be the same. If the length is measured in feet and the width in yards, one unit must be changed to that of the other.*

Example: Find the perimeter of an office that is 30 m long and 16 m wide.

$$\begin{aligned} p &= 2l + 2w \\ &= 2(30 \text{ m}) + 2(16 \text{ m}) \\ &= 60 \text{ m} + 32 \text{ m} \\ &= 92 \text{ m} \end{aligned}$$

The perimeter is 92 m.

Example: Find the amount of fencing required to close in a space that is 400 yd wide and 1500 ft long.

Known:
l = 1500 ft
w = 400 yd = 1200 ft   400 yd x3 = 1200 ft

Find perimeter (P)

$P = 2(l + w)$
$= 2 (1500 \text{ ft} + 1200 \text{ ft})$
$= 2(2700 \text{ ft})$
$= 5400 \text{ ft}$

The space will require 5400 ft of fencing.

To find the perimeter of an irregular shape, measure and add all the lengths together. Just make sure all the measurements are in the same units.

Finding the Length of an Unknown Side When the Perimeter Is Known

If you know the perimeter of a rectangle and the length of one side, you can find the other side.

1. Manipulate (or rearrange) the variables in the formula for perimeter so the letter for length or width is by itself on the left side.
2. Solve to find the unknown side.
3. *Remember, whatever you do to one side of the formula, you need to do to the numbers and letters on the other side.*

Example: The perimeter of an opening is 3 m. The length is 1 m. What is the width?

Known:
P = 3 m
l = 1 m

Find w

$P = 2l + 2w$
$3 = 2(1) + 2w$
$3 = 2 + 2w$

$2 + 2w = 3$ Reverse the equation.

$2 - 2 + 2w = 3 – 2$ Subtract 2 from both sides.

$2w = 1$ Divide both sides by 2.

$w = ½$ m   write in the unit

The width is ½ m.

FINDING THE AREA

*The **area** of a polygon is the measure of the surface inside the boundary*. The units of area are squared units.

Area of a Rectangle
The area of a rectangle is the amount of surface enclosed within its boundaries of ***length*** and ***width***.

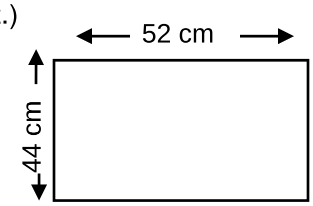
Example: The area of a room is the amount of floor space it has.

Area is calculated by multiplying the length of the rectangle times its width.
The formula for area is:

$A = lw$

Remember: *When finding the area of a rectangle, the units used to measure the length and the width must be the same. If the length is in meters, the width must also be in meters. If the units are different, one must be converted to the other before you can multiply.*

Example: Find the area of a rectangle that is 52 cm long and 44 cm wide.
(The units are the same so we don't have to convert.)



Draw the rectangle

Known:
l = 52 cm
w= cm

Find:
Area
Use A = lw

$A = lw$
$= 52 \text{ cm} \times 44 \text{ cm}$
$= 2288 \text{ cm}^2$

Note: When two of the same units are multiplied together, such as the centimeters in our example, they become square units. Instead of writing square centimeters, you can use the short form of cm^2 or sq cm. (Sq is the short form for square.) Four square feet is written 4 sq ft or 4 ft^2.

Example: Find the area of a space with length 5 m and width 142 cm.

We must convert one of the units so both are the same.

Known:	*Find:*
l = 5 m	area
w = 142 cm or 1.42 m	Use A = lw

A = *lw*
A = 5 m x 1.42 m

Example: Find the floor space of a room that measures 60 feet long by 32 feet wide by 10 feet high.
(The information on height is not needed to answer this question.)

Known
l = 60 ft
w = 32 ft

Find
Area
Use A= lw

A = *lw*
= 60 ft x 32 ft
= 1920 sq ft

Area of a Square
The four sides of a square are all the same length. To find the area of a square, square the length. (To square a number, multiply it by itself. Three squared is 3 x 3 = 9.)

Example: Find the area of a square with sides 5 ft long.

Known: l = 5 ft
w = 5 ft

Find A

A = *lw* or l^2
A = 5 ft x 5 ft
A = 25 sq ft

Finding the Length of an Unknown Side When the Area Is Known
If you know the area of a rectangle and the width of one side, you can find the length of the other side.

1. Manipulate the variables in the formula for area so the letter for length ends up by itself on the left side.
2. Substitute the known measurements for area and width.
3. Solve to find the unknown side.

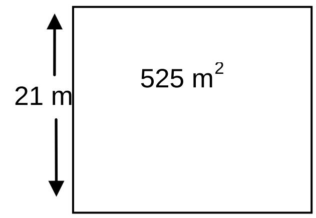
You can substitute the given measurements either before or after you manipulate the formula.

Example: If the area of a rectangle is 525 m^2 and the width is 21 m, what is the length?

Known: Find l

w = 21 m

A = 525 m^2



$A = lw$

$525m^2 = l \times 21m$

$l \times 21\text{ m} = 525m^2$ Reverse the equation.

$$l \times \frac{\overset{1}{\cancel{21m}}}{\underset{1}{\cancel{21m}}} = \frac{\overset{25}{\cancel{525m^2}}}{\underset{1}{\cancel{21m}}}$$

Divide both sides by 21 m to isolate l on the left.

When you divide a squared unit by a linear unit, such as square meters by meters, the meters on the bottom cancel one of the units on the top, leaving meters in the answer.

$l = 25$ m

The length is 25 meters.

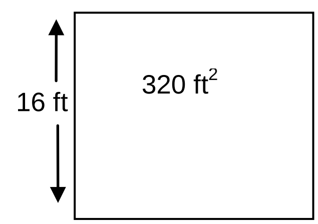
Example: Find the length of a room that has an area of 320 sq ft and a width of 16 ft.

Known:

w = 16 ft

A = 320 ft^2

Find l



First rearrange the letters of the formula so l is by itself on the left.

$A = lw$

$lw = A$ Reverse the equation.

$l = A/w$ Divide both sides by w.

$l = \frac{320\text{ ft }(\cancel{\text{ft}})}{16\ \cancel{\text{ft}}}$ Fill in the given amounts. Divide. Cancel the units where possible.

$l = 20$ ft

The length is 20 ft.

CALCULATING THE COST WHEN THE PERIMETER OR AREA IS KNOWN

You can find the total cost of the material used in an installation when the size and the cost per unit length are known. Multiply the size by the cost per unit. The units of length cancel, leaving only the dollar sign to indicate the total cost.

Example: You need to install 248 meters of wire in a building. The cost of the wire is $7.59 per meter. What is the total cost of the wire?

Known:
Length of wire = 248 m
Price of wire = $759/m

Find total cost of wire
To find the total cost of the wire, multiply the length in meters by the cost per meter.

248 ~~meter~~ x $7.59/~~meter~~ = $1882.32

The cost of the wire is $1882.32.

Example: Find the cost of installing a protective metal equipment room cover that measures 9.6 feet and 3.5 feet. The metal costs $8.99 per square foot.

Known:
Cover l = 9.6 ft
Cover w = 3.5 ft
Price of metal $ 8.99/ft^2

Find:
1. First find the area of the cover. (A= lw)
2. Cost of the material (A x price)

A = *lw*
= 9.6 ft x 3.5 ft
= 33.6 sq ft

Now multiply the ***number of square feet*** (the area) times the ***price per square foot*** to find the total cost:

33.6 ~~sq ft~~ x $8.99/~~sq ft~~ = $302.06

(Notice that the square feet cancel.)

The cost is $302.06.

Answer the following questions about geometric figures. Answers are on the last page.

1. Find the perimeter of a room that is 4.5 m by 6 m.

2. Find the perimeter of an office that measures 45 feet by 20 yards.

3. Find the area of a room that is 7.2 m long and 4.7 m wide.

4. Find the area of a rectangle that is 18 meters long and 400 centimeters wide.

5. Find the area of a square with sides that are 16 yards long.

6. Find the amount of cable needed to surround an area that is 30 feet by 10 yards.

7. A heating duct must run along the length of a room that is 45 feet long, then run 22 feet along the end wall, with 9 feet running to the termination point. How much duct work is needed? If the duct costs $15.50 per foot, what will the total cost of the duct work be?

8. Find the cost of installing a metal roof on an furnaceroom that measures 18.4 m long by 3.1 m wide. The steel costs $35.95 per square meter.

9. If a sheet of metal measures 4 ft by 8 ft, find the number of sheets required to make a rectangular metal tool box that has a height of 24 in, a width of 24 in and a length of 48 in. (First change the inches to feet.)

PART II
CALCULATING THE AREA OF CIRCLES

Terms Used To Describe A Circle

Degrees in a Circle
When the minute hand of a clock makes a complete circle around the face, moving from 12 around to 12 again, it travels 60 minutes or 1 hour. The clock face is divided into 60 divisions of one minute each. The face of a clock is usually in the shape of a circle.

In the same way that a clock is divided into different sections:
- ♦ A circle is divided into **degrees**.
- ♦ *A circle is divided into **360** equal parts that measure 1 degree each.*
- ♦ This means that a degree is 1/360th of the outside boundary or the *circumference* of a circle.
- ♦ To indicate the unit degree, we write the symbol·°

Each degree is further divided into 60 equal parts called minutes.

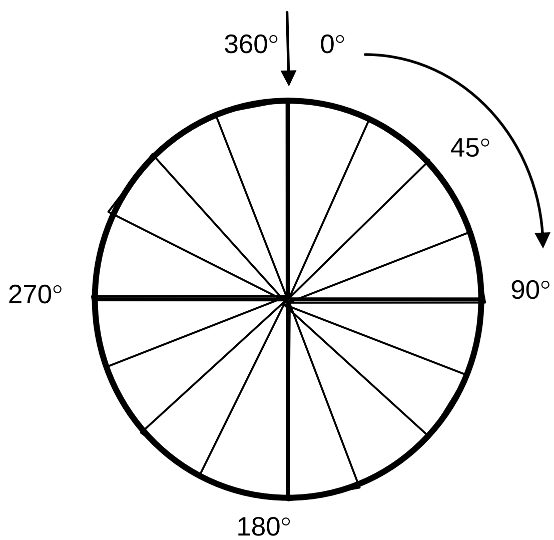


FIGURE 2: Some of the angles formed by dividing a circle into 360°

Circumference of a Circle
*The **circumference** (C) of the circle is the outside boundary of the circle.*
- ♦ Any partial section of the circumference is called an ***arc.***
- ♦ A circle has 360 arcs of equal size, each measuring one degree.
- ♦ T*he actual circumference of a circle in meters, inches, centimeters, etc, varies with the size of the circle.*
  - A larger circle will have a greater distance around its circumference than a smaller circle.

Examine Figure 2, and note the following:

- One-fourth of the circumference of a circle is equal to an arc of 90°.
    - If you draw lines from the end points of a 90° arc to connect in the centre of the circle, the lines form a 90° (right) angle.

- One-half the circumference of a circle is equal to an arc of 180°.
    - If you draw lines from the end points of a 180° arc to connect in the centre of the circle, the lines form an angle of 180°, which is also a straight line.

- An arc covering three-quarters of the circle is equal to 270°.
    - Lines drawn from the ends of this arc to the centre form an angle of 270°.

- If you travelled twice around the outside of a circle, you would travel 360° x 2 = 720° .
    - If you travelled three times around, you would travel 360° x 3 = 1080°.

Every circle is divided into 360 degrees around its circumference. However, *the actual distance in meters, inches, centimeters, etc, around the outside of a circle varies with the size of the circle.* A larger circle will have a greater distance around its circumference than a smaller circle.

- This distance could be measured by placing a flexible measuring tape around the circumference of the circle however, it is difficult to measure accurately this way.

Diameter and Radius

The ***diameter*** *(d) of a circle is a line that goes from one edge of the circle to the other edge and passes through the centre.*

- It is easy to measure the diameter of a circle accurately because it is a straight line, as you can see in Figure 3.

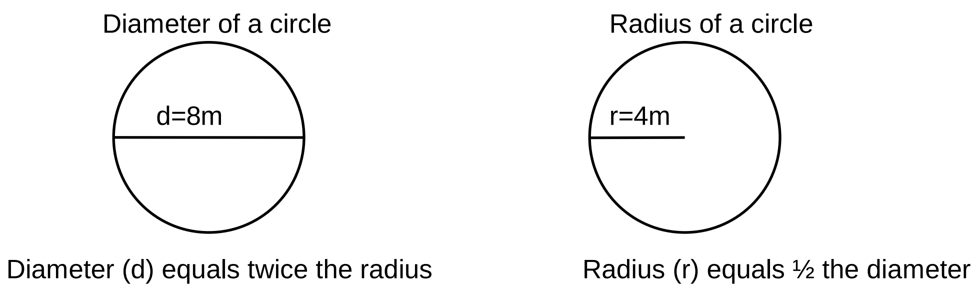


FIGURE 3: The diameter and radius of a circle

The ***radius*** *(r) of a circle is a line that goes from the centre of the circle to the circumference.*

- It is equal to one half the length of the diameter.

If you know either the diameter or the radius, you can find the other:

$r = {}^{1}/_{2}\,d$ <u>and</u> $d = 2r$

Example: If the diameter is 14 inches, what is the radius?

r = 1/2 d
= 1/2 x 14
= 7 in

If you place a circle inside a square so that the edges of the circle touch the square in four places, the length of the side of the square is equal to the diameter of the circle. So if you know the length of a side of the square, you also know the diameter of the circle.

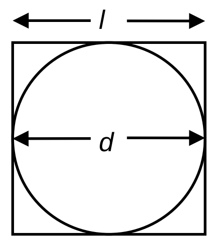


FIGURE 4: The length of a square is the same as the diameter of a circle placed inside it.

Conversely, if you know the radius of the circle, you can find the length of the sides of the square. Multiplying the radius by two gives the diameter. We know that the diameter is equal to the length of the sides. See Figure 4.

Example: If the length of a side of a square is 16 cm, what is the diameter of a circle that is inside the square and touching all four sides, as in the diagram above? What is the radius?

The diameter is the same as the length of the side of the square, which is 16 cm.
The radius is one-half of the diameter,

½ x 16 = 8 cm

The Meaning of Pi (π)

We use what we know about the diameter and radius of circles to find the circumference of a circle and to find its area. In order to do these calculations we use a ratio called pi (which sounds like pie).

Pi is *written using the symbol* ***π***

- *π =22/7* when it is written as a fraction.
- *π =3.14159…* When it its written as a decimal, the decimal places go on forever.
  - it is usually rounded off to two decimal places.
  - *π =3.14* rounded off to two decimal places

FINDING THE CIRCUMFERENCE OF A CIRCLE

The circumference is the distance around the boundary of a circle. To find the circumference, you must know either the diameter or the radius.

Circumference (C) is equal to π times the diameter (d). The formula for finding circumference is:

$C = \pi d$

or, since the diameter is twice the radius (r),

$C = 2 \pi r$

Example: Find the circumference of a circle with a radius of 32 cm.

$C = 2 \pi r$
C = 2 x 3.14 x 32 cm
C = 201 cm

Example: Find the circumference of a circle with a diameter of 8 m.

$C = \pi d$
= 3.14 x 8 m
= 25 m

Example: A piece of metal is to be formed into a duct that has a diameter of 20 cm. How much metal will be used?

The amount of metal used will be equal to the circumference of the duct.

Known
D = 20 cm

Find C

$C = \pi d$
= 3.14 x 20
= 62.8 cm

62.8 cm of metal are needed to make the duct.

FINDING THE AREA OF A CIRCLE

The area of a circle is the amount of space enclosed within the boundary of the circle.

***Area** (A) is equal to pi (π) times the radius squared.* The formula for finding the area of a circle is:

$A = \pi r^2$ Remember $\pi = 3.14$ or $\frac{22}{7}$

Squaring a number: To find the area of a circle, you need to know how to square a number. *To **square** a number, multiply it by itself.* For example, to square 11, multiply 11 x 11 to get 121.

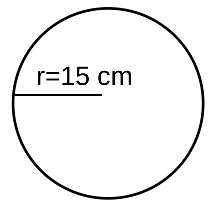
If you square a number that has units attached, the units are also squared. For example, the square of 5 meters (5m x 5m) is 25 meters squared. To show that meters, or any units, have been squared, we write a small two after the unit and slightly above it, like this: m^2. (The small raised 2 is called an exponent.) 5 m squared is equal to 25 m^2.

To square a number and its unit of measurement, we put brackets around *both of them* and write the exponent ^{2} after the brackets. So $(9\text{ m})^2$ means you square 9 m by multiplying 9 m by 9 m.

$$(9\text{ m})^2 = 9\text{ m} \times 9\text{ m} = 81\text{ m}^2$$

Now let's use the formula $A = \pi r^2$ to find the area for the following circle.

Example: Find the area of a circle with a radius of 15 cm.



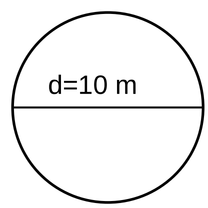
Known:
r = 25 cm
$\pi = 3.14$

Find A

$$\begin{aligned} A &= \pi r^2 \\ &= 3.14 \times (15\text{ cm})^2 \\ &= 3.14 \times 225\text{ cm}^2 \\ &= 706.5\text{ cm}^2 \end{aligned}$$

Note that the units are squared in the answer. Whenever you find the area of a circle, a square, a rectangle etc, the units are squared.

Example: Find the area of a circle with a diameter of 10 m.



Known
d = 10 m
r = 10 m ÷ 2 = 5m
$\pi = 3.14$

Find
A

$$\begin{aligned} A &= \pi r^2 \\ A &= 3.14 \times (5\text{ m})^2 \\ A &= 3.14 \times 25\text{ m}^2 \\ A &= 78.5\text{ m}^2 \end{aligned}$$

Using $A = \pi r^2$ to find the radius: If you know the area of a circle, you can find its radius by manipulating or changing around the position of the variables (the letters and symbols) in the formula. Rewrite the formula so that r^2 is by itself on the left. The letters in equations can be moved around just like numbers as long as you follow the rules of basic algebra.

Example: Find the radius of a circle with an area of 78.5 cm^2. First change the formula so that r^2 is by itself on the left:

Known
A = 78.5 cm^2

Find r

Area = πr^2	reverse the equation
$\pi r^2 = A$	divide by π to isolate r^2
$r^2 = 78.5\ cm^2 \div 3.14$	
$r^2 = 25\ cm^2$	Find the square root of r^2 and 25 cm^2
$r = \sqrt{25}$	
r = 5 cm	

Answer the following questions about geometric figures. Answers are on the last page.

1. Find the radius of a circle with a diameter of 24 inches.

2. Find the diameter of a circle with a radius of 3 feet.

3. Square the following numbers.
   a) 7 b) 31 c) 4 km d) 15 in e) 2 cm

4. Find the circumference of a steam pipe that has a diameter of 12 cm.

5. How much tape will be needed to go around the outside of a circular area that has a diameter of 1.5 m?

6. What is the cost of putting a metal strip around a circular opening with a radius of 24 inches if the metal costs $5.50 a foot? (Change the inches to feet first.)

7. A ball on a string that is 4 meters long is swung around in a circle two and a half times. How far did the ball travel? (Find the circumference and multiply it by 2 1/2)

8. Find the area of a circle with a radius of 28 cm.

9. Find the area of a circle that has a diameter of 14 m.

10. A circle has an area of 28.26 cm^2 . Find the radius, using 3.14 as π. (After dividing by π, find the square root of the answer. Use a calculator if you have one.)

11. After a circular hole with a radius of 20 cm is cut out of a sheet of metal measuring 100 cm by 200 cm, how much metal is left?

12. If a circle with a diameter of 10 cm is cut in half, what is the area of each half.

13. You must paint a circular area that has a diameter of 1.2 m. If a can of spray paint covers 2 m^2, how many cans will you need? (First calculate the area of the circle.)

14. What is the cost of making a metal cover for a barrel that is 2 yards in diameter if the cost of the metal is $5.00 a square yard? The waste metal can be reused, so only the amount used for the cover needs to be calculated.

ANSWER PAGE

Answers to Part I

1. p = 2*l* + 2*w*
   = 2(4.5 m) + 2(6 m)
   = 9 m + 12 m
   = 21 m

2. Change yd to ft: 20 yd = 60 ft
   p = 2*l* + 2*w*
   = 2(60) + 2(45)
   = 120 + 90
   = 210 ft

3. A = l x w
   = 7.2 m x 4.7 m
   = 33.84 m^2

4. Change cm to m. 400 cm = 4 m
   A = l x w
   = 18 x 4
   = 72 m^2

5. A = l^2
   = 16 x 16
   = 256 sq yd

6. Change feet to yards. 30 ft = 10 yd
   p = 2(10 yd) + 2(10 yd)
   = 40 yd

7. Length of duct work = 45 + 22 + 9
   = 76 ft
   Cost of duct work = 76 x $15.50
   = $1178.00

8. A = 18.4 m x 3.1 m
   = 57.04 m^2
   Cost = 57.04 m^2 x $35.95/$m^2$
   = $2050.59

9. First change inches to feet. 24 in = 2 ft, 48 in = 4 ft
   Area of top and bottom = 2 x 2 x 4 = 16 sq ft
   Area of long sides = 2 x 2 x 4 = 16 sq ft
   Area of short sides = 2 x 2 x 2 = 8 sq ft
   Find the total area and divide that by the area of 1 sheet of metal.
   Total surface area of the box = 16 + 16 + 8 = 40 sq ft
   Area of a sheet of metal = 4 x 8 = 32 sq ft
   Number of sheets needed = 40 ÷ 32 = 1.25 sheets

Answers to Part II

1. r = 12 in.

2. d = 6 ft.

3. a) 49 b) 961 c) 16 km^2 d) 225 in^2 e) 4 cm^2

4. C = π d = 37.68 cm

5. C = 4.71 m Tape needed = circumference of circle

6. r = 24 in ÷ 12 = 2 ft
   C = 12.56 ft
   Cost = 12.56 x $5.50 = $69.08

7. r = 4 m Don't forget that the ball went around not once, but 2.5 times
   C= 2 x 3.14 x 4 m So, we have to multiply the circumference of 25.12 m by 2.5
   = 25.12 m
   25.12 m x 2.5 = 62.8 m. The ball traveled 62.8 m.

8. A = π r^2
   = 3.14 x $(28\text{ cm})^2$
   = 3.14 x 784 cm^2
   = 2461.76 cm^2

9. Radius = diameter ÷2
   = 7 m
   Area = 3.14 x 7^2
   = 153.86 m^2

10. r^2 = area ÷ π
    = 28.26 cm^2 ÷ 3.14
    = 9 cm^2

11. Area of circular hole = 1256 cm^2
    Original area of the sheet = 20,000 cm^2
    Area of metal left = 20,000 cm^2 - 1256 cm = 18,744 cm^2

12. Area = 78.5 cm^2
    ½ of area = 39.25 cm^2

13. One can covers 2 m^2. This is more than the area to be covered (1.11 m^2), so 1 can is enough.

14. A = 3.14 sq. yd.
    Cost of one square yard = $5.00
    Cost of 3.14 sq. yd. = 3.14 x $5.00 = $15.70