EVALUATING
ACADEMIC READINESS
FOR APPRENTICESHIP TRAINING
Revised for
ACCESS TO APPRENTICESHIP

MATHEMATICS SKILLS
POWERS OF TEN

AN ACADEMIC SKILLS MANUAL
for
The Construction Trades: Mechanical Systems

This trade group includes the following trades:
Electrician (Construction, Maintenance & Industrial),
Network Cabling Specialist,
Plumber, Refrigeration & Air Conditioning Mechanic,
Sprinkler &Fire Protection, and Steamfitter,

Workplace Support Services Branch
Ontario Ministry of Training, Colleges and Universities

Revised 2011

In preparing these Academic Skills Manuals we have used passages, diagrams and questions similar to those an apprentice might find in a text, guide or trade manual.

This trade related material is not intended to instruct you in your trade. It is used only to demonstrate how understanding an academic skill will help you find and use the information you need.

MATHEMATICS SKILLS
POWERS OF TEN

An academic skill required for the study of the Construction Trades: Mechanical Systems

INTRODUCTION

This skill manual looks at evaluating powers of ten. Powers of ten can help organize very large or very small numbers. In your trades, you use some very big and very small numbers that are sometimes written as powers of ten or in scientific notation. Writing numbers as powers of ten can make it easier to see the relative size of numbers. For example, when the prefix mega is written as the power of ten,10^6, and the prefix giga as 10^9, it is easier to see which is bigger.

The very large and very small numbers you encounter in your textbooks are sometimes written in scientific notation because they are easier to understand and work with in this form. When a number is written in scientific notation, there is only one digit in front of the decimal point. This number is then multiplied by a power of ten. The size of the number is shown by the exponent used with the power of ten. The speed of light, written in scientific notation, is 3×10^{10} cm/sec. It is easier to read, although less accurate, than the long form, 29,979,280,000 cm/sec.

The following topics concerning powers of ten:

- ♦ Place value
- ♦ Exponents
- ♦ Powers of ten, including writing numbers as powers of ten
- ♦ Decimal numbers as powers of ten including writing decimal numbers as powers of ten
- ♦ Multiplying and dividing powers of ten, including working with integers

PLACE VALUE

We can write any number we want to with only ten symbols, or ***digits***. The digits are:

0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

We can write any amount we choose, from the largest to the smallest using only these nine digits. Here’s how:

Our number system is based on the idea of ***place values***. We use the number symbols or ***digits*** from 0 to 9 to write numbers. *The place or position of a digit in the number indicates its place value*. (See Table 1 for place values up to a million.)

Let's look at the number 785.

- The digit on the far right hand side is in the ones place. 785 has 5 ones.
- The digit to the left of the ones place is in the tens place. 785 has 8 tens.
- And the next digit to the left is in the hundreds place. 785 has 7 hundreds.
- (See Table 1 for place values up to a million.)

Whole numbers: When we write a number such as six thousand, four hundred and eighty-two using digits instead of words, we don't show the thousands, the hundreds, the tens and the ones separately.

Instead, we put:

- the 2 in the ones place,
- the 8 in the tens place,
- the 4 in the hundreds place, and
- the 6 in the thousands place.

The written number looks like this: 6 482.

We are so familiar with this system that we know automatically that in a number like 57 213, the digit 2 (three places from the right) stands for two hundred.

The number 4 782 951 has seven digits. Table 1 shows the place value of each digit:

Table 1: Place Values from Ones to Millions

millions	hundred-thousands	ten-thousands	thousands	hundreds	tens	ones
4	7	8	2	9	5	1

We read this number as four million, seven hundred eighty-two thousand, nine hundred fifty-one.

When we refer to a specific digit in a larger number, we use the place value column to find the value the digit represents.

Example: In the number 356, the digit 5, two places from the right side, represents fifty. We know the 5 represents fifty because it is in the tens column.

Decimal numbers: Place values are also used to write decimal numbers. A ***decimal number*** indicates a partial amount that is less than one. A period called a ***decimal point*** is written after the ones place to signify the decimal portion of the number.

- ♦ The first place after the decimal point (to the right) is called the tenths place.
  - *The digit in the tenths place is the same as the numerator of a fraction with the denominator 10.*

Example: The decimal .7 is the same as the fraction 7/10.

- ♦ The next place to the right is called the hundredths place.
  - The digit 3 in the decimal number .538 is in the hundredths place.
- ♦ The place value to the right of the hundredths place is called the thousandths place.
  - The 8 in the decimal number above is in the thousandths place.

Refer to Table 2 below which shows place values after the decimal (and also the digit 4 in the ones place value). Notice that *th* is used to indicate decimal place values.

Table 2: Place Values after the Decimal

ones	**decimal**	tenths	hundredths	thousandths	ten thousandths	hundred thousandths	millionths
4	.	6	2	8	3	7	5

In a number such as 4.628375, the digit 2, in the hundredths column represents 2/100. The digit 3, in the ten thousandths column represents 3/10 000 and so on.

Putting all the place value fractions together gives the fraction:

$$\frac{628\ 375}{1\ 000\ 000}$$

This is the same as the decimal .628375.

EXPONENTS

We can organize numbers in many different ways. For example, when we have to multiply any number by itself many times, we use ***exponents*** to indicate this kind of multiplication.

A short form of indicating that 5 is to be multiplied by itself (5 x 5) is writing 5^2.

- This is read "*five squared*" or "*five times five*" or "*five to the second power*".
- The whole number, in this case five, is called the ***base*** or the factor.
- The small number 2 written after the base and slightly above it is called the ***exponent***.

The exponent tells how many times to multiply the base by itself. The number 2^4, written with an exponent, tells us that the base 2 is to be multiplied by itself four times, like this: 2 x 2 x 2 x 2.

The base 8 with the exponent 3, written 8^3, tells us that eight is to be multiplied by itself three times, like this: $8^3 = 8 \times 8 \times 8$

The exponent ^{0} is unusual. Any number with the exponent ^{0} is equal to 1.

$8^0 = 1$
$2^0 = 1$
$10^0 = 1$

POWERS OF TEN

Numbers such as 100, 1000, 10 000, and 100 000 can all be calculated by multiplying 10 times itself.

100 = 10 x 10
1 000 = 10 x 10 x 10
10 000 = 10 x 10 x 10 x 10 and so on.

These numbers can be written in an exponential form. Ten is the base, written with the required exponent, such as 10^2, 10^3, or 10^4 and so on. *Numbers with exponents that use ten as the base are called* ***powers of ten***.

The term "power of ten" *can refer to these kinds of numbers whether they are written with an exponent (10^3) or with the multiplication indicated (10 x 10 x 10) or in long form (1000).* Any of these forms can be referred to as a power of ten. However, for this skills manual, "power of ten" will refer to the exponential form, 10^2, 10^4, 10^6, and so on.

We interpret powers of ten as follows:

10^0 = 1

10^1 = ten to the first power, which is 10
10^1 = 10

10^2 = 10 x 10, it is read as ten squared
10^2 = 100

10^3 = 10 x 10 x 10, it is read as ten to the third
10^3 = 1 000

10^4 = 10 x 10 x 10 x 10, it is read as ten to the fourth
10^4 = 10 000

10^{10} = 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10, it is read ten to the tenth
10^{10} = 10 000 000 000

Look at the examples again. Notice that in each case the number of zeros after the one in the answer is the same as the exponent. So 10^7 will be a one with seven zeros after it (10 000 000).

Writing Powers of Ten

To write a power of ten in the exponential form:

1. Count the number of zeros after the first digit, 1.
2. The number used as an exponent after the base 10 is the same as the number of zeros.

Example: Write 100 000 as a power of ten with an exponent.

There are 5 zeros after the digit 1.
The exponent will be the same number as the number of zeros.
Write the number 10 as the base with the exponent 5.

100 000 written as a power of ten is 10^5.

Example: Write 10 000 000 as a power of ten.

There are seven zeros after the first digit, 1.
Write the number 10 as the base with the exponent 7.

10 000 000 as a power of ten = 10^7.

Example: Write 100 as a power of ten.

There are two zeros after the first digit, 1.

100 as a power of ten = 10^2

Writing a power of ten in the long form

To reverse the process and write a power of ten in the long form:

1. Write a 1.
2. Then write as many zeros after the 1 as the number that forms the exponent.
3. *Do not add the zeros to the base 10. Add the zeros to the digit 1.*

Example: Write 10^4 in long form.

The exponent is 4 so there will be four zeros.
Write a 1 with four zeros after it.

10^4 written in long form (or multiplied out) is 10 000.

Example: Write 10^6 in the long form.

The exponent is six. Write 1 with six zeros after it.

10^6 = 1 000 000

DECIMAL NUMBERS AS POWERS OF TEN

So far we have looked at how to write whole numbers as powers of ten. Decimal numbers like .0001 can also be written as powers of ten. The method of writing decimal powers of ten with an exponent is a little different from writing whole numbers.

First let's look at some decimal powers of ten:

10^{-1} is read as ten to the negative one and equals .1

10^{-2} is read as ten to the negative two and equals .01

10^{-3} is read as ten to the negative three and equals .001

10^{-4} is read as ten to the negative four and equals .0001

10^{-5} is read as ten to the negative five and equals .00001

If you look at these examples carefully, you see that, in each one, *the number of **decimal places** in the answer is the same as the number forming the exponent*. So 10^{-7} has seven decimal places.

Writing Decimal Powers of Ten

To write a decimal number consisting of zeros followed by the digit 1 as a power of ten:

1. First count all the decimal places *including the 1*.
2. The number forming the exponent after the base ten is the same as the number of decimal places.
3. *There is one important difference.* Write the exponent *with a **negative sign** in front to indicate that the number is a decimal number***.**

Example: Write .00001 as a power of ten.

Write ten as the base.
There are 5 decimal places after the decimal point. The exponent is the same as the number of decimal places.
Put a negative sign in front of the exponent, so it becomes $^{-5}$.

.00001 written as a power of ten is 10^{-5}

Example: Write .0000001 as a power of ten.

There are seven decimal places after the decimal point. Write ten with the exponent -7.

$.0000001 = 10^{-7}$

Example: Write .1 as a power of ten.

There is one decimal place.

$.1 = 10^{-5}$

MULTIPLYING AND DIVIDING POWERS OF TEN

Powers of ten written with exponents are easier to read and keep track of than the longer forms of the numbers. As a bonus, the exponents written with the base ten have some special characteristics. When you multiply and divide powers of ten, you can use a shortcut.

Multiplying powers of ten

To multiply powers of ten:

1. *Add the exponents* together, and
2. use that addition answer as the new exponent of the multiplication answer.

Example: Multiply $10^3 \times 10^5$

1000 x 10 0000 = 100 000 000
$10^3 \times 10^5 = 10^8$

You can multiply 1000 (10^3) x 100 000 (10^5) to get 100 000 000 and rewrite it as 10^8. *Or you can simply add the exponents together*. This gives you the exponent of the multiplication answer. To multiply $10^3 \times 10^5$, add the exponents 3 + 5, which equals 8. The exponent of the answer is 8.

$10^3 \times 10^5 = 10^8$

The answers are the same but the second method is faster and it is easier to keep track of the zeros.

Example: Multiply 10^7 x 10^9.

(7 + 9 = 16) Add the exponents together. The addition answer becomes the new exponent of the multiplication answer. You can add the exponents in your head.

$10^7 \times 10^9 = 10^{16}$

Dividing powers of ten

To divide using powers of ten:

1. *Subtract the exponent of the divisor* from the exponent of the dividend; and,
2. use that answer as the new exponent.

In the division 100÷10, 100 is the ***dividend***, the number you divide into, and 10 is the ***divisor***, the number you divide by.

Example: $10^{12} \div 10^6$

You could divide the long forms and rewrite the answer as 10^6.

$$1\,000\,000 \overline{)\,1\,000\,000\,000\,000} = 1\,000\,000 = 10^6$$

Or you could subtract the exponents and use the subtraction answer as your new exponent.

$10^{12} \div 10^6 = 10^6$ (12 - 6 = 6)

Example: $10^{15} \div 10^4$

Subtract the exponents and use your subtraction answer as the new exponent. You can do the subtraction of exponents in your head.

$10^{15} \div 10^4 = 10^{11}$ (15 - 4 = 11)

Remember: *Decimal numbers expressed as powers of 10 have a negative sign in front of the exponent.*

To multiply and divide powers of ten with negative exponents, you need to know the rules for adding integers.

Working with Integers

Integers *include all numbers with* ***positive*** *or* ***negative*** *signs.*

- A negative number always has a negative sign in front of it.
- If there is no sign in front of a number, it is assumed to be positive

When working with integers, we consider addition as another positive sign and subtraction as another negative sign. If you are adding or subtracting integers, a − or + sign is placed in front of the number to be added or subtracted. This number will already have a positive or negative sign.

Before doing the operation, simplify the terms so there is only one sign in front of each number.

Here are the rules that tell you what sign to leave in front of each number:

- If there is both a − sign and a + sign, the sign becomes −.
- If there are two + signs, the sign is +.
- If there are two − signs, the sign becomes +.

Example: Simplify $-8 + (-5) - (-7)$

Use the rules given so that there is only one sign in front of each number.

– 8 stays the same
+(– 5) becomes – 5
−(– 7) becomes +7
So
$-8 + (-5) - (-7) = -8 - 5 + 7$

Once you simplify the equation so there is only one sign in front of each number, the operation is considered as ***addition*** with positive and negative numbers. You then add the signed numbers following the rules below for adding integers.

Adding integers

Rule 1: *To add two positive numbers,* ***find the sum of their values and write the answer***. The positive sign in the answer doesn't have to be shown.

Add $4 + 7 = 11$

Rule 2: *To add two negative numbers, add the* ***value of the numbers*** *together and write the answer with a* ***negativ****e sign in front.*

Add

$$\begin{array}{r} -8 \\ \underline{-14} \\ -22 \end{array}$$

or $-8 + (-14) = -22$

Rule 3: *To add a negative and a positive number,* ***subtract the number with the smaller value from the number with the larger value***. Give the answer the sign of the number with the ***larger*** value.

Add

$$\begin{array}{r} -8 \\ \underline{5} \\ -3 \end{array} \qquad \begin{array}{r} 10 \\ \underline{-7} \\ 3 \end{array} \qquad \begin{array}{r} 8 \\ \underline{-9} \\ -1 \end{array}$$

Or

$-8 + 5$
$= -3$

$10 + (-7)$
$= 10 - 7$
$= 3$

$8 + (-9)$
$= 8 - 9$
$= -1$

Example: $10^{-8} \times 10^{2}$

Multiply the powers of ten by adding the exponents −8 + 2.
Follow Rule 3: When adding a positive and a negative number, subtract the number with the smaller value (2) from the number with the larger value (-8). (8 − 2 = 6) Give the answer the sign of the number with the larger value. Since -8 has the larger value, the sign of the answer will be negative.

−8 + 2 = − 6

Write the answer to the addition question as the exponent to the base 10, keeping the negative sign.

$10^{-8} \times 10^{2} = 10^{-6}$

Example: $10^{-3} \div 10^{-6}$

Both exponents are negative numbers, follow the rules for simplifying signs and adding integers.

First write the exponents with their signs as a subtraction question.
− 3 − (− 6)

Write each number with one sign, following the rules for simplifying signs.

− 3 + 6

Follow Rule # 3 for adding integers.

−3 + 6 = 3

Write the answer as the exponent to base 10.

$10^{-3} \div 10^{-6} = 10^{3}$

Here are some questions on powers of ten. Answers are at the end of the skills manual.

1. In the number 3^2, the 3 is called a _______________ or _______________, and the small 2 to the right of and slightly above the three is called the __________________ .

2. 2^5 can be written as ___ x ___ x ___ x ___ x ____ .

3. Numbers with exponents that use ten as their base are called _____________ of ______ .

4. 10^4 is read as ten to the ______________ .

5. 10^2 means to multiply _____ x _____. 10^2 is equal to _______ .

6. To write 10^4 in long form, we write a one with _________ zeros after the one. 10^4 written in long form is _____________ .

7. 10^7 written in long form is ________________ .

8. 1 000 000 written as a power of ten is ______________ .

9. 1 000 written as a power of ten is _____________ .

10. 10^{-2} written in long form is _____________ .

11. 10^{-5} written in long form is _______________ .

12. 10^{-1} written in long form is ____________ .

13. .001 written as a power of ten is _____________ .

14. .000001 written as a power of ten is _______________ .

15. 10 000 000 000 written as a power of ten is ______________ .

16. Multiply (write your answers as powers of ten):

a) $10^3 \times 10^4$ = ___________ b) $10^1 \times 10^5$ = ___________

c) $10^6 \times 10^{-4}$ = __________ d) $10^0 \times 10^8$ = ___________

17. Divide (write your answers as powers of ten):

a) $10^{10} \div 10^7$ = _________ b) $10^5 \div 10^2$ = ___________

c) $10^9 \div 10^{-7}$ = _________ d) $10^5 \div 10^0$ = ___________

ANSWER PAGE

1. factor or base, exponent

2. 2 x 2 x 2 x 2 x 2

3. powers of ten

4. fourth

5. 10 x 10, 100

6. four, 10 000

7. 10 000 000

8. 10^6

9. 10^3

10. .01

11. .00001

12. .1

13. 10^{-3}

14. 10^{-6}

15. 10^{10}

16. a) 10^7 b) 10^6

    c) 10^2 d) 10^8

17. a) 10^3 b) 10^3

    c) 10^{16} d) 10^5