EVALUATING
ACADEMIC READINESS
FOR APPRENTICESHIP TRAINING
Revised for
ACCESS TO APPRENTICESHIP

MATHEMATICS SKILLS
SCIENTIFIC NOTATION

AN ACADEMIC SKILLS MANUAL
for
The Construction Trades: Mechanical Systems

This trade group includes the following trades:
Electrician (Construction, Maintenance & Industrial),
Network Cabling Specialist,
Plumber, Refrigeration & Air Conditioning Mechanic,
Sprinkler &Fire Protection, and Steamfitter,

Workplace Support Services Branch
Ontario Ministry of Training, Colleges and Universities

Revised 2011

In preparing these Academic Skills Manuals we have used passages, diagrams and questions similar to those an apprentice might find in a text, guide or trade manual.

This trade related material is not intended to instruct you in your trade. It is used only to demonstrate how understanding an academic skill will help you find and use the information you need.

MATHEMATICS SKILLS
SCIENTIFIC NOTATION

An academic skill required for the study of the Construction Trades: Mechanical Systems

INTRODUCTION

Mechanical construction workers sometimes have to deal with very large or very small numbers which are difficult to keep track of and awkward to write. Very large or very small numbers can be written in a simplified form called ***scientific notation.*** Numbers in scientific form are more convenient to work with.

> **Example:** The electrical charge on an object is determined by the number of electrons it possesses in excess of the number of its protons. The amount of charge on an object is measured in coulombs (Q).
>
> 1 Q = 6 250 000 000 000 000 000 electrons
>
> If you have to use this number in your calculations, it will be difficult to keep track of all the zeros. If you are using a calculator, you may not be able to do it at all because the number is so long. Here is the same number written in scientific notation:
>
> 1 Q (coulomb) = 6.25×10^{18} electrons
>
> Expressing the value of a coulomb using scientific notation is a more convenient form.

You can also express very small numbers in scientific notation.

> **Example:** To find the linear expansion of hard steel for each degree of Celsius temperature increase, you multiply the length of the steel by .0000132. If you are using this number to do calculations, it can be difficult to keep track of all the decimal places. In scientific notation, this number is 1.32×10^{-5}.

This skill manual will introduce scientific notation. It covers the following topics:

- Significant figures and absolute value
- Writing numbers in scientific notation
- Changing numbers from scientific notation to the long form
- Working with numbers in scientific notation.

SIGNIFICANT FIGURES and ABSOLUTE VALUE

In order to write a number in scientific notation, we must first write its significant digits, or figures. We must also understand the absolute value of any number. So, before we learn how to express numbers in scientific notation, we need to be familiar with these ideas.

Significant Figures

When you determine the precision of a measurement, the number of significant figures in the measurement gives information on how accurate you are. A measurement of 30 centimeters is less precise than a measurement of 30.00 centimeters. For this reason, you should be aware of what the concept of significant digits means.

The concept of significant figures is easiest to understand by looking at examples. The rules listed below state what digits are significant.

In any number:

- All non zero digits are significant.

  6 957.443 has seven significant figures. They are the digits 6, 9, 5, 7, 4, 4, and 3.

- All zeros between two non zero digits are significant.

  105.009 contains six significant figures.

- Unless otherwise stated, all zeros after or to the right of a non zero digit are not significant if the decimal point is not shown.

  215 000 contains three significant figures.
  3 400. shows the decimal point so it has four significant figures.

- Zeros to the right of the decimal point with no whole numbers in front of the decimal and before the non zero digits are not significant.

  .0076 contains two significant figures.

- Zeros to the right of a decimal in a number which has a whole number preceding the decimal point and non zero digits after the zeros are considered significant digits.

  8.0076 has 5 significant figures.

- When a decimal ends in zeros, these zeros are significant.

  .3200 has 4 significant figures.

Absolute Value

All numbers can be positive (+) or negative (–) in value. Remember that we do not have to show the + sign in front of a positive number but we have to always show the – sign. If there is no sign in front of a number, the number is positive.

The ***absolute value****. of any number is its positive value.*

- ♦ The absolute value of a positive number is its positive numerical value:
  - The absolute value of +15, or 15, is 15.
  - The absolute value of 9 is 9.

- ♦ The absolute value of a negative number is its positive numerical value:
  - The absolute value of – 15 is also 15.
  - The absolute value of – 9 is 9.

Example: The number 8, or +8, is positive. Its absolute value is 8.

Example: The number - 74 is negative. Its absolute value is 74.

WRITING A NUMBER IN SCIENTIFIC NOTATION

Numbers like 65 000 or .0065 can be awkward to work with. To make them more manageable, we write them in *scientific notation,* using significant figures and what we know about powers of ten.

To write any number in scientific notation:

1. *Rewrite the number,* ***shifting the decimal point*** *until it is directly after or to the right of* ***the first significant digit on the left side of the number.***

   a. In the number 65 000, shift the decimal point four places to the left so it is to the right of the first significant digit, 6.

2. *Write any other significant digits after the decimal point.*

   a. In the number 65 000, the digits 6 and 5 are the only significant digits.
   b. The result now looks like this: 6.5

3. *Next write a multiplication sign* ***(x)*** *followed by a power of ten.*

   a. The number will now look like this:

      6.5 x 10

4. *Determine the **exponent** for the power of ten by counting **the number of places you shifted the decimal point** in # 1. above.*

   a. The decimal point was shifted four places in the example.
   b. The exponent for the power of ten is four.

   The number is written 6.5×10^4.

5. *If you shift the decimal point to the **left**, the exponent is **positive**.*

   a. In our example we moved the decimal 4 places to the right to find the significant figures, so the exponent 4 is positive.

6. *If you shift the decimal point to the **right**, the exponent is **negative.***

   We start this time with .00065.
   We again move the decimal directly after the digit 6.
   Our significant figure is again 6.5.
   Because the decimal was moved four places, the exponent is again 4.
   But, we moved it to the *right* so it will be a *negative exponent*.
   The number is written 6.5×10^{-4}.

Example: Write 38 500 000 in scientific notation.

Here are the steps to follow:

3.	Shift the decimal point until it is directly to the right of the first significant digit, 3.
3.85	Write the other significant figures after the decimal point:
3.85 x 10	Write this number followed by a multiplication sign and a power of ten
3.85×10^7	The value of the exponent is the number of places you shift the decimal point . • You shifted the decimal point seven places to the left to put it directly after the first digit. • The exponent is seven. The exponent is positive as you shifted the decimal point to the left,

In scientific notation 38 500 000 = 3.85×10^7

Example: Write .009 73 in scientific notation.

	Move the decimal point so it is directly to the right of the 9.
9.73	Write the other significant figures after the decimal point
9.73 x 10	Follow by a multiplication sign and a power of ten:
9.73×10^{-3}	The decimal point was moved three places to the right. -3 is the exponent as the decimal was shifted to the right.

Example: Write 1 000 000 in scientific notation.

1. x 10	Put the decimal point to the right of the 1.
	There are no other significant figures
	Write the multiplication sign and a power of ten:
1. x 10^6	The decimal point was moved six places to the left, so the exponent is positive.

So, in scientific notation 1 000 000 = 1. x 10^6

When the only significant figure is a 1, the number can simply be written as a power of ten.

- So the number above can be written without the 1 as: 1 000 000 = 10^6.

Example: Write .000 01 in scientific notation.

1. x 10	Put the decimal point to the right of the 1.
	There are no other significant figures
	Write the multiplication sign and a power of ten:
1. x 10^{-5}	The decimal point was moved 5 places to the right, so the exponent is negative.

.000 01= 1. x 10^{-5}

or

.000 01 = 10^{-5}

CHANGING FROM SCIENTIFIC NOTATION TO LONG FORM

You should also know how to change a number that is in scientific notion back to the longer form. We use significant figures, absolute value, and what we know about powers of ten to change scientific notation to long form.

Here are the steps to follow:

1. Look at the number forming the exponent. If it is positive, you move the decimal point in the number to the right.

2. The decimal point is moved the same number of places to the right as the value of the number that forms the exponent.

3. If the number forming the exponent is negative, you move the decimal place to the left.

4. The decimal point is moved to the left the same number of places as the absolute (positive) value of the number forming the exponent.

5. Use zeros wherever necessary as place holders.

Example: Write 6.87×10^8 in the longer form.

1. The exponent is positive so the decimal point is moved to the right.
2. The value of the exponent is eight. The decimal point is moved eight places to the right.
3. Six zeros are needed as place holders:

$6.87 \times 10^8 = 687\ 000\ 000$

Example: Write 10^5 in the longer form.

10^5 can be written as 1×10^5. The decimal point is after the one.

1. Since the exponent is 5, the decimal point is moved five places.
2. Since it is positive, it is moved to the right.
3. Five zeros are needed as place holders:

$1. \times 10^5 = 100\ 000$

Example: Write 9.38×10^{-4} in the longer form.

1. The exponent is negative so the decimal point is moved to the left.
2. The absolute (positive) value of the exponent is four. The decimal point is moved four places to the left.
3. Three zeros are needed as place holders:

$9.38 \times 10^{-4} = .000\ 938$

Example: Write 10^{-7} in the longer form.

1. 10^{-7} can be written as $1. \times 10^{-7}$. The decimal point is after the one.
2. The exponent has an absolute value of 7. The decimal point is moved seven places.
3. Move the decimal to the left since the exponent is negative.
4. Six zeros are needed as place holders:

$1 \times 10^{-7} = .000\ 0001$

WORKING WITH NUMBERS IN SCIENTIFIC NOTATION

Multiplication and division can be carried out directly with numbers written in scientific notation, but the final result must be expressed in the proper notation with the decimal point directly after the first digit.

Multiplication

To multiply using scientific notation, follow these steps:

1. Multiply the significant number parts of the numbers together.
2. Determine the power of ten by adding the exponents together
3. The answer is the result of the multiplications:
4. New significant number x $10^{\text{new exponent}}$

Remember: Multiplication can be indicated by a times sign (x), a raised dot ($\cdot$) or by writing one number in front of another number enclosed in brackets. Both numbers can also be enclosed in brackets.

7 x 3
$7 \cdot 3$
7(3)
(7)(3)

Operations to be done inside the brackets are done first, before the multiplication.

7(2+1)
7(3)

If several numbers are to be multiplied, you can do the multiplication in any order.

- *If two numbers in scientific notation are to be multiplied, you can group the decimal notation part of each number together and the powers of ten together*.
  - Multiply the decimal numbers and then multiply the powers of ten.

Example: Multiply $(4.3 \times 10^5)(7. \times 10^6)$

4.3 x 7 = 30.1	1. Multiply the decimal notation parts together.
$10^5 \times 10^6$	2. Group and multiply the powers of ten. Add the exponents: 5 + 6 = 11
$10^6 \times 10^5 = 10^{11}$	3. Use the addition answer as the exponent in your multiplication answer
	4. Put the two answers together with a times sign between them.

So $(4.3 \times 10^5)(7. \times 10^6)$
$= 4.3 \times 7 \times 10^5 \times 10^6$
$= (4.3 \times 7)(10^5 \times 10^6)$
$= 30.1 \times 10^{11}$

Notice that we can group numbers to be multiplied in different ways. We can write:
$4.3 \times 10^5 \times 7 \times 10^6$

We can rearrange the order of the numbers to be multiplied:
$4.3 \times 7 \times 10^5 \times 10^6$

We can group numbers in brackets to show what numbers we will multiply first:
$(4.3 \times 7)(10^5 \times 10^6)$

Now write the answer in correct scientific notation. (Reread page 2 to remember how.)

$(3.01 \times 10^1) \times 10^{11}$	Move decimal to the right of the first significant digit. Follow by a multiplication sign and the power of ten.
3.01×10^{12}	Combine the powers of ten.

Round the answer off to the correct number of significant figures:

1. If the answer has more significant figures than the original decimal numbers, round off the answer.
2. The answer should have the same number of significant figures as the original decimal number with the least number of significant figures.

Look at the example $(4.3 \times 10^5)(7. \times 10^6)$.

- The number 4.3 has two significant figures, but the number 7 has only one significant figure.
- Since 3.01 rounded off to one significant figure is 3, the final answer is: $3. \times 10^{12}$

If only one number has a power of ten, multiply the decimal numbers and write the original power of ten with the multiplication answer.

Example: $(4.2 \times 10^2)2.5$

$4.2 \times 2.5 = 10.50$	Multiply the decimal numbers:
10.50×10^2	Put the exponent with the answer:
$1.050 \times 10^1 \times 10^2$	Write in scientific notation:
$= 1.050 \times 10^3$	
$(4.2 \times 10^2)2.5 = 1.1 \times 10^3$	Round off to two significant figures:

Example: $(6.75 \times 10^8)(3.9 \times 10^5)$

$6.75 \times 10^8 \times 3.9 \times 10^5$
$= (6.75 \times 3.9)(10^8 \times 10^5)$ multiply
$= 26.325 \times 10^{13}$

Now write the answer in correct scientific notation.

$26.325 = 26$	3.9 has two significant figures so round off 26.325 so it has are only two significant figures rounded off to two places
$26 = 2.6 \times 10^1$	Move the decimal point so it is directly after the first digit 2.

The question now looks like this:
$2.6 \times 10^1 \quad \times 10^{13}$

$2.6 \times 10^1 \times 10^{13}$ Multiply the two powers of ten together by adding the exponents to get your final answer.

$= 2.6 \times 10^1 \times 10^{13}$
$= 2.6 \times 10^{14}$

Division
Note: *Division with numbers in scientific notation is usually indicated by writing the division question as a fraction. The dividend (the number you are dividing into) is written on top of the fraction line and the divisor (the number you are dividing by) is written underneath the fraction line.*

To divide using scientific notation follow these steps:
1. Divide the decimal numbers first.
2. Divide the powers of ten by subtracting the exponent of the divisor (the number under the fraction line) from the exponent of the dividend (the number on top) .
3. The answer should be expressed in proper scientific notation with the decimal directly after the first digit.
   - As in multiplying, round the decimal answer off to the same number of significant figures as the decimal number with the least number of significant figures in the question.

Example: Divide $9.66 \text{ x}10^{10}$ by $2.3\text{x}10^4$

$\frac{9.66 \text{ x}10^{10}}{2.3\text{x}10^4}$ $9.66 \div 2.3 = 4.2$ $10^{10} \div 10^4 = 10^6$

$=4.2 \times 10^6$

Addition and Subtraction
Note: *You can only add and subtract numbers in scientific notation if the powers of ten are the same.*

A. If both numbers have the same power of ten, follow these steps:
1. Add or subtract the decimal numbers.
2. Write the answer with the same power of ten as in the question.

Example: Add: $3.45 \times 10^{11} + 5.31 \times 10^{11}$

$$\begin{array}{r} 3.45 \times 10^{11} \\ +\ 5.31 \times 10^{11} \\ \hline 8.76 \times 10^{11} \end{array}$$

B. If the numbers do not have the same power of ten:

1. Write the numbers in long form.
2. Add or subtract.
3. Write the answer in correct scientific notation.

Example:

$$\begin{array}{r} 3.6 \times 10^{8} \\ \underline{+\ 4.5 \times 10^{13}} \end{array}$$

360 000 000	Rewrite in long form.
+ 45 000 000 000 000	Add.
45 000 360 000 000	Write the answer in correct scientific notation.

The answer is 4.50036×10^{13}

Order of Operations

When doing operations with several numbers in scientific notation, follow the proper order of operations. You can use the term BEDMAS to help you remember the correct order. When doing questions involving several mathematical operations, you do the operations in the following order:

1. **B**rackets. Do any operations in brackets first, except multiplying powers of ten. If there are brackets within brackets, do the innermost brackets first.
2. **D**ivision and
3. **M**ultiplication in the order in which they appear from left to right, but group the powers of ten where possible.
4. **A**ddition and
5. **S**ubtraction, in the order in which they appear from left to right.

And, finally: Always express your final answer in proper scientific notation. Round off to the correct number of significant figures and write the answer with the decimal point directly after the first digit.

Example:

Simplify: $\dfrac{3.78 \times 10^{3} \cdot \pi\ (6.10 \times 10^{8})}{5.42 \times 10^{4}}$

Multiplication is indicated by the x sign, the raised dot and by brackets in this question and π equals 3.14

$\dfrac{3.78 \times 10^{3} \times \pi \times 6.10 \times 10^{8}}{5.42 \times 10^{4}}$ Multiply above the division line first.

$= \dfrac{(3.78 \times 3.14 \times 6.1) \times (10^{3} \times 10^{8})}{5.42 \times 10^{4}}$ Group and multiply the decimal numbers
Group and multiply powers of ten by adding exponents.

$= \frac{72.402 \times 10^{11}}{5.42 \times 10^{4}}$

Group the decimal numbers and the powers of ten and then divide. Since 6.10 in the original question has three significant figures, you need three significant figures in the answer. Divide to four places.

$\frac{72.407}{5.42} \times \frac{10^{11}}{10^{4}}$

$= 13.35 \times 10^{7}$

Round off to three significant figures.

$13.35 \times 10^{7} = 13.4 \times 10^{7}$

Express in proper scientific notation.

$(1.34 \times 10^{1})10^{7}$

$= 1.34 \times 10^{8}$

Answer the following questions on scientific notation. The answers are on the last page.

1. Write each of these numbers in scientific notation:

a) 826 b) 300

c) 4050 d) 520 000

e) 3 667 000 f) 721 000 000 000

g) .45 h) .07932

i) .0081 j) .000546

k) .000 000 006 l) 100,000

m) 100 n) .000 01

o) .01 p) .000 000 001

2. Write the following numbers in their long form:

a) 5.446×10^4 b) 2.9×10^6

c) 1×10^5 d) 10^7

e) 10^2 f) 9.99×10^{-3}

g) 1.5×10^{-5} h) 1×10^{-4}

i) 10^{-6} j) 10^{-2}

3. The distance from the sun to the earth is about 93,000,000 miles. Write this distance in scientific notation. ______________________

4. The speed of light is 29,979,280,000 cm/s. Write this speed in scientific notation.
____

5. The size of a bacterial cell is .005 micrograms. Write this size in scientific notation.
____

6. The average distance between gas molecules in a room is .00000001 metres. Write this in scientific notation. ___________________

ANSWER PAGE

1. a) 8.26×10^{2} b) 3.0×10^{2}

   c) 4.05×10^{3} d) 5.2×10^{5}

   e) 3.667×10^{6} f) 7.21×10^{11}

   g) 4.5×10^{-1} h) 7.932×10^{-2}

   i) 8.1×10^{-3} j) 5.46×10^{-4}

   k) 6.0×10^{-9} l) 1×10^{5} or 10^{5}

   m) 1×10^{2} or 10^{2} n) 1×10^{-5} or 10^{-5}

   o) 1×10^{-2} or 10^{-2} p) 1×10^{-9} or 10^{-9}

2. a) 54,460 b) 2,900,000

   c) 100,000 d) 10,000,000

   e) 100 f) .00999

   g) .000015 h) .0001

   i) .000001 j) .01

3. 9.3×10^{7} mi.

4. 2.997928×10^{10} cm/s

5. 5×10^{-3} micrograms

6. 1×10^{-8} or 10^{-8} m