

**EVALUATING
ACADEMIC READINESS
FOR APPRENTICESHIP TRAINING**
Revised for
ACCESS TO APPRENTICESHIP

**MATHEMATICS SKILLS
ORDER OF OPERATIONS**

**AN ACADEMIC SKILL MANUAL
for
The Precision Machining and Tooling Trades:**

This trade group includes the following trades:
General Machinist, Tool & Die Maker,
Mould Maker, Pattern Maker, and
Machine-Tool Builder Integrator

*Workplace Support Services Branch
Ontario Ministry of Training, Colleges and Universities*

Revised 2011

In preparing these Academic Skills Manuals we have used passages, diagrams
and questions
similar to those an apprentice might find in a text, guide or
trade manual.

**This trade related material is not intended to instruct
you in your trade. It is used only to
demonstrate how understanding an academic skill will help
you find and use the information you need.**

MATHEMATICS SKILLS

ORDER OF OPERATIONS

*An academic skill required for the study of the
Precision Machining and Tooling Trades*

INTRODUCTION

In the machining trades, you need to know how to cut and shape pieces of metal, and to measure, fabricate and attach different pieces. You also need skills that don't depend on your ability to work with metal. You have to make estimates, read blueprints, and keep track of your expenses and income.

Example: To estimate the cost of a job, you calculate the cost materials used in the process and then determine the cost of labour. To determine the cost of making two identical parts, for example, you multiply the cost per part by two and add in the cost of fittings, finishings, and other materials. Then you calculate the cost of labour by multiplying the hourly rate by the number of hours the job will take. You are required to add, subtract, multiply and divide.

Does the order in which you do the addition, subtraction, multiplication or division make any difference? Let's look at an example of an equation that requires several operations to find the answer:

$$24 - 4 \times 5 + 10 \div 2 = ?$$

First, we will do the operations in the order they occur. The operations being done on each line are in brackets.

$$\begin{aligned} & (24 - 4) \times 5 + 10 \div 2 \\ & = (20 \times 5) + 10 \div 2 \\ & = (100 + 10) \div 2 \\ & = (110 \div 2) \\ & = 55 \end{aligned}$$

Second, we will do all multiplication and division first, then addition and subtraction:

$$\begin{aligned} & 24 - (4 \times 5) + (10 \div 2) \\ & = (24 - 20 + 5) \\ & = 9 \end{aligned}$$

Third, let's add and subtract first, then multiply and divide:

$$\begin{aligned} & (24 - 4) \times (5 + 10) \div 2 \\ & = (20 \times 15) \div 2 \\ & = (300 \div 2) \\ & = 150 \end{aligned}$$

Order Matters

We ended up with three different answers to the same question. The order in which we do the math operations affects the answers. To make sure everyone gets the same answer for the same math question, rules for the **order of operations** were developed.

In this skills manual we will describe these rules of order so you can accurately solve questions like the one above. The topics include:

- ◆ Order of operations for basic calculations,
- ◆ Brackets and exponents
- ◆ Decimal numbers
- ◆ Fractions

ORDER OF OPERATIONS FOR BASIC CALCULATIONS

Multiplication and Division, Addition and Subtraction

Here are the first two rules for **Order of Operations**:

1. *First, do all multiplication and division in the order in which they appear from left to right.*
2. *Next, do all addition and subtraction in order from left to right.*

According to this rule, the second example above gave the correct answer. Multiplication and division were done first in the order that they appeared; then addition and subtraction were done in the order that they appeared.

This means that every time you have to make a calculation where there are several kinds of operations, you follow this order.

Example:

$$24 - 4 \times 5 + 10 \div 2 = ?$$

- Work from left to right through the question
- Multiply the numbers on each side of any multiplication sign and divide the numbers on each side of the division signs.

$$24 - 20 + 5 = ?$$

- Rewrite the question, putting the multiplication and division answers in the question where they occur.

$$4 + 5 = 9$$

- Leave all addition and subtraction signs in their place.
- Add and subtract the numbers until you have a final answer.

Caution! *Be careful to read each question from beginning to end before you start making calculations. Then do the questions one step at a time. Write each step as you do it. Pay careful attention to the signs as you copy them. If you write down a subtraction sign instead of an addition sign, or a division sign instead of a multiplication sign, the error will make your answer incorrect. Be sure you are accurate.*

The correct order of operation is a basic math skill you will use all the time.

Example: You need to estimate the cost of a small job. In order to do the job, you need 15 feet of 2x4 inch steel bar that costs \$27.52 per foot. Other materials will cost \$90.49. You think the job will take 1½ hours at \$45.00 an hour.

First, write the calculation. Work from left to right through the question.

$$15 \times \$27.52 + \$90.49 + 1 \frac{1}{2} \times \$45$$

$$= \$412.80 + \$90.49 + \$67.50$$

$$= \$570.79$$

- You need to calculate the cost of the stock (**15 multiplied by \$27.52**) plus \$90.49 for other materials plus the labour cost (**1½ times \$45.00**).
- Multiply first. Multiply the numbers on each side of any multiplication sign and divide the numbers on each side of the division signs.
- Rewrite the question, putting the multiplication and division answers in the question where they occur.
- Leave all addition and subtraction signs in their place.
- Add and subtract the numbers until you have a final estimate.

You use your understanding of order of operations several times to make this estimate.

Let's look at some examples that illustrate the first two rules for order of operations. The operations of multiplication and division are highlighted with bold print to show they are done first.

Example:

$$\begin{aligned} 56 \div 7 - 6 + 5 \times 4 \div 2 \\ = 8 - 6 + 20 \div 2 \\ = 8 - 6 + 10 = 12 \end{aligned}$$

Example:

$$\begin{aligned} 9 + (36 \div 6) - (5 \times 3) \\ = 9 + 6 - 15 \\ = 0 \end{aligned}$$

BRACKETS AND EXPONENTS

The above rules for order of operations cover the basic situations. However, you might have to solve other questions that include brackets and exponents. We will look at the rules for order of operations concerning these elements.

Working with Brackets

Brackets, or parentheses, can be written as (), [] or { }. They are used to set off certain numbers, or to indicate multiplication.

In the second of the two examples above, brackets were used to indicate which operations were being done on each line. In this case, the brackets were simply removed as you completed each step in order to continue. However, questions with brackets are often more complicated.

Brackets affect the order of operations

Numbers grouped inside brackets in an equation affect the order of operations. When a question includes numbers inside brackets, use the following steps:

- ◆ **All operations inside the brackets are done first.**
 - This means do everything in the brackets – *even before you do any multiplication or division which may be outside of the brackets.*
 - Inside the brackets, follow the first rule for Order of Operations. First do multiplication and division in the order they appear, then do addition and subtraction.
- ◆ **Brackets can mean multiplication**
 - Any number directly in front of another number in brackets indicates that the two numbers are to be multiplied. *The brackets are considered a multiplication sign.*

Example:

$$5(21) \quad \text{means 5 multiplied by 21}$$
$$5(21) = 105$$

Sometimes there are several numbers and operations to be done inside brackets, and there is also a number in front of the bracket. You need to multiply by the number outside the brackets, **but** only after all the operations inside the brackets have been done, leaving a single number.

Example: Simplify $8(6 + 12 \div 4 - 1)$.

First, do all operations in the brackets, *in order*,. Then multiply by the number beside the brackets.

- | | |
|------------------------|--|
| $8(6 + 12 \div 4 - 1)$ | <ul style="list-style-type: none">• First do all operations inside the brackets.<ul style="list-style-type: none">▪ Multiply and divide first: $12 \div 4 = 3$▪ Rewrite the question – keep the brackets, and leave all addition and subtraction signs in their place |
| $8(6+3-1)$ | |
| $8(8)$ | <ul style="list-style-type: none">• Stay inside the brackets to do all addition and subtraction: $6+3-1$<ul style="list-style-type: none">▪ Rewrite the question – keep the results inside the brackets.. |
| 64 | <ul style="list-style-type: none">• Multiply those results by the number beside the brackets to get rid of the brackets.• This result is your final answer. |

Parentheses inside brackets

If there are parentheses inside brackets, *always start with the innermost brackets and work your way out*. Let's look at the steps used in this situation:

Example: Simplify $4\{3 + [7 + (4 - 2)]\}$

- | | |
|--------------------------|--|
| $4\{3 + [7 + (4 - 2)]\}$ | • First do the innermost grouping: $(4 - 2) = 2$ |
| $4\{3 + [7 + 2]\}$ | • Then do the next grouping: $[7 + 2] = 9$ |
| $4\{3 + 9\}$ | • Do the last grouping: $\{3 + 9\} = 12$. |
| $4\{12\} = 48$ | • There is now a single number in brackets.
• Multiply the 12 in brackets by the number in front. |

Example: Simplify $4(6 + 3) - (6 - 3)$.

- | | |
|---------------|---|
| $4(9) - (3)$ | • Do operations in each set of brackets first. |
| $36 - 3$ | • Only the number inside the first set of brackets needs to be multiplied by the number in front.
• Simply remove the brackets from the second number: $4 \times 9 = 36$ |
| $36 - 3 = 33$ | • Finally, subtract. |

Example: Simplify $9[14 - 3(2 + 1) \div 3]$.

- | | |
|---------------------------|---|
| $9[14 - 3(2 + 1) \div 3]$ | • Start with the inside brackets: $2 + 1 = 3$ |
| $= 9[14 - 3(3) \div 3]$ | • Multiply and divide inside the brackets: $3(3) = 9$ |
| $= 9[14 - 9 \div 3]$ | • Then do the next grouping: $9 \div 3 = 3$ |
| $= 9[14 - 3]$ | • Subtract inside the brackets. |
| $= 9[11]$ | • $14 - 3 = 11$ |
| $= 99$ | • Finally, multiply to get the final answer. |

As you gain confidence, you can combine several steps in the same line, but keep the order of operations straight and be accurate with the operation signs in each step. In the next example, everything in the inner bracket is done in one step, then everything in the outer bracket is done in one step.

Example:

$$\begin{aligned} & 15 + 3[4 + 2(45 \div 5 - 6) - 8] \\ & = 15 + 3[4 + 2(3) - 8] \\ & = 15 + 3[2] \\ & = 21 \end{aligned}$$

In the following examples, we have highlighted the operations in each step so you can follow the order. Sometimes more than one step is done in a line.

Example:

$$\begin{aligned} & 42 \div 14 + 2(16 - 18 \div 3) - 16 \\ & = 42 \div 14 + 2(16 - 6) - 16 \\ & = 42 \div 14 + 2(10) - 16 \\ & = 3 + 20 - 16 \\ & = 23 - 16 \\ & = 7 \end{aligned}$$

Example:

$$\begin{aligned} & 5[189 \div 9 (17 - 7 \times 2) - 1] \div 2 \\ & = 5[189 \div 9(17 - 14) - 1] \div 2 \\ & = 5[189 \div 9(3) - 1] \div 2 \\ & = 5[21(3) - 1] \div 2 \\ & = 5(63 - 1) \div 2 \\ & = 5(62) \div 2 \\ & = 310 \div 2 \\ & = 155 \end{aligned}$$

Note: Formulas use letters to represent different quantities whose amounts can vary. If these quantities are to be multiplied the two letters representing the quantities are written right beside each other, instead of using a multiplication sign or putting one number in brackets. A common example is the formula for area of a rectangle, usually written as: $A = lw$

A stands for area, l stands for length and w stands for width. To find the area, multiply the length and width. This is indicated in the formula by writing the letters for length and width beside each other, as lw .

Example: At work, you are welding three metal panels. One panel measures 3.75 feet by 1.5 feet, one measures 2.5 feet by 1.75 feet and one measures .6 feet by 1.45 feet. You want to calculate the total area of metal.

To find the total area, the formula is:

$$\text{Total area} = (lw)_1 + (lw)_2 + (lw)_3$$

$$A = (3.75 \times 1.5) + (2.5 \times 1.75) + (.6 \times 1.45)$$

$$A = 5.625 + 4.375 + .87$$

$$A = 10.87$$

- First fill in the given amounts.
 - Remember that letters written beside each other in the formula (lw) indicate multiplication.
- Using the rules for order of operations, do the calculations inside the brackets first.

The total area is 10.87 square feet.

Here are some examples for you to fill in the blanks:

1. $64 \div 8 + 10 \times 3$

= $\underline{\quad}$ + $\underline{\quad}$

= 38

(Did you get 8 + 30 in the blanks?)

2. $4(45 \div 9 - 3) + 8(6 - 3) - 12$

= $4(\underline{\quad}) + 8(\underline{\quad}) - 12$

(Did you get 2 and 3 in the blanks?)

= $\underline{\quad} + \underline{\quad} - 12$

= 20

Exponents

An exponent is a short way of indicating how many times a number is to be multiplied by itself. It is written as a small number to the right of and slightly above the number to be multiplied. In the expression 5^2 , the exponent 2 tells you to multiply the 5 by itself *two* times, as $5 \times 5 = 25$.

You use exponents in some formulas. For example, to calculate the area of a circular patch, you use the formula:

$$\text{Area} = \pi r^2,$$

which contains the exponent r^2 .

If there is an exponent in an equation, the following rule determines what order to use:

Do the operations inside any brackets first, then multiply any exponents.

Example: $9 + 7 - 4^2 \div 2$

= $9 + 7 - 16 \div 2$

There are no brackets so multiply the exponent first: $4^2 = 4 \times 4 = 16$

= $9 + 7 - 8$

Continue, following order of operations

= 8

If there is an exponent just after and outside brackets, do the operations inside the brackets and multiply that answer by the exponent.

Example:

$8(4 - 1 + 2)^2 + 9$

brackets first

= $8(5)^2 + 9$

exponent next, $5^2 = 5 \times 5$

= $8(25) + 9$

Multiply

= $200 + 9$

Add

= 209

DECIMAL NUMBERS

The rules for order of operations apply whether you are working with whole numbers, decimal numbers or fractions.

Example:

$$\begin{aligned} & 1.5(5.6 - 2.6) + (5 - 3) - 2.5 \div .5 \\ & = 1.5(3) + (2) - 5 \\ & = 4.5 + 2 - 5 \\ & = 1.5 \end{aligned}$$

FRACTIONS

Remember that a fraction line indicates division.

Treat the fraction line like a set of brackets

Do the operations above and below the line *before* you divide the answer above the line by the answer below the line.

Example:

$$\frac{6(20 \div 5) + 48 \div 8 - 2 \times 5}{16 \div 8 + 2(9) - 10}$$

$$\frac{6(20 \div 5) + 48 \div 8 - 2 \times 5}{168 + 2(9) - 10}$$

Do inside brackets first, above and below the fraction line.
Then divide and multiply.

$$= \frac{6(4) + 6 - 10}{2 + 18 - 10}$$

$$= \frac{24 + 6 - 10}{20 - 10}$$

Add and subtract.

$$= \frac{20}{10}$$

Now divide

$$= 2$$

SUMMARY

You use the rules for order of operations in all your calculations, especially those involving formulas. Here is a summary of the rules for order of operations:

1. **Brackets:** If there are brackets, do all operations inside the brackets first, following the rules below.
 - a. If there are rounded brackets () within square brackets [], work out answers for the innermost brackets first.
2. **Exponents:** Multiply out numbers with exponents.
3. **Division and Multiplication:** Do all division and multiplication (remember brackets can mean multiplication) in the order in which they are written from left to right.
4. **Addition and Subtraction:** Do all addition and subtraction next in the order in which they are written from left to right to get a final answer.
5. **If numbers are presented on both sides of a fraction line,** work out a final answer on each side, then divide the top number by the bottom one to get a final answer.

And remember, in each step, be accurate with the operations signs.

Bedmas

Some people use the word **bedmas** to remember correct order. The letters in **bedmas** list the first letters of the operations in the order in which they should be done.

b	brackets
e	exponents
d	division
m	multiplication
a	addition
s	subtraction

Note: Division is listed before multiplication in bedmas, but do these operations in the order in which they appear from left to right. Addition is listed before subtraction, but do these operations in the order in which they appear.

RULES FOR ORDER OF OPERATIONS

BEDMAS

1. **Brackets:** Do all operations in parentheses first, starting with the innermost group, using the rules for the order of operations below.
2. **Exponents:** Multiply any exponents.
3. **Division and Multiplication:** Do all multiplication and division next in the order in which they appear from left to right, remembering that a number in front of a number inside brackets indicates the two numbers are to be multiplied.
4. **Addition and Subtraction:** Do all addition and subtraction in the order in which they appear from left to right.
5. **Pay attention to the signs.**

Find the value of each of the following. Answers are on the last page.

1. $64 \div 8 + 5 \times 4 =$

2. $6 + 4 \times 7 - 15 =$

3. $9 \times 6 - 24 + 40 \div 8 + 3 \times 2 =$

4. $36 \div 6 + 3(5 \times 5) =$

5. $7(42 \div 6) + 18 - 2(14 - 9) =$

6. $\$5.59 + \$4.60(3 - 6 \div 3 + 2) - \$9.99 =$

7. $7.2 \div 8 + 3.6(6.6 - 1.1) + 1.2 =$

8. $\frac{3(25 \div 5) + 2 \times 9 - 12 \div 4}{6 \times 3 - 49 \div 7 + 2(2)} =$

ANSWER PAGE

1. $64 \div 8 + 5 \times 4$
 $= 8 + 20$
 $= 28$

2. $6 + 4 \times 7 - 15$
 $= 6 + 28 - 15$
 $= 19$

3. $9 \times 6 - 24 + 40 \div 8 + 3 \times 2$
 $= 54 - 24 + 5 + 6$
 $= 41$

4. $36 \div 6 + 3(5 \times 5)$
 $= 6 + 3(25)$
 $= 6 + 75$
 $= 81$

5. $7(42 \div 6) + 18 - 2(14 - 9)$
 $= 7(7) + 18 - 2(5)$
 $= 49 + 18 - 10$
 $= 57$

6. $5.59 + \$4.60(3 - 6 \div 3 + 2) - \9.99
 $= \$5.59 + \$4.60(3 - 2 + 2) - \$9.99$
 $= \$5.59 + \$4.60(3) - \$9.99$
 $= \$5.59 + \$13.80 - \$9.99$
 $= \$9.40$

7. $7.2 \div 8 + 3.6(6.6 - 1.1) + 1.2$
 $= 7.2 \div 8 + 3.6(5.5) + 1.2$
 $= .9 + 19.8 + 1.2$
 $= 21.9$

8. $\frac{3(25 \div 5) + 2 \times 9 - 12 \div 4}{6 \times 3 - 49 \div 7 + 2(2)}$
 $= \frac{3(5) + 18 - 3}{18 - 7 + 4}$
 $= \frac{30}{15}$
 $= 2$