EVALUATING
ACADEMIC READINESS
FOR APPRENTICESHIP TRAINING
Revised for
ACCESS TO APPRENTICESHIP

MATHEMATICS SKILL
CALCULATION OF PERIMETER, AREA AND VOLUME OF GEOMETRIC FIGURES

AN ACADEMIC SKILLS MANUAL
for
The Precision Machining And Tooling Trades

This trade group includes the following trades:
General Machinist, Tool & Die Maker,
Mould Maker, Pattern Maker, and
Machine-Tool Builder Integrator

Workplace Support Services Branch
Ontario Ministry of Training, Colleges and Universities

Revised 2011

In preparing these Academic Skills Manuals we have used passages, diagrams and questions similar to those an apprentice might find in a text, guide or trade manual.

This trade related material is not intended to instruct you in your trade. It is used only to demonstrate how understanding an academic skill will help you find and use the information you need.

MATHEMATICS SKILLS: CALCULATION OF PERIMETER, AREA & VOLUME OF GEOMETRIC FIGURES

An academic skill required for the study of the Precision Machining and Tooling Trades

INTRODUCTION

An important part of a machinist's job involves measurements.

The different challenges of your trade require you to measure various shapes. For example, to install a new panel, you first have to measure the length carefully and compare your measurement to the reference values. If they match, you attach the panel temporarily and then compare the height with that on the other side. Then you have to continue by checking diagonal and width measurements until you are sure the panel is positioned correctly. Only then do you weld it in place.

While at work, you will read and scribe measurements from drawings to fabricate parts. You might use measurements to calculate the perimeters of shapes as you build or repair them. Or you will calculate the area of the side of a workpiece.

This skill sheet reviews the steps in finding the perimeter, area and volume of simple two and three dimensional geometric figures, including:

- Two dimensional figures
- Finding the perimeter
- Finding the area
- Three dimensional figures
- Finding the surface area
- Finding the volume
- Calculating the cost of covering an area

TWO DIMENSIONAL GEOMETRIC FIGURES

A simple, closed, two dimensional (flat) figure with three or more straight sides is called a **polygon**.

- Triangles, squares, rectangles, and parallelograms (figures with 2 pair of opposite sides parallel) are all examples of polygons.

A **circle** is also a flat, closed figure but it is a curve, consisting of points that are all the same distance from the center.

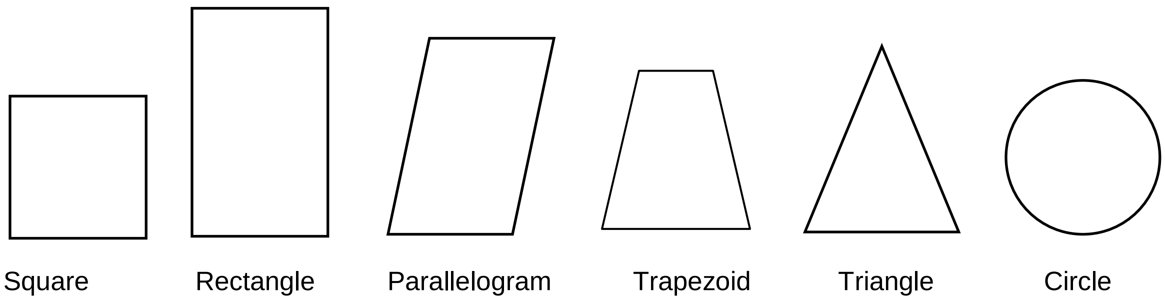


FIGURE 1: Some Simple Geometric Shapes

These figures can be measured in different ways.

- Whenever we use measurements to make calculations with geometric figures, all measurements must be in the same linear units.
  - The units might be meters or centimeters, but they can't be a mix of meters and centimeters.

FINDING THE PERIMETER

*The **perimeter** (P) of any polygon is the distance around its boundary*. Perimeter is found by adding together the lengths of the sides.

Perimeter of a Rectangle

*A **rectangle** is a polygon with four 90°angles (right angles) and with each pair of parallel sides the same length* (see Figure 1).

- This means that we can find the perimeter of a rectangle by adding the lengths of the two long side to the lengths of the two shorter side.

The perimeter of a rectangle equals twice the length (*l*) added to twice the width (*w*). The formula is written in two forms:

$P = 2l + 2w$ where P is the perimeter, *l* is the length and *w* is the width of the rectangle.

or $P = 2(l + w)$

Remember: *When finding perimeter, all units must be the same. If the length is measured in feet and the width in yards, one unit must be changed to that of the other.*

Example: Find the perimeter of a sheet of aluminum that is 4 feet long and 2 ½ feet wide.

$$\begin{aligned} P &= 2l + 2w \\ &= 2(4\text{ ft}) + 2(2\tfrac{1}{2}\text{ ft}) \\ &= 8\text{ ft} + 5\text{ ft} \\ &= 13\text{ ft} \end{aligned}$$

The perimeter is 13 ft.

Example: Find the perimeter of a shop that is 30 m long and 16 m wide.

P = 2(*l* + *w*)
= 2(30 m + 16 m)
= 2(46 m)
= 92 m

The perimeter is 92 m.

Example: Find the amount of fencing required to close in a space that is 400 yd wide and 1500 ft long.

Known:
l = 1500 ft
w = 400 yd = 1200 ft   400 yd x3 = 1200 ft

Find perimeter (P)

P = 2(*l* + *w*)
= 2 (1500 ft + 1200 ft)
= 2(2700 ft)
= 5400 ft

The space will require 5400 ft of fencing.

Perimeter of a Square
A square is a rectangle with all four sides the same length.

To find the perimeter of a square, multiply the length by 4.

Perimeter of a square = 4*l*

Perimeter of a Parallelogram and a Trapezoid
A ***parallelogram*** has two pair of opposite sides which are parallel. See Figure 1.

- The pairs of opposite sides are equal in length.

A ***trapezoid*** has only one pair of opposite sides that are parallel.

- The other pair of opposite sides are not parallel.
- None of the sides are the same length.

To find the perimeter of a parallelogram or a trapezoid, measure the lengths of the four sides and add them together.

In the same way, ***to find the perimeter of any irregular shape***, measure and add all the lengths together. Just make sure all the measurements are in the same units.

Finding the Length of an Unknown Side When the Perimeter Is Known

If you know the perimeter of a rectangle and the length of one side, you can find the other side.

1. Manipulate (or rearrange) the variables in the formula for perimeter so the letter for length or width is by itself on the left side.
2. Solve to find the unknown side.
3. *Remember, whatever you do to one side of the formula, you need to do to the numbers and letters on the other side.*

Example: The perimeter of an opening is 3 m. The length is 1 m. What is the width?

Known:
P = 3 m
l = 1 m

Find w

$P = 2l + 2w$
$3 = 2(1) + 2w$
$3 = 2 + 2w$

$2 + 2w = 3$	Reverse the equation.
$2 - 2 + 2w = 3 – 2$	Subtract 2 from both sides.
$2w = 1$	Divide both sides by 2.
$w = ½$ m	Write in the unit

The width is ½ m.

FINDING THE AREA

The ***area*** *of a polygon is the measure of the surface inside the boundary*. The units of area are squared units.

Area of a Rectangle

The area of a rectangle is the amount of surface enclosed within its boundaries of ***length*** and ***width***.

Example: The area of a room is the amount of floor space it has.

Area is calculated by multiplying the length of the rectangle times its width.
The formula for area is:

$A = lw$

Remember: *When finding the area of a rectangle, the units used to measure the length and the width must be the same. If the length is in meters, the width must also be in meters. If the units are different, one must be converted to the other before you can multiply.*

Example: Find the area of a rectangle that is 52 cm long and 44 cm wide.
(The units are the same so we don't have to convert.)

Draw the rectangle

Known:
l = 52 cm
w= cm

Find:
Area
Use A = lw

A = *lw*
= 52 cm x 44 cm
= 2288 cm^2

Note: When two of the same units are multiplied together, such as the centimeters in our example, they become square units. Instead of writing square centimeters, you can use the short form of cm^2 or sq cm. (Sq is the short form for square.) Four square feet is written 4 sq ft or 4 ft^2.

Example: Find the area of a space with length 5 m and width 142 cm.

We must convert one of the units so both are the same.

Known:
l = 5 m
w = 142 cm or 1.42 m

Find:
Area
Use A = lw

A = *lw*
A = 5 m x 1.42 m
A = 7.1 m^2

Example: Find the floor space of a box that measures 60 inches long by 40 inch wide by 20 inches high.
(The information on height is not needed to answer this question.)

Known
l = 40 in
w = 20 in

Find
Area
Use A= lw

A = *lw*
= 60 x 40
= 2400 sq in

Example: Find the floor space of a garage that measures 10 m long by 5 m. wide.

A = *lw*
= 10 x 5
= 50 m^2

Area of a Square

The four sides of a square are all the same length. To find the area of a square, square the length. (To square a number, multiply it by itself. Three squared is 3 x 3 = 9.)

Example: Find the area of a square with sides 5 ft long.

Known: l = 5 ft
w = 5 ft

Find A

A = lw or l^2
A = 5 ft x 5 ft
A = 25 sq ft

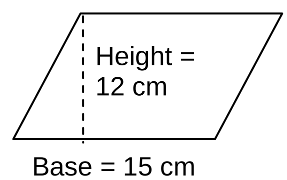
Area of a Parallelogram and a Trapezoid

Parallelogram: The area of a parallelogram is equal to the altitude or height times the base. The formula is:

A = ab or bh

Example: Find the area of a parallelogram with a height of 12 cm and a base of 15 cm.

Draw and label a parallelogram

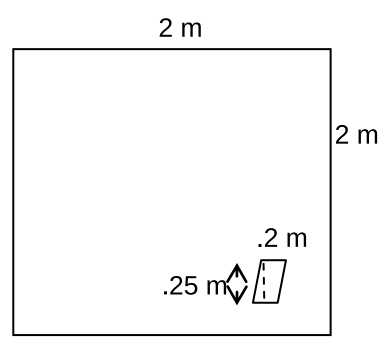


Known: b = 15 cm
h = 12 cm

Find A

A = bh
= 15 cm x 12 cm
= 180 cm^2

Example: A parallelogram with a base of 20 cm and a height of 25 cm is cut from a square sheet of metal that is 2 m on each side. How much of the sheet will be left over?



Draw and label the shapes.

Known:
sheet of metal sides = 2m
parallelogram b = 20 cm = .20 m
h = 25 cm = .25 m

Find: Area of the metal sheet
Area of the parallelogram
Amount of sheet metal that will be left
(Area of sheet – Area of parallelogram)

The area of the sheet is:

$A = l^2$
$= 2^2$
$= 4 \text{ m}^2$

Now find the area of the parallelogram. The cm should be changed to m.

20 cm = .2 m
25 cm = .25 m
$A = bh$
$= .2 \times .25$
$= .05 \text{ m}^2$

The area of the parallelogram is subtracted from the area of the sheet of metal.

$4 \text{ m}^2 - .05 \text{ m}^2 = 3.95 \text{ m}^2$

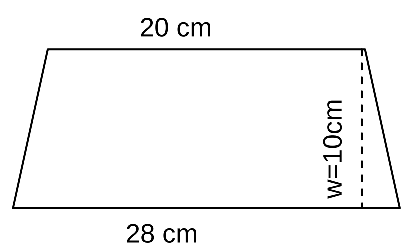
There are 3.95 m^2 left over.

Trapezoid: The area of a trapezoid is equal to the width times the average of the two parallel sides (bases). The formula is:

$$A = w\left(\frac{b_1 + b_2}{2}\right)$$

Example: Find the area of a trapezoid with parallel bases of 20 cm and 28 cm and a width of 10 cm.

Draw and label the trapezoid.



Known: Find the area
$b_1 = 20$ cm
$b_2 = 28$ cm
w = 10 cm

$$A = w\left(\frac{b_1 + b_2}{2}\right)$$

$$A = 10 \text{ cm}\left(\frac{20 \text{ cm} + 28 \text{ cm}}{2}\right)$$

$$A = 10\left(\frac{48}{2}\right)$$

$= 10 \times 24$
$= 240 \text{ cm}^2$

Finding the Length of an Unknown Side When the Area Is Known

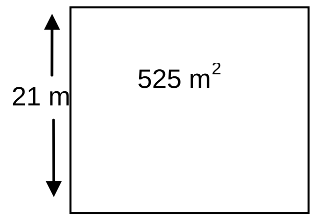
If you know the area of a rectangle and the width of one side, you can find the length of the other side.

1. Manipulate the variables in the formula for area so the letter for length ends up by itself on the left side.
2. Substitute the known measurements for area and width.
3. Solve to find the unknown side.

You can substitute the given measurements either before or after you manipulate the formula.

Example: If the area of a rectangle is 525 m^2 and the width is 21 m, what is the length?

Known: Find l
w = 21 m
A = 525 m^2



A = lw

$525m^2 = l \times 21m$.

$l \times 21\ m = 525m^2$ Reverse the equation.

$l \times \frac{\overset{1}{\cancel{21m}}}{\underset{1}{\cancel{21m}}} = \frac{\overset{25}{\cancel{525m^2}}}{\underset{1}{\cancel{21m}}}$ Divide both sides by 21 m to isolate l on the left.

When you divide a squared unit by a linear unit, such as square meters by meters, the meters on the bottom cancel one of the units on the top, leaving meters in the answer.

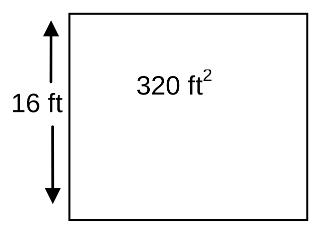
$l = 25$ m

The length is 25 meters.

Example: Find the length of a room that has an area of 320 sq ft and a width of 16 ft.

Known:
w = 16 ft
A = 320 ft^2

Find l



First rearrange the letters of the formula so l is by itself on the left.

A = lw
lw = A Reverse the equation.

l = A/w Divide both sides by w.

$l = \frac{320\text{ ft }(\cancel{\text{ft}})}{16\ \cancel{\text{ft}}}$ Fill in the given amounts. Divide. Cancel the units where possible.

l = 20 ft
The length is 20 ft.

THREE DIMENSIONAL FIGURES

A closed, solid geometric figure has three dimensions. It has length, width and height or depth. Some solid figures are the cube, the rectangular solid, the cylinder, the cone and the sphere.

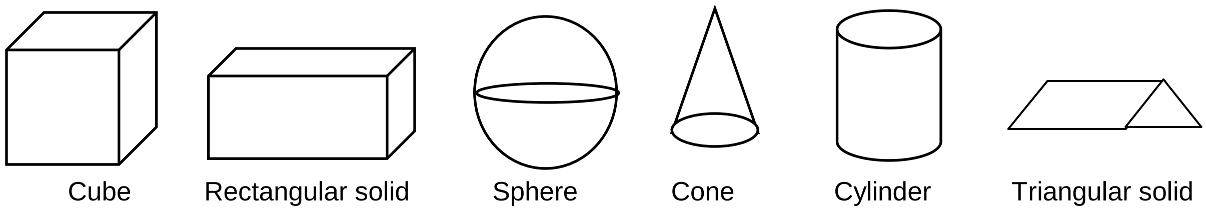


FIGURE 2: Solid Geometric Figures

SURFACE AREA OF THREE DIMENSIONAL FIGURES

The surface area of a three dimensional figure is the combined areas of all the outside surfaces or faces of the figure. When finding the surface area, all measurements must be in the same linear units. The answer will be in square units.

Finding the surface area of a rectangular solid

To find the total area of the outside surface of a rectangular solid, we have to find the areas of each face of the figure.

1. First find the area of the front surface by multiplying the length times the height.
   - The back surface is the same area, so multiply that answer by 2.
2. Next find the area of one side by multiplying the width times the height.
   - Since the opposite side is the same, multiply the answer by 2.
3. Now find the base by multiplying the length times the width.
   - The top is the same as the base, so multiply that answer by 2 also.

The formula is:

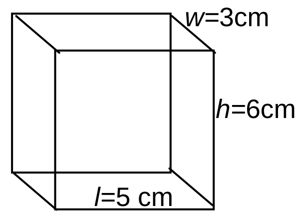
$A = 2lh + 2wh + 2lw$

or $A = 2(lh + wh + lw)$

or $A = 2(lh + wh + lw)$

Example: Find the total area of the outside surface of a rectangular solid 5 cm long, 3 cm wide and 6 cm high.

Draw and label the solid



Known:
l = 5 cm
w = 3 cm
h = 6 cm

Find:
Outside surface area of the solid A = 2(lh + wh + lw)

$A = 2\,(lh + wh + lw)$
$= 2(5cm \times 6cm + 3cm \times 6cm + 5cm \times 3cm)$
$= 2(30\ cm^2 + 18\ cm^2 + 15\ cm^2)$
$= 2(63\ cm^2)$
$= 126\ cm^2$

Finding the surface area of a cube

A cube is made of six identical squares. Each edge is the same length, each side has the same area.

To find the area of a cube:

1. Find the area of one side (l^2) and multiply it by 6.

The formula is:

$$A = 6\,(l^2)$$

Example: Find the total surface area of a cube whose edges measure 10 in.

Known:
Edges of cube = 10 in

Find:
Surface area of cube $A = 6(l^2)$

$A = 6\,(l^2)$
$= 6(10^2)$
$= 6(100)$
$= 600$ sq in.

Finding the surface area of a cylinder

The surface area of a cylinder consists of the outside curved surface, which is actually a rectangle if it is straightened, and the circular areas at the top and bottom.

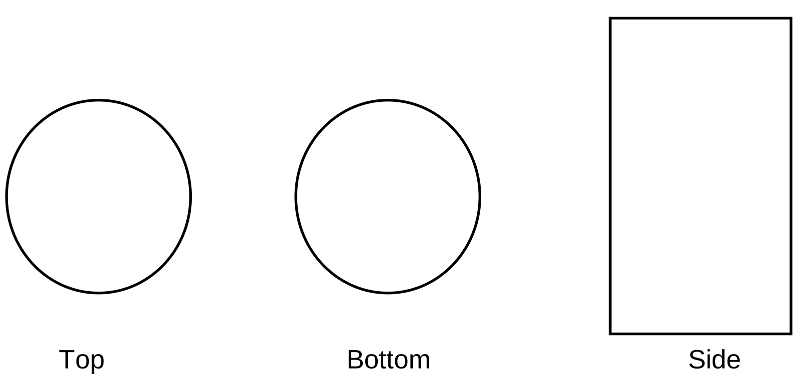


FIGURE 3: Finding the surface area of a cylinder

To find the surface area of a cylinder:

1. Find the area of each of the top and bottom circles.

2. Find the area of the rectangular side:
3. Add the areas together.

1. **To find the area of the top and bottom:** Use the formula $A = \pi r^2$. A cylinder has two circles (the top and the bottom), so we need to find the two areas, $2\pi r^2$.

Remember: *$\pi = 3.14$*

2. **To find the area of the side of the cylinder (a rectangle):** Multiply the length times the width.

   The formula is: $A = lw$.

   This rectangle has a width equal to the height of the cylinder so substitute height (*h*) for the width.

   The formula is now: $A = 2\ lh$.

   The length of the rectangle is the same as the perimeters of the circles at the top and bottom. We find the perimeter of a circle using the formula $P = 2\pi r$. Substitute this formula for the length of the rectangle.

   The formula becomes $A = 2\pi rh$.

3. **To find the area of the cylinder** add the areas of the top and bottom ($2\pi r^2$) to the area of the rectangle ($2\pi rh$).

   $A = 2\pi r^2 + 2\pi rh$.

4. **The formula is rearranged** to become:

   $A = 2\ \pi r(r + h)$

   Example: Find the surface area of a cylinder when its radius is 8 ft and its height is 20 ft.

   Known:
   r of cylinder = 8 ft
   h of cylinder = 20 ft

   Find the surface area of the cylinder

   $A = 2\ \pi r(r + h)$
   $= (2 \times 3.14 \times 8)(8 + 20)$
   $= (50.24)\ (28)$
   $= 1406.72$ sq ft

Finding the surface area of a sphere
A sphere is a ball. The surface area of a sphere is equal to 4 times π times the radius squared. The formula is:

$A = 4 \pi r^2$

Example: Find the surface area of a sphere with a radius of 5 cm.

Known
r = 5 cm

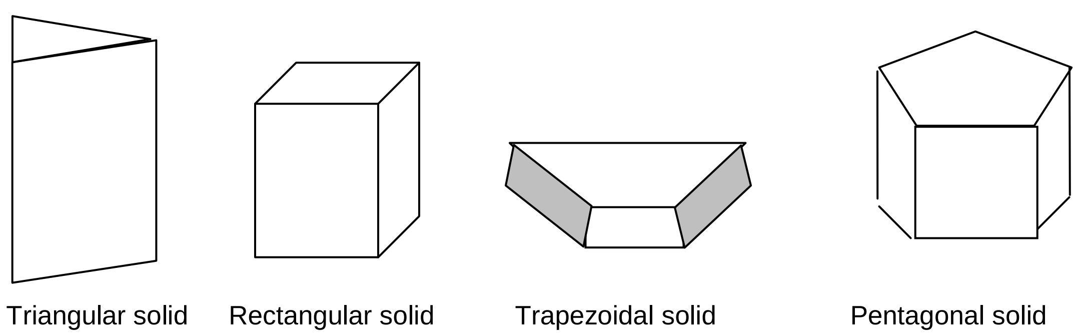
Find the surface area of the sphere

$A = 4 \pi r^2$
$= 4 \times 3.14 \times 5^2$
$= 314 \text{ cm}^2$

VOLUME OF THREE DIMENSIONAL FIGURES

The ***volume*** *or capacity* of a solid figure is the amount of space contained within its boundaries.

Regular solids
A regular solid is a three dimensional object with straight sides. A solid is named for the shape of its base.



To calculate volume of a solid, multiply the area of its base by its height.

Since each linear measurement has a unit, the units in the answer become cubic units. Area (with units meters x meters) x height (with units meters) equal cubic meters. The short form for cubic units such as cubic inches is in^3 or cu in.

Volume of a Rectangular Solid
The volume of a rectangular solid equals the length times the width times the height. The formula is:

$V = lwh.$

Example: Find the volume of a rectangular solid 9 cm long, 4 cm wide and 3 cm high.

$V = lwh$
$= 9 (4) (3)$
$= 108 \text{ cm}^3$

Volume of a Cube

The volume of a cube equals the length of one edge cubed. The formula is:

$V = l^3$

Example: Find the volume of a cube whose length measures 2 m.

$V = l^3$
$= 2^3$
$= 8 \text{ m}^3$

Volume of a Triangular Solid

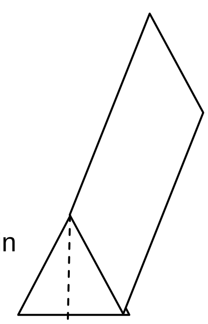
To find the volume (V) of a triangular solid, multiply the length (*l*) by the area (*A*) of the triangle. The area of the triangle equals ½ base times height.

$V = ½ bhl$

or,

$V = lA$

Example: Find the volume of the triangular solid in the diagram.



Known:
b = 2 in
h = 3 in
l = 7 in

You can solve this problem by using either one of the formulae.

$V = 1/2\ bhl = ½(2\text{in} \times 3\text{in} \times 7\text{in})$
$V = 1/2\ (42\text{in}^3)$
$V = 21 \text{ in}^3$

or

$V = lA$

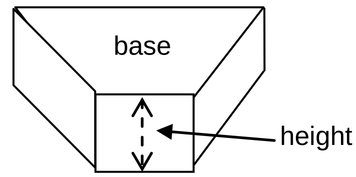
$A = 1/2\ bh$
$= ½(2" \times 3")$
$= 3 \text{ in}^2$

$V = l(Area)$
$= 7\text{in}(3\text{in}^2)$
$= 21 \text{ in}^3$

Volume of a Trapezoidal Solid

The volume of a trapezoidal solid is equal to the height times the area of the base. The formula is:

$V = Ah$



Since the area of a trapezoid is:

$$A = w\left(\frac{b_1 + b_2}{2}\right)$$

the formula can be written:

$$V = w\left(\frac{b_1 + b_2}{2}\right)h$$

Example: Find the volume of a trapezoidal solid with the following dimensions:

width = 8 cm
length of the first base = 10 cm
length of second base = 16 cm
height = 20 cm

$$V = w\left(\frac{b_1 + b_2}{2}\right)h$$

$$V = 8\left(\frac{10 + 16}{2}\right)20$$

= 8 x 13 x 20

= 2080 cm^3

Volume of a Cylinder

The volume of a cylinder equals π times the square of the radius of the base times the height. The formula is:

$V = \pi r^2 h$

Example: Find the volume of a cylinder with a radius of 12 ft and a height of 72 in.

72 in ÷ 12 = 6 ft   Change the units of height to feet by dividing by 12.

Now use the formula.

$V = \pi r^2 h$
$= 3.14\ (12^2)\ (6)$
= 2713 cu ft

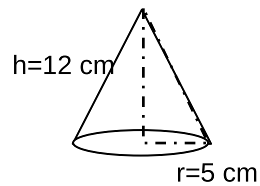
Note: *A volume of 1000 cm^3 of liquid is equivalent to 1 liter of the liquid. If you know the volume, you can calculate the number of liters of liquid.*

Volume of a Cone
The base of a cone forms a circle. The volume of a cone is equal to 1/3 times the area of the circular base times the height. The formula is:

$V = 1/3\ \pi\ r^2\ h$

Example: Find the volume of a cone with a height of 12 cm and a radius of 5 cm.

$V = 1/3\ \pi\ r^2\ h$
$= 1/3\ (3.14)\ (5^2)\ (12)$
$= 1/3\ (3.14)\ (25)\ (12)$
$= 314\ cm^3$



Volume of a Sphere
The volume of a sphere equals 4/3 times *π* times the cube of the radius. The formula is:

$$V = \frac{4\pi r^3}{3}$$

Example: Find the volume of a sphere with a radius of 10 inches.

$$V = \frac{4\pi r^3}{3}$$

$$V = \frac{4(3.14)(10^3)}{3}$$

$= 4186.67$ cu in

CALCULATING THE COST OF COVERING AN AREA

Once you have found the area of a surface, you might have to calculate the cost of covering it with a material like steel, or flooring, or paint. The cost of the material is usually given as a rate, such as $16.95 per square meter, or $12.45 per can where a can of paint covers 150 sq ft.

To find the cost of covering an area:
1. First calculate the area to be covered.
2. Then multiply the area by the cost per unit area.

Example: Find the cost of installing a metal top on a storage box that measures 4.6 m long and 3.5 m wide. The metal costs $24.99 per square meter.

Known:
Top l = 4.6 m
w = 3.5 m
Cost of metal 24.99 per m^2

Find:
Area of the top (A = lw)
Cost of metal (Area x $24.99/$m^2$)

First find the area:

A = *lw*
= 4.6 m x 3.5 m
= 16.1 sq m

Find the cost of the metal:

16.1 sq m x $24.99/sq m = $402.34
(Notice that the square meters cancel.)

The cost to cover the box is $402.34.

Example: It takes 3 cans of spray paint to cover the outside of a metal box. Each can covers approximately 10 sq ft. What is the approximate surface area of the box? What will it cost to cover the box if each can costs $4.99.

Known:
1 can covers 10 ft^2
It takes 3 cans to cover the box
1 can paint costs $4.99

Find:
The approximate surface area of the box.
The cost to cover the box.

If one can of paint covers 10 ft^2, 3 cans will cover three times as much.

3 x 10 = 30 ft^2

The surface area of the box is about 30 sq ft.

If 1 can costs $4.99, 3 cans will cost:

3 x $4.99 = $14.97

Answer the following questions about geometric figures. Answers are at the end of this manual.

1. Find perimeter of a rectangle that measures 4.5 m by 6 m.

2. Find the amount of metal needed to go around a filter that is 48 inches by 24 inches.

3. Find the area of a piece of metal that measures 15 inches by 20 inches.

4. Find the area of a rectangle that is 3 m long and 100 centimeters wide.

5. Find the area of a parallelogram that is 7.2 cm long and 4.7 cm wide.

6. Find the area of a square with sides that are 16 inches long.

7. Find the cost of putting a metal floor on a trailer that is 8 feet long and 5 1/2 feet wide if the sheet metal costs $5.95 per square foot.

8. If a sheet of metal measures 4 ft by 8 ft, find the amount required to make a rectangular metal box that has a height of 48 in, a width of 48 in and a length of 60 in. (First change the inches to feet.)

9. Find the amount of material required to make a rectangular metal box that has a height of 40 in, a width of 20 in and a length of 60 in.

10. What is the volume of the box in *question 9*?

11. Find the volume of a cube whose sides measure 1 meter.

12. What is the surface area of a cylinder with a radius of 100cm and a height of 150 cm?

ANSWER PAGE

1. P = 2 L + 2 W
 = 2(4.5 m) + 2(6 m)
 = 9 m + 12 m
 = 21 m

2. P = 2(48 + 24)
 = 144 inches

3. A = 15 x 20
 = 300 sq in

4. Change 100 cm to 1 m
 A = 3 x 1
 = 3 m^2

5. A = 7.2 cm x 4.7 cm
 = 33.84 cm^2

6. A = 16 x 16
 = 256 sq in

7. A = 8 x 5 1/2
 = 44 sq ft
 Cost = 44 sq ft x $5.95/sq ft = $261.80

8. First change inches to feet. 48 in = 4 ft, 60 in = 5 ft
 Area of top and bottom = 2 x 4 x 5 = 40 sq ft
 Area of long sides = 2 x 4 x 5 = 40 sq ft
 Area of short sides = 2 x 4 x 4 = 32 sq ft
 Find the total area and divide that by the area of 1 sheet of metal.
 Total surface area of the box = 40 + 40 + 32 = 112 sq ft
 Area of one sheet of metal = 4 x 8 = 32 sq ft
 Number of sheets needed = 112 ÷ 32 = 3.5 sheets

9. Area of top and bottom = 2(60 x 20) = 2(1200) = 2400 sq in
 Area of long sides = 2(40 x 60) = 2(2400) = 4800 sq in
 Area of short sides = 2(40 x 20) = 2(800) = 1600 sq in
 Total surface area = 2400 + 4800 + 1600 = 8800 sq in

10. Vol = *lwh*
 = 60 x 40 x 20
 = 48 000 cu in

11. $V = l^3$
$= 1^3$
$= 1$ cu m

12. A $= 2\pi r\ (r + h)$
$=(2 \times 3.14 \times 100)(100 + 150)$
$= (628)(250)$
$= 157\ 000$ cc (cubic centimeters)