EVALUATING
ACADEMIC READINESS
FOR APPRENTICESHIP TRAINING
Revised for
Access To Apprenticeship

MATHEMATICS SKILL
TRIGONOMETRY

AN ACADEMIC SKILLS MANUAL
for
The Precision Machining And Tooling Trades
This trade group includes the following trades:
General Machinist, Tool & Die Maker,
Mould Maker, Pattern Maker, and
Machine-Tool Builder Integrator

Workplace Support Services Branch
Ontario Ministry of Training, Colleges and Universities

Revised 2011

In preparing these Academic Skills Manuals we have used passages, diagrams and questions similar to those an apprentice might find in a text, guide or trade manual.

This trade related material is not intended to instruct you in your trade. It is used only to demonstrate how understanding an academic skill will help you find and use the information you need.

MATHEMATICS SKILL
TRIGONOMETRY

An academic skill required for the study of the Precision Machining and Tooling Trades

INTRODUCTION

One of the most important skills you will learn in the machining trades is the ability to accurately lay out a pattern on a piece of stock material. You will need to be able to take information from a drawing and lay it out to the correct size and shape. And, you need to know that when you have fabricated the piece it is precisely accurate in its size and shape, so you will test it against the tolerances set out in the drawing. You will use a part of mathematics called *trigonometry* to do these tasks successfully.

***Trigonometry** means "triangle measure." It uses the relationship or ratio between the length of the sides and the size of the angles of a right triangle to find an unknown side or angle.*

In this skills manual, we will look at the basic trigonometric functions and how to use them to find unknown values of lengths of sides or angles. The skills manual includes:

- A description of right triangles
- Ratios of lengths of sides of a right triangle
- The trigonometric functions
  - sine
  - cosine
  - tangent
- Using trigonometric tables
- Using the sine ratio
- Using the cosine ratio
- Using the tangent ratio
- Deciding what ratio to use
- Using a calculator

RIGHT TRIANGLES

Your work in precision machining will include right triangles in layouts. What is a right triangle? *A **right triangle** has one right angle, which is 90°, and two acute angles.* Acute angles are greater than 0 ° but less than 90° The sum of the three angles of any triangle is 180°.

Finding an unknown angle in a right triangle

If you know one of the acute angles in a right triangle, you can then find all the angles.

1. We know that the sum of all the angles is 180 ° .
2. We know the right angle is 90°.
3. The sum of the two acute angles will also be 90°
4. If one acute angle is 60°, we subtract 60° from 90° to get 30°, the value of the other acute angle.

Example: If one acute angle in a right triangle measures 50°, what is the third angle?

90°– 50° = 40°

Parts of a Right Triangle Used in Trigonometry

The sides of a right triangle have been given names. The longest side of a right triangle is always opposite the right angle. It is called the ***hypotenuse***. The side on the bottom is the ***base*** and the side that extends up from the right angle is the ***altitude or height***.

In trigonometry, like geometry, the angles of a right triangle are usually named with capital letters and the sides are usually named with lower case letters.

Triangle ABC in Figure 1 shows the way a right triangle is normally labeled. Angle C is a right angle and angles A and B are acute angles. Notice that each side is named by the small letter name of the angle opposite it.

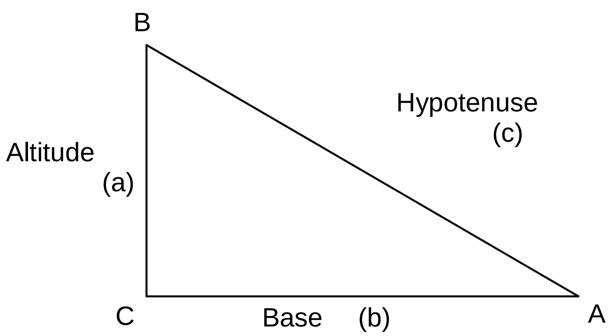


FIGURE 1: Labelling a right triangle

In Triangle ABC

The side **opposite angle A** is side **a**.

The side **opposite angle B** is side **b**.

The side **opposite angle C** is side **c**.
Side c is the hypotenuse.

The side **adjacent to** (or beside) **angle A** (but not the hypotenuse) is side **b**.

The side **adjacent to angle B** (but not the hypotenuse) is side **a.**

RATIOS OF LENGTHS OF SIDES IN RIGHT TRIANGLES

Trigonometry is based on the ratios of the sides of a right triangle to each other.

Remembering ratio: A ratio is a comparison of two quantities by division.

- The ratio of the length of side a to the length of side b is written a : b or $\frac{a}{b}$.
- The ratio of 3 centimeters to 4 centimeters is written 3 : 4 or $\frac{3}{4}$ or as the decimal .75.

Remembering about triangles:

- *The sum of all the angles in a triangle is 180 °*. In right triangles, we always know that one angle is 90°. That means that the sum of the other two angles has to be 90°.
- Similar triangles have the same shape but are different in size. The corresponding angles in similar triangles are the same but the lengths of the corresponding sides are different.

In similar triangles the ratios of corresponding sides are equal.

You only need to know one acute angle in a right triangle to find the other acute angle by subtraction. In trigonometry, you only need to know the measurements of some of the angles and some of the lengths to find unknown angles or sides by using ratios.

Trigonometry uses ratios based on the relationships between the sides and the angles of a right triangle to find the length of an unmeasured side or an unknown angle.

The 30°/ 60°/ 90° Triangle
It is easiest to understand trigonometric ratios by looking at right triangles that have acute angles of 30° and 60°as the other two angles. We can see that they are similar triangles and we can easily find the corresponding sides and angles.

We will first look at the relationships between the side opposite the 30° angle and the hypotenuse. Each triangle in Figure 2 has a 30° angle at the endpoint of the base.

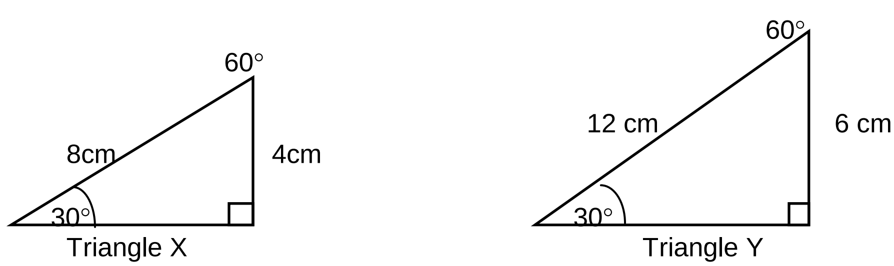


Figure 2: The hypotenuse is always twice the length of the side opposite the 30° angle in a 30° – 60° right triangle.

In Triangle X:

- The side opposite the 30° angle, is 4 cm long.
- The hypotenuse is 8 cm long, or twice as long.

In Triangle Y:

- The side opposite the 30° angle, is 6cm.
- The hypotenuse is 12cm, or twice as long.

When one length is two times longer than another length, we say the ratio of the first length to the second length is 2:1

In triangles X and Y, the ratio of the side opposite the 30° angle to the hypotenuse is 1:2

$$\frac{side\ opposite}{hypotenuse} = \frac{1}{2}$$

$$\frac{hypotenuse}{side\ opposite} = \frac{2}{1}$$

Conversely, in triangles X and Y, the ratio of the length of the hypotenuse to the side opposite the 30° angle is 2:1.

These ratios are true, ***or constant***, no matter what the actual lengths of the sides are.

So, in any 30 ° / 60 ° *right triangle, with regard to the sides* ***relative to the 30°angle:***

$$\frac{the\ side\ opposite\ the\ 30°angle}{the\ hypotenuse} = \frac{1}{2}\ or\ .5000$$

And,

$$\frac{the\ hypotenuse}{the\ side\ opposite\ the\ 30°\ angle} = 2$$

Because this is true, we know that the third side of the 30°/ 60°/ 90° triangle must also have a pair of *constant* ratios when compared to the hypotenuse. This third side is the side adjacent to the 30° angle (but not the hypotenuse)

The ratios formed by comparing this side adjacent to the 30° angle and the hypotenuse do not form whole numbers but they remain constant for all 30°/ 60°/ 90° triangles. They are approximately 8 : 6.85 and 6.85 : 8)

In any 30°/ 60° right triangle, with regard to the sides ***relative to the 30° angle:***

$$\frac{side\ adjacent}{hypotenuse} = about\ \frac{6.85}{8} = about\ \frac{.8563}{1}\ or\ \ about\ .8563$$

And,

$$\frac{hypotenuse}{side\ adjacent} = about\ \frac{8}{6.85} = about\ \frac{1.168}{1}\ or\ about\ 1.168$$

And, finally, we know that the ratios comparing the sides opposite and adjacent to the 30° angle will also have constant ratios.

In any 30° / 60° right triangle, with regard to the sides adjacent and opposite ***to the 30° angle:***

$$\frac{side\ opposite\ \ the\ angle}{side\ adjacent\ to\ the\ angle} = about\ \frac{.5774}{1}\ or\ about\ .5774$$

And,

$$\frac{side\ adjacent\ to\ the\ angle}{side\ opposite\ the\ angle} = about\ \frac{1}{.5774} = about\ \frac{1.732}{1}\ \ or\ about\ 1.7320$$

In triangles X and Y the sides related to the 60° angle also form a set of ratios.

In any 30°/ 60° right triangle, with regard to relationships ***between the side opposite the 60° angle and the hypotenuse:***

$$\frac{side\ opposite}{hypotenuse} = about\ \frac{6.85}{8} = about\ \frac{.8563}{1}\ or\ \ about\ .8563$$

And, in any 30°/ 60° right triangle,

$$\frac{hypotenuse}{side\ opposite} = about\ \frac{8}{6.85} = about\ \frac{1.168}{1}\ or\ about\ 1.168$$

In any 30°/ 60° *right triangle, with regard to relationship* ***between the side adjacent to the 60° angle and the hypotenuse:***

$$\frac{side\ adjacent}{hypotenuse} = \frac{4}{8} = about\ \frac{1}{2}\ or\ .5000$$

And,

$$\frac{hypotenuse}{side\ adjacent} = \frac{8}{4} = about\ \frac{2}{1}\ or\ a2.0000$$

And, finally:

In any 30°/ 60° *right triangle, with regard to the relationships* ***between the sides opposite and adjacent to the 60° angle:***

$$\frac{side\ opposite\ the\ angle}{side\ adjacent\ to\ the\ angle} = about\frac{1.732}{1}\ or\ about\ 1.732$$

And,

$$\frac{side\ adjacent\ to\ the\ angle}{side\ opposite\ the\ angle} = \frac{1}{1.732} = about\frac{.5774}{1}\ or\ about\ .5774$$

Other Right Triangles

Other right triangles do not have acute angles of 30° and 60°. But all right triangles do have a 90° angle and two acute angles. In another triangle the acute angles could be 40 and 50 degrees, or they could be 17 and 73 degrees. Still, ***the ratios which compare the lengths of the sides of any right triangle which has that particular pair of acute angles will always be the same, or constant.***

Examples:

In all right triangles with acute angles of 20° and70°, the ratios which compare the relative lengths of the sides will always be constant, no matter how long those sides are.

In all right triangles with acute angles of 56° and 34°, the ratios which compare the relative lengths of the sides will always be constant, no matter how long those sides are.

THE TRIGONOMETRIC FUNCTIONS

The Six Possible Ratios

For any acute angle in any right angle triangle, the ratios of the hypotenuse to the side opposite and to the side adjacent to the angle are as follows:

1. Side opposite : hypotenuse = a constant,
2. Side adjacent : hypotenuse = a constant,
3. Hypotenuse : side opposite = a constant, and
4. Hypotenuse : side adjacent = a constant.

And, the ratios of the side adjacent to the side opposite the angle to each other will always be the same.

5. Side opposite : side adjacent = a constant, and
6. Side adjacent : side opposite = a constant.

These six ratios are the basis of trigonometry. *The ratios, known as **the trigonometric functions of angles in a triangle, or trigonometric values**, are used to calculate unknown measurements of sides or angles in triangles.*

We can use the functions of angles (the ratios of the lengths of sides that result from the acute angles in a right triangle) to find unknown sides or angles.

- To use the functions we need to depend on measurements we do know.
- The ratios of the lengths of the sides of all the acute angles in a right triangle have been calculated.
- Most of the ratios are numbers like the ratio for adjacent side to hypotenuse in the 30°/ 60 ° / 90 ° triangle (the ratio 6.85 : 8).
- These, ratios forming trigonometric functions, are divided out and written as decimals to several decimal places. (6.85 ÷ 8 = .85625)

Ratios for the acute angles of a right triangle: The trigonometric functions for all of the possible ratios listed above for every acute angle have been calculated.

Example: To look at the functions of a 35° angle, we talk about the ratios of the side adjacent to that angle, about the side opposite that angle, and about the hypotenuse of the triangle.

- The altitude and base are named as adjacent or opposite to whatever acute angle is being considered, depending on their position to that angle.
- The hypotenuse, always being opposite the 90° angle, is always called the hypotenuse.

The six trigonometric ratios or functions for the acute angles have been given names:

- sine,
- cosine,
- tangent,
- cotangent,
- secant, and,
- cosecant.

Sine, Cosine and Tangent
In basic trigonometry, we will concentrate on only three of the functions – ***sine, cosine, and tangent.***

Sine: *In a right triangle, the ratio of the length of the side opposite an acute angle to the hypotenuse is called the* ***sine*** *of the angle (abbreviated to* ***sin****).* For every right triangle, there are two acute angles, which we will call A and B, that can form sine ratios.

The sine of an angle is always named in relation to that angle.

Sin A means the ratio of the side opposite ∠A to the hypotenuse. The ratio is written:.

$$\sin \angle A = \frac{side\ opposite\ \angle A}{the\ hypotenuse}$$

Sin B is the ratio of the side opposite angle B to the hypotenuse. The ratio is written:

$$\sin \angle B = \frac{side\ opposite\ \angle B}{The\ hypotenuse}$$

Cosine: *In a right triangle, the ratio of the length of the side adjacent to an acute angle to the hypotenuse is called the* ***cosine*** *of that angle (abbreviated to* ***cos****).*

The cosine of an angle is always named in relation to that angle.
Cos A means the ratio of the side adjacent to angle A to the hypotenuse. The ratio is written:.

$$\cos A = \frac{side\ adjacent}{the\ hypotenuse}$$

Cos B means the ratio of the side adjacent to angle B to the hypotenuse. The ratio is written:.

$$\cos B = \frac{side\ adjacent}{the\ hypotenuse}$$

Tangent

In a right triangle, the ratio of the length of the side opposite an acute angle to the side adjacent to the angle is called the ***tangent*** *of that angle (abbreviated to* ***tan****).*

The tan of an angle is always named in relation to that angle.
Tan A means the ratio of the side opposite to angle A and the side adjacent to angle A. The ratio is written:

$$\tan\ A = \frac{side\ opposite\ \angle A}{side\ adjacent\ \angle A}$$

Tan B means the ratio of the side opposite to angle B and the side adjacent to angle B. The ratio is written:

$$\tan\ B = \frac{side\ opposite\ \angle B}{side\ adjacent\ \angle B}$$

Application

The values of these ratios, sin, cos, and tan, can be used to find the unknown length of a side or to find the value of an unknown acute angle.

Let's go back to the 30°/60° right triangle and look at what we learned about the function

$$\frac{side\ opposite}{hypotenuse} = \frac{1}{2}\ or\ .5000$$

This function is the sine of a 30° angle.

In the sample triangles when the side opposite the 30° angle was 4 the ratio looked like this:

$$\frac{4}{8} = \frac{1}{2}\ or. 5000$$

If the length of the side opposite is 5 and we do not know the length of the hypotenuse, we use *c* to represent the hypotenuse. We get this:

$$\frac{5}{c} = .5$$

We can find the length of the hypotenuse if we cross multiply.

$.5(c) = 5$
$.5\ c = 5$ divide both sides by.5
$c = 10$

The hypotenuse is 10.

TABLES OF TRIGONOMETRIC FUNCTIONS

When we looked at the ratios in the right angle triangle, we divided the ratios to get decimal numbers that are equivalent to the ratios. These functions of sine, cosine and tangent, for all the acute angles from 1° to 89° in right angle triangles are expressed as decimal numbers and listed in the ***Tables of Trigonometric Functions*** or ***Tables of Trigonometric Values***.

We can substitute these values for the tan, sin and cos of the angles in any right triangle to find unknown lengths or angles.

Here is a partial table of functions. It shows the tangent, sine and cosine values for some acute angles. A complete table would show values to at least four decimal places for all the angles and parts of angles (called minutes) between 0° and 90°.

TABLE OF SOME TRIGONOMETRIC VALUES

Angle	Sin	Cos	Tan
10°	.1736	.9848	.1763
15°	.2588	.9659	.2679
25°	.4226	.9063	.4663
30°	.5000	.8660	.5774
37°	.6018	.7986	.7536
45°	.7071	.7071	1.0000
52°	.7880	.6157	1.2799
60°	.8660	.5000	1.7321
66°	.9135	.4067	2.2460
75°	.9659	.2588	3.7321
80°	.9848	.1736	5.6713

USING TABLES OF TRIGONOMETRIC FUNCTIONS

Before solving problems, we will practice finding the value of sin, cos and tan using the table. Then we will use the table to find an unknown angle when the sin, cos or tan of that angle is known.

Finding the Value of a Function in the Tables

Example: Find the value of:

a) tan 30
b) cos 66
c) sin 45

Use the table to look up the required values.

Find the given angle in the table by looking in the left hand column of angles.

1. **To find the sin of an angle**, move across from the given angle to the sin column and record that value.
2. **To find cos of an angle**, move across from the angle to the cos column; record the cos value.
3. **To find tan of an angle**, move across from the angle to the tan value.

Answers:

a) tan 30°= .5774
b) cos 66° = .4067
c) sin 45° = .7071

The next step is to find the unknown angle when the value of sin, cos or tan is given.

Finding an Unknown Angle When a Function of it Is Given

Example: Find angle A when:

a) cos A = .8660
b) tan A = 1.2799
c) sin A = .6018

1. To find cos angle A, go to the cos column and search down it until you come to the given value.
2. Move across to the left to the angle column to find the required angle.
3. To find tan or sin repeat steps 1 and 2 using the tan or sin column.

Answers:

a) If cos A = .8660, angle A = 30°
b) If tan A = 1.2799, angle A = 52°
c) If sin A = .6018, angle A = 37°

Example: Find angle B when:

a) sin B = .4226
b) tan B = 1.0000
c) cos B = .2588

Answers:

a) If sin B = .4226, angle B = 25°
b) If tan B = 1.0000, angle B = 45°
c) If cos B = .2588, angle B = 75°

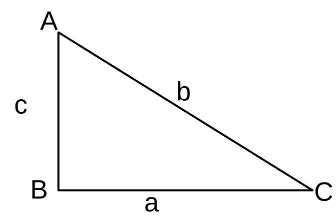
Solving problems with trigonometric functions

To find the unknown length of a side or an unknown acute angle to solve a problem, follow these steps:

1. Draw a right triangle and label it as below so the angles and sides are named as they are given in the problem. (Sides are named by the small letters of the angles opposite them.) Write in all the dimensions given in the problem.

In Triangle ABC

Side a is opposite ∠A and adjacent to ∠C.
Side b is the hypotenuse opposite the right angle ∠B
Side c is opposite ∠C and adjacent to ∠A



2. Select the function (either sin, cos, or tan) that will enable you to solve the problem.
3. Solve the resulting equation, first substituting the values from the table of trigonometric values

We have used triangles labeled ABC in this manual. Often though, triangles are given other labels such as XYZ or LMN or RST. For this reason, ***we refer to an unknown angle by using the symbol θ (theta).*** *When we are solving a problem, we can change θ to the name of whatever angle we are working with.*

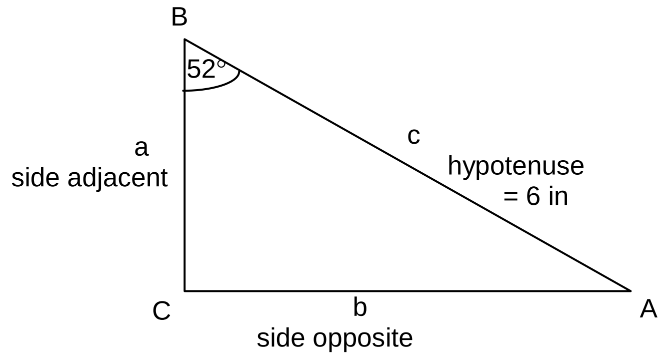
Here are the functions. We will use θ to represent the angle.

$$\sin\theta = \frac{side\ opposite}{hypotenuse} \qquad \cos\theta = \frac{side\ adjacent}{hypotenuse} \qquad \tan\theta = \frac{side\ opposite}{side\ adjacent}$$

USING THE SINE RATIO

The sine ratio enables us to find unknown angles or sides using the Table of Trigonometric Functions.

Example 1: In Triangle ABC, find AC, or side b, if angle B = 52° and the hypotenuse = 6 in. Use the sine ratio to find the length of the unknown side to the nearest tenth.



1. **Draw and label a right triangle.** Put in all the information you have been given. We are given the following information:
   $\angle B = 52^\circ$
   The hypotenuse is 6 in.
   We need to find side b , the side opposite $\angle B$.

2. **Select the function.** We know the size of the angle and the length of the hypotenuse and we have to find the length of the side opposite B. The function which compares side opposite and the hypotenuse is the sine function.

   We use the sine function.

   $$\sin\theta = \frac{side\ opposite}{hypotenuse}$$

3. **Solve the equation.**

$$\sin 52^\circ = \frac{b}{6\ in}$$

Now, use the table to look up sin 52°, which is .7880.

Substitute this value and the length of the hypotenuse in the equation

$$.7880 = \frac{\text{b}}{6\text{ in}}$$

Rearrange the equation so that the unknown quantity is on its own on the left.

$$.7880 \times 6\text{ in} = \frac{\text{b}}{6\text{ in}} \times 6\text{ in}$$

Multiply both sides by 6 in.

$$.7880 \times 6\text{ in} = \text{b}$$

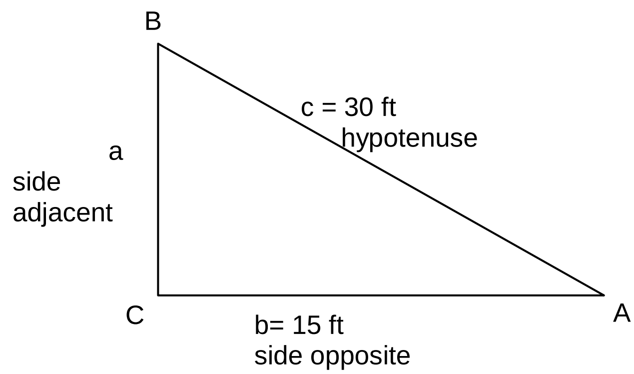
Reverse the equation and finish the calculations.

b = .7880 x 6 in
= 4.728 in

You have now found the length of the unknown side using the sine ratio.

Example 2: Use the sin ratio to find angle B if b is 15 ft. and the hypotenuse is 30 ft.

1. **Draw and label a right triangle.** Put in all the information you have been given.
   We are given the following information:
   b (side opp) = 15 ft
   The hyp = 30 ft
   We are asked to find the value of ∠B



2. **Select the function.** We have to find the value ∠B and we know the size the lengths of the hypotenuse and the side opposite ∠B.

   We choose the sine function, which compares side opp and hyp.

$$\sin\theta = \frac{side\ opposite}{hypotenuse}$$

3. **Substitute known values and solve the equation**

$$\sin B = \frac{15\ ft}{30\ ft}$$

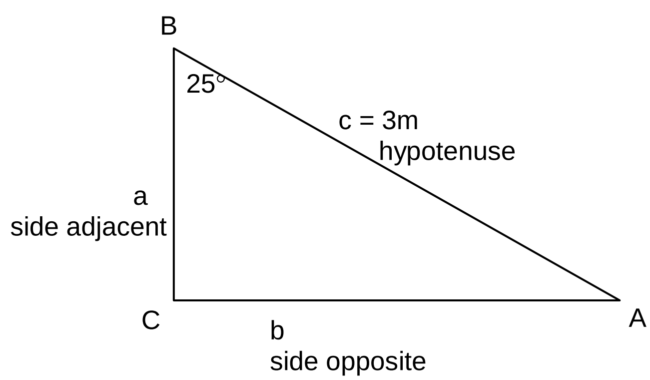
sin B = .5000 Under the sine column, look for the function.5000
angle B = 30°

USING THE COSINE RATIO

Example 1: Find the side adjacent to angle B if angle B = 25^{o} and the hypotenuse = 3m. Use the cosine ratio to find the length of the unknown side to the nearest tenth. Draw and label the triangle first.

1. **Draw and label a right triangle.** Put in all the information you have been given.
   We are given the following information:
   ∠B = 25°
   The hyp = 3 m

   We are asked to find the length of side a.
   Side a is adj to ∠B



2. **Select the function.** We have to find the value of a. We know the size of ∠B and the length of the hypotenuse.

   We choose the cosine function, which compares side adj and hyp.

3. **Substitute known values and solve the equation.**

$$\cos \theta = \frac{\text{side adj}}{\text{hyp}}$$

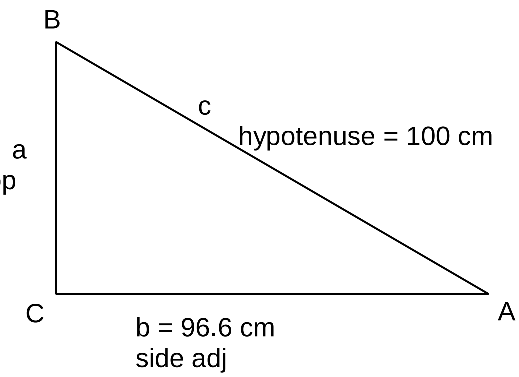
$$\cos 25° = \frac{\text{side adj}}{3\text{ m}}$$ Look for cos 25 in the table.

$$.9063 = \frac{\text{side a}}{3\text{ m}}$$

side a = .9063 x 3 m
= 2.7 m

Example 2: Use the cos ratio to find angle A if the side adjacent = 96.6 cm and the hypotenuse = 100 cm.

1. **Draw and label a right triangle.** Put in all the information you have been given.
   We are given the following information:
   b = 96,6 cm
   The hyp = 100cm
   We are asked to find ∠A
   Side b is adj to ∠A



2. **Select the function.** We have to find the value ∠A. We know the size of b and the length of the hypotenuse.

   We choose the cosine function, which compares adj and hyp.

$$\cos A = \frac{\text{side adj}}{\text{hyp}}$$

3. **Substitute known values and solve the equation**

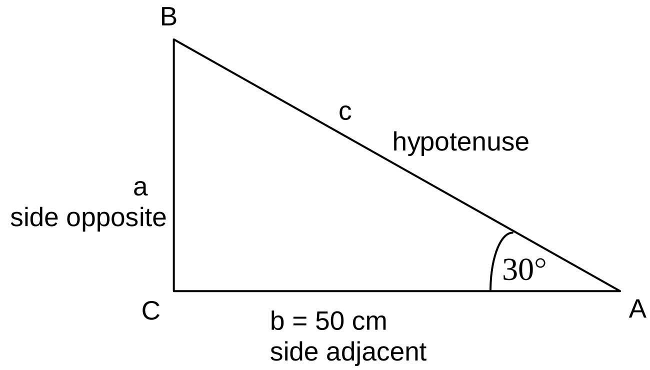
$$\cos A = \frac{96.6\text{ cm}}{100\text{ cm}}$$

cos A = .9660

angle A = 15°

USING THE TANGENT RATIO

Example 1: Find the length of side a if angle A = 30° and the side adjacent to angle A = 50 cm. Use the tangent ratio to find the length of the unknown side to the nearest tenth.



1. **Draw and label a right triangle.** Put in all the information you have been given.
   We are given the following information:
   ∠A = 30°
   Side adj = 50cm
   We are asked to find side a
   Side a opp to ∠A

2. **Select the function.** We know ∠A and the length of b. We have to find the value of a.

   We choose the tan function, which compares sides opp and ad j.

   $\tan \theta = \frac{\text{side opp}}{\text{adj}}$

3. **Substitute known values and solve the equation**

   $\tan A = \frac{\text{side a}}{50 \text{ cm}}$ Find tan 30° and substitute it in the equation.

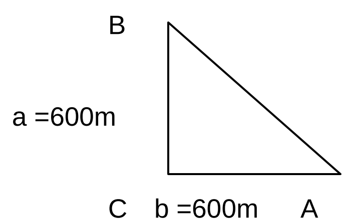
   $.5774 = \frac{\text{side a}}{50 \text{ cm}}$

   $.5774 \times 50 \text{ cm} = \frac{\text{side a}}{50 \text{ cm}} \times 50 \text{ cm}$ Multiple both sides by 50 cm.

   $.5774 \times 50 \text{ cm} = \text{side a}$
   side a = .5774 x 50 cm
   = 28.9 cm

Side a = 50 cm.

Example 2: Find angle A if side a = 600 m and side b = 600 m. Use the tangent ratio in the following questions to find the unknown angle to the nearest tenth.



1. **Draw and label a right triangle.** Put in all the information you have been given.

   We are given the following information:
   A= 600 m
   b = 600 m
   We are asked to find ∠A
   Side a is opp ∠A
   Side b is adj to ∠A

2. **Select the function.** We have to find the value ∠A.

   We know the sizes of a and b.

   We choose the tan function, which compares adj and opp.

   $\tan\theta = \frac{\text{side opp}}{\text{adj}}$

3. **Substitute known values and solve the equation**

   $\tan A = \frac{600\text{ m}}{600\text{ m}}$

   $= 1.000$ Find 1.000 in the column for tan ratios.

   ∠A = 45°.

DECIDING WHAT RATIO TO USE

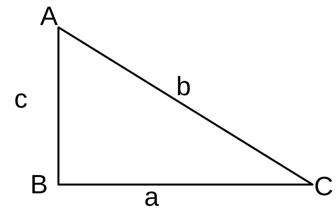
The next step is to solve problems that require you to decide which ratio to use. You do this by examining the given information. If the triangle is unnamed, assume it is named ABC. It always helps to draw and label the triangle, as we did in the examples using tangent.

To find the unknown length of a side or an unknown acute angle to solve a problem, follow these steps:

1. Draw a right triangle and label it as below so the angles and sides are named as they are given in the problem. (Sides are named by the small letters of the angles opposite them.) Write in all the dimensions given in the problem.

   In Triangle ABC

   Side a is opposite ∠A and adjacent to ∠C.
   Side b is the hypotenuse opposite the right angle ∠B
   Side c is opposite ∠C and adjacent to ∠A



2. Select the function (either sin, cos, or tan) that will enable you to solve the problem.
3. Solve the resulting equation, first substituting the values from the table of trigonometric values

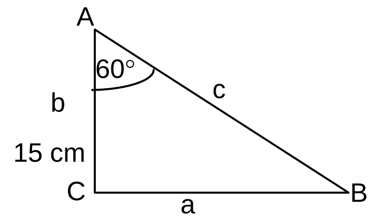
Here are the functions again.

$$\sin\theta = \frac{side\ opposite}{hypotenuse} \qquad \cos\theta = \frac{side\ adjacent}{hypotenuse} \qquad \tan\theta = \frac{side\ opposite}{side\ adjacent}$$

Example: Find side a when angle A is 60° and side b is 15 cm. We first must decide what formula to use. Angle A and side b are given, and we need to find side a. Follow the steps.

1. Draw a right triangle and label it so the angles and sides are named as they are given in the problem. Write in all the dimensions given in the problem.

We are given the following information:
∠A = 60°
Side b = 15cm
We are asked to find side a
a is opp ∠A
b is adj ∠A



2. Select the function (either sin, cos, or tan) that will enable you to solve the problem.
   We see that the tan function compares opp to adj.
   We choose the tan function
3. Solve the resulting equation, first substituting the values from the table of trigonometric values

$$\tan \theta = \frac{opp}{adj}$$

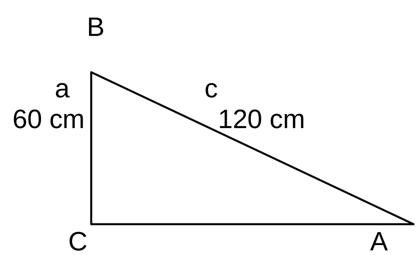
$$\tan A = \frac{\text{side a}}{\text{side b}}$$

side a = tan A x side b
side a = 1.7321 x 15 cm
= 26 cm

Example : Find angle B in the triangle below, if side a = 60 cm and side c = 120 cm.

1. **Draw and label a right triangle.** Put in all the information you have been given.

   We are given the following information:
   a= 60 cm
   c = 120 cm
   We are asked to find ∠B
   Side a is adj to ∠B
   Side c is the hypotenuse



2. **Select the function.** We have to find the value ∠B.

   We know the sizes of a and c.

   We choose the cos function, which compares adj and hyp.

$$\cos \theta = \frac{\text{side adj}}{\text{hyp}}$$

3. **Substitute known values and solve the equation**

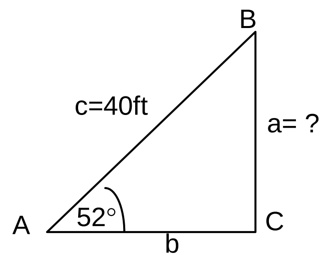
$$\cos B = \frac{\text{side a}}{\text{side c}}$$

$$= \frac{60\text{ cm}}{120\text{ cm}}$$

$= .5000$ Find .5000 under the cos function

$\angle B = 60^{o}$

Example: Find side a in the following triangle. Notice again that the letter of the side is the same as the letter of the angle that is opposite to it.



1. We know:
   $\angle A = 52°$
   side c = 40 ft

   We need to find side a.
   Side c is the hyp
   Side a is opp

2. Sin is the function that compares side opp to hyp.

$$\sin \theta = \frac{\text{side opp}}{\text{hyp}}$$

3. Substitute and solve.

$$\sin A = \frac{\text{side a}}{\text{side c}}$$

$$\sin 52° = \frac{\text{side a}}{40\text{ ft}}$$

side a = .7880 x 40 ft

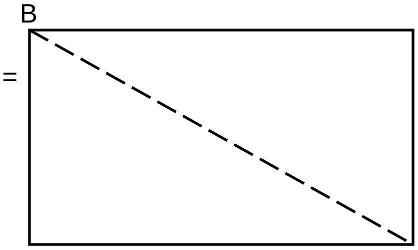
= 31.5 ft

Example: A rectangle has sides of 6.8in. and 9in. Draw a diagonal and calculate the angles in one triangle.

1. Make a diagram and label it.

   We know
   side a = 6.8 in
   side b = 9 in

   We need to find∠ A and ∠B.



2. To find angle A
   Side a is opp
   Side b is adj
   We choose the function that uses opp and adj sides.

   $\tan A = \frac{\text{side opp}}{\text{side adj}}$

3. Substitute and solve
   $\tan A = \frac{\text{side a}}{\text{side b}}$

   tan A = 6.8/9 = .75

   Angle A = 37°

   Remember that the angles in a triangle equal 180^0. If angle A = 37^{o} and angle C = 90° we know that ∠A + ∠B = 90°

   ∠ B = 90° – 37°
   = 53°

   The angles in the triangle are:
   ∠ A = 37°
   ∠ B = 53°
   ∠ C = 90°

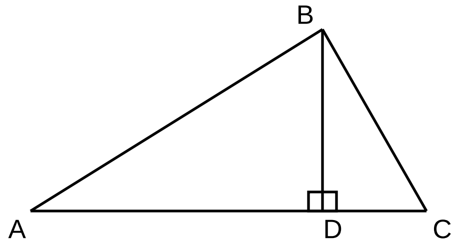
You could also find angle B by calculating tan B using side a and side b.

SUMMARY

Trigonometric tables contain ratios calculated from the measurements of the lengths of the sides of right triangles. The ratios are named by their relationship to the right angle and the other angle being considered. Since the right angle is 90°, the other two angles must vary between 0° and 90 °. For each of these angles between 0°and 90°, ratios that compare the lengths of the triangle's sides have been determined. These values are called trigonometric ratios or functions. The functions we have discussed here are sine, cosine, and tangent.

And something new to think about
You should also realize that by drawing a line perpendicular to a side of any triangle so that the new line goes through the opposite angle you can create two new triangles, both of which are right angled. This means we can use trigonometry to find information about any triangle.

See the example below:



Triangle ABC has now become two right angle triangles ABD and DBC. Sides AB and BC are their hypotenuses.

If we know any of the following things:
- the length of two of the sides of either triangle,
- the length of one side and the value of one angle in either triangle, or,
- the height of triangle ABC and any one of the following: angle A, angle C, angle ABD or angle CBD,

then we can use the functions of sin, cos, and tan to find out all the rest of the triangle's measurements and angles.

USING A CALCULATOR TO FIND TRIGONOMETRIC VALUES

If you solve many problems involving trigonometric functions, you can find the trigonometric ratios with a scientific calculator instead of looking them up in the tables. These calculators are programmed with every value from the table. When you push the correct key, the required sin, cos or tan function appears in the window of the calculator. This is faster than looking up the values in the tables.

Finding Trigonometric Functions

Here are the steps to find the trigonometric functions using the calculator:

1. Make sure the calculator is in degree mode. ***Deg*** (or maybe **D**) in small letters will appear at the top of the window to indicate that the calculator is in degree mode.
2. Enter the value of the angle in degrees into the calculator.
3. Press the key with the ratio you require, either **sin**, **cos** or **tan**.
4. The trigonometric value will appear in the window, usually showing seven or eight decimal places.
5. Round off the number to however many decimal places you need for the problem you are solving.

Example: Find sin 58° using a calculator.

With the calculator in degree mode, enter 58 and then press the sin key. The decimal number will appear.

Sin 58° = .8480481

If you want sin 58° to three significant figures, round off .8480481.

Sin 58° = .848

Example: Find tan 61° using a calculator.

Enter 61, then press the **tan** key. The decimal number will appear.

tan 61° = 1.8040478

If you want tan 61° to four significant figures, round off

tan 61° = 1.804

Finding Angles

If you are given a trigonometric function and you need to find the angle, you follow these steps:

1. Key in the function number.
2. Press the **INV** (or maybe **Shift)** key first.
3. Then press the correct function key.
4. The number that appears in the window represents the angle you are looking for.

Example: Find the angle, given that cos θ = .8387

Key in .8387, press INV first and then press cos. The number will appear.

θ = 32.996904

Round it off to 33°.

SUMMARY OF STEPS IN SOLVING PROBLEMS USING TRIGONOMETRIC RATIOS:

1. Draw the triangle. Label the sides so that each angle and the side opposite it are named with the same letter. Use a small letter to name the sides and a capital letter to name the angles. Write in whatever quantities you are given.

2. Decide what angle or length of side you are required to find.

3. Choose the function (tan, sin, or cos) which includes the three quantities you know and what you need to find.

4. Fill in the values in the equation for the known quantities, using values from the trigonometric tables when necessary.

5. Solve the equation to find the unknown quantity.

Here is a summary of the trigonometric ratios:

$\sin\theta = \frac{opposite}{hypotenuse}$ $\cos\theta = \frac{adjacent}{hypotenuse}$ $\tan\theta = \frac{opposite}{adjacent}$

Answer the following questions based on the relationships of the angles and sides in a right triangle. Use the Partial Table of Trigonometric Functions to find the ratios. The answers are on the last page.

1. From the table or using a calculator, find the following ratios:
   a) tan 15° b) sin 60° c) cos 37°

   d) sin 30° e) cos 45° f) tan 75°

2. Find angle A when:
   a) sin A = .4226 b) cos A = .6947 c) tan A = 1.2799

   d) cos A = .9659 e) tan A = 1.7321 f) sin A = .9135

3. Find angle B when:
   a) tan B = .7536 b) sin B = .2588 c) cos B = .4067

   d) tan B = 1.0000 e) cos B = .1736 f) sin B = .5000

Find the following answers to the nearest tenth.

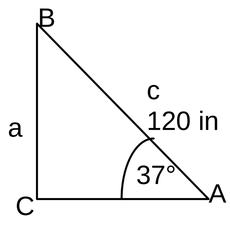
4. Use the tangent ratio to find side a if angle A = 30° and side b = 5 ft.

5. Use the sin ratio to find angle B if side b = 86.6 cm and side c = 100 cm.

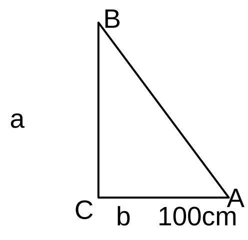
6. Use the cosine ratio to find side c if angle A = 52° and side b = 12 m.

7. Find the indicated part of the following right triangles:

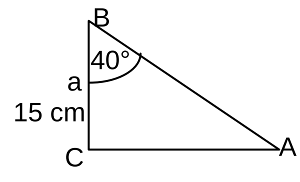
   a) Find side b using sin ratio

   b) Find angle A using tan ratio

   c) Find side c using cos ratio

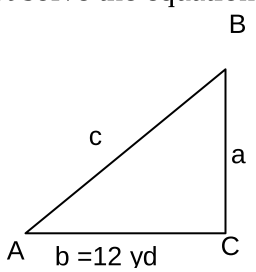
   

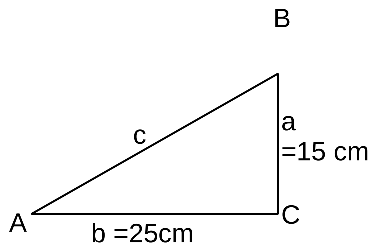
8. If you were given side b and angle B, and you were required to find side a, what ratio (either sin, cos or tan) would you use?

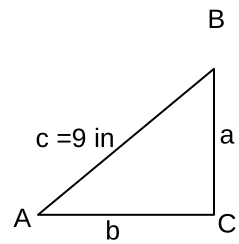
9. If you were given angle A and side b, and you were required to find side c, what ratio would you use?

10. If you were given side b and side c, and you were required to find angle B, what ratio would you use?

11. In the following triangles, what trigonometric function (sin, cos, or tan) would you use? (Do not solve the equations)







a) Find side a. b) Find Angle B. c) Find side b.

12. Find side b if angle A = 60°and side c = 40 ft.

13. Find side c if angle B = 37° and side b = 25 cm.

14. Find angle A if side a = 914 in and side c = 1000 in.

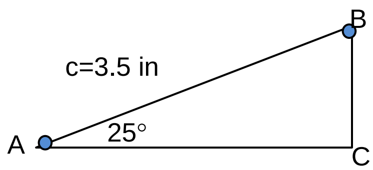
15. Find side a if angle A = 45° and side b = 18 m.

If necessary, make a diagram of the right triangle in the following questions. Label it in the same way as the triangle in Figure 1.

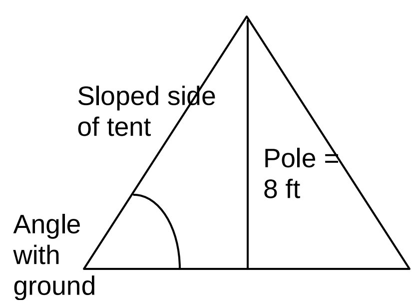
16. How high is a kite if 150 feet of string is let out and the string forms an angle of 60° with the ground?

17. An incline is at a 30° slope. How long must it be to reach a height of 20 ft?

18. To drill holes in a metal plate at A and B in the diagram below, the table movements of the milling machine must be known. The movements are AC and BC. Find these lengths.



19. The pole holding up a tent in the middle of each end is 8 ft high and the distance across the tent along the ground is 16 ft as shown in the diagram. What angle does the side of the tent make with the ground?



20. The legs of a pair of dividers are each 12 cm long and are opened at an angle of 60°. Find the distance between their points.

ANSWER PAGE *(If you are using a calculator your answers might be slightly different because you will not be rounding off as early as when you are using the tables.)*

1. a) .2679 b) .8660 c) .7986 d) .5000 e) .7071 f) 3.7321

2. a) angle A = 25° b) angle A = 45° c) angle A = 52°

   d) angle A = 15° e) angle A = 60° f) angle A = 66°

3. a) angle B = 37° b) angle B = 15° c) angle B = 66°

   d) angle B = 45° e) angle B = 80° f) angle B = 30°

4. $\tan \mathrm{A} = \frac{\mathrm{a}}{\mathrm{b}}$

   side a = tan 30° x 5 ft
   = .5774 x 5 ft
   = 2.9 ft

5. $\sin \mathrm{B} = \frac{\mathrm{b}}{\mathrm{c}}$

   $\sin \mathrm{B} = \frac{86.6}{100}$

   $\sin \mathrm{B} = .866$
   ∠ B = 69°

6. $\cos A = \frac{b}{c}$

   $side\ c = \frac{12m}{cos\ 52°}$

   $side\ c = 12m \div .6157$
   = 19.5m

7. a) side b = 109.6 in.
   b) angle A = 60°
   c) side c = 18.8 cm

8. tan B

9. cos A

10. sin B

11. a) $\sin A = \frac{a}{b}$

b) $\tan B = \frac{b}{a}$

c) $\sin C = \frac{b}{c}$

12. $\cos A = \frac{b}{c}$

$\text{side}B = \cos 60° \times 40\text{ft}$

$= .500 \times 40\text{ft}$
$= 20$ ft

13. $\sin B = \frac{b}{c}$

$\text{side } c = \frac{25 \text{ cm}}{\sin 37°}$

$= 25 \text{ cm} \div .602$
$= 41.5$ cm

14. $\sin A = \frac{a}{c}$
$\sin A = \frac{914 \text{ in}}{1000 \text{ in}}$
$= .914$
if $\sin A = .914$, $\angle A = 66°$

15. $\tan A = \frac{a}{b}$

$\text{side } a = \tan 45° \times 18\text{m}$

$= 1.000 \times 18$ m
$= 18$ m

16. We need to find side a (the height of the triangle)
We know side c (150 ft) and angle A (60°)
$\sin A = \frac{a}{c}$
side a = sin 60° x 150 ft
= .866 x 150 ft
= 130 ft
The height of the kite is 130 ft.

17. We need to find side c (the slope of the triangle).
We know the height or side a (20 ft) and angle A (30°).
$\sin A = \frac{a}{c}$
$\text{side } c = \frac{a}{\sin 30°}$
$= \frac{20 \text{ ft}}{.500}$
= 40 ft The slope must be 40 ft long.

18. Side AC is side b
We know angle A (25°) and side c (3.5 in.)

$$\cos A = \frac{b}{c}$$

side b = cos 25° x 3.5 in
= .907 x 3.5 in
= 3.2 in

Side BC is side a
We know angle A (25°) and side c

$$\sin A = \frac{a}{c}$$

side a = sin 25° x 3.5 in
= .423 x 3.5 in
= 1.5 in

19. The base of the triangle (side b) is 8 ft.
The height (side a) is also 8 ft.
We need to find angle A.

$$\tan A = \frac{a}{b}$$

$$\tan A = \frac{8\ \text{ft}}{8\ \text{ft}}$$

$$= 1.0$$

If tan a = 1.0, angle A = 45°.

20. Draw a triangle formed by the two legs and the distance between their end points. Divide the triangle in half by a perpendicular line extending from the base to the point of the 60° angle between the legs. Two right triangles are formed. Each triangle has an angle of 30° at the top (1/2 of angle B). The length of each leg (12 cm) is also the length of side c. We can now find side b, which is half the distance between the two points of the divider.

$$\sin B = \frac{b}{c}$$

side b = sin 30° x 12 cm
= .500 x 12 cm
= 6 cm

The distance between the points is 2 x side b.
= 2 x 6 cm
= 12 cm