

**EVALUATING
ACADEMIC READINESS
FOR APPRENTICESHIP TRAINING**
Revised for
ACCESS TO APPRENTICESHIP

**MATHEMATICS SKILLS
CARTESIAN COORDINATE SYSTEM AND
INTERPRETATION OF GRAPHS**

**AN ACADEMIC SKILLS MANUAL
for
The Precision Machining And Tooling Trades**

This trade group includes the following trades:
General Machinist, Tool & Die Maker,
Mould Maker, Pattern Maker, and
Machine Tool Builder Integrator

*Workplace Support Services Branch
Ontario Ministry of Training, Colleges and Universities*

Revised 2011

In preparing these Academic Skills Manuals we have used passages, diagrams and questions similar to those an apprentice might find in a text, guide or trade manual.

This trade related material is not intended to instruct you in your trade. It is used only to demonstrate how understanding an academic skill will help you find and use the information you need

MATHEMATICS SKILLS: CARTESIAN COORDINATE SYSTEM AND INTERPRETATION OF GRAPHS

*An academic skill required for the study of the
Precision Machining and Tooling Trades*

INTRODUCTION

Graphs present facts visually, making it easier to find and interpret information. You can see the relationship between different quantities, rather than having to analyse numbers.

Machinists work with diagrams of different layouts. These diagrams are laid out on a grid system based on the *Cartesian coordinate system*. This system is the basis of mathematical graphs. Before you do any computer layout, you need a basic understanding of how the coordinate system works. If you have to program a CNC machine the positions are set up using Cartesian coordinates.

In your training, a graph can bring together work you do using geometry, or using trigonometry. In trigonometry, ratios of the lengths of the sides of a right triangle are used to find unknown lengths and angles. A right triangle can be placed on a coordinate system. This will let you name every part of the triangle as a point on the grid. This is how a computer is given measurement information.

This skills manual looks at tables and graphs, including the following topics:

- ◆ Tables
- ◆ Graphs
- ◆ The coordinate system
- ◆ Graphing a linear relationship
- ◆ Blueprint drawings as points on a grid

TABLES

Usually the information, or *data*, presented in a graph is first organized into a table, such as Table 1. A simple table lists two sets of numbers or measurements in a way that shows the connection between them. The data is arranged in rows and columns.

In your work, you will have to lift heavy loads. If you are using a swivel to attach the load to a rigging, you need to check that the swivel will be strong enough to support the load. Table 1 shows the safe working load that different diameters of swivels can support. For each size of swivel, the maximum load that can be lifted is listed.

TABLE 1: Safe working load for different diameters of forged alloy steel swivels

| Diameter in inches | Max. safe working load in pounds |
|--------------------|----------------------------------|
| 1/4 | 850 |
| 5/16 | 1250 |
| 3/8 | 2250 |
| 1/2 | 3600 |
| 3/4 | 7200 |
| 7/8 | 10,000 |
| 1 1/8 | 15,200 |

A table can also show the relationship between two quantities in a formula. For example, the formula for the area of a circle is:

$$\text{Area} = \pi r^2$$

In this formula:

- One quantity in the formula, area, varies as the length of the radius changes. and
- For every value of radius in the formula, there is one value of area, found by multiplying π (3.14) times the radius squared.

Table 2 compares area with different values of radius.

TABLE 2: Value of area as the radius changes

| Radius | Area (πr^2) |
|--------|-----------------------|
| 2 cm | 12.6 cm ² |
| 3 cm | 28.3 cm ² |
| 4 cm | 50.2 cm ² |
| 5 cm | 78.5 cm ² |
| 6 cm | 113.0 cm ² |

Instead of a specific formula such as that for area of a circle, you can have a general equation such as $y = 2x + 3$. This equation, called a **function**, relates the value of y to every different value of x from 1 to 4.

- To find values for y , you substitute different values of x in the equation and solve for y .

**TABLE 3: Value of x and y for the equation
 $y = 2x + 3$**

| x | y ($2x + 3$) |
|---|------------------------|
| 1 | $(2 \times 1 + 3) = 5$ |
| 2 | $(2 \times 2 + 3) = 7$ |
| 3 | $(2 \times 3 + 3) = 9$ |

GRAPHS

A graph is a diagram that is used to show the changes of quantities in relation to each other.

Example: A graph could be used to show the relationship between the diameters of swivels and safe working loads in Table 1.

THE CO-ORDINATE SYSTEM

A **grid** system is used to locate points on a graph. A **grid** consists of a horizontal line crossed by a vertical line.

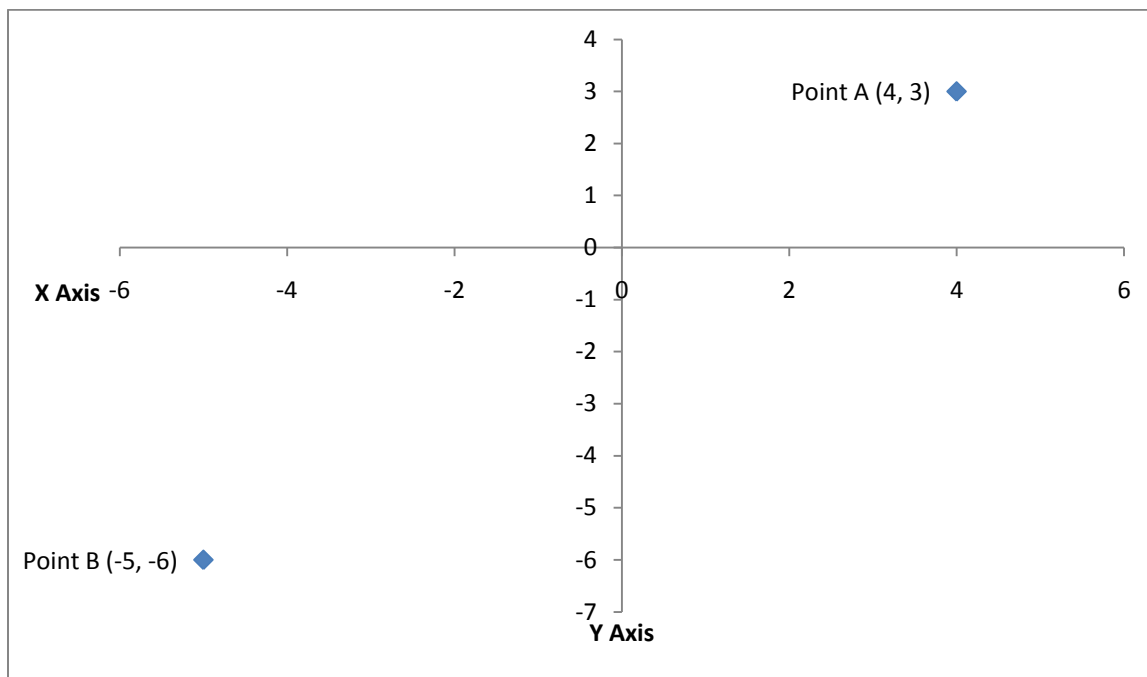
Graph 1, below, shows a grid.

- ◆ The horizontal line is called the **x-axis**.
- ◆ The vertical line is called the **y-axis**.
- ◆ The x-axis represents one set of measurements and the y-axis represents another set.

The x- and y-axis each have a **scale**, a graduated series of numbers that include the measurements of the quantities used on the graph.

The point where the x and y-axis cross, called the point of intersection or the **origin**, usually has the value of 0.

- ◆ Numbers to the right of 0 on the x-axis are positive, while numbers to the left of the origin are negative.
- ◆ Numbers above 0 on the y-axis are positive, while numbers below the origin are negative.



GRAPH 1: A grid with an X axis and a Y axis

Often only positive quantities are shown on a graph.

- A graph like this will not show negative numbers.
- The y-axis will be at the left hand side and the x-axis is at the bottom of the graph.

Points on a grid

A vertical line can be drawn from any point on the x-axis to meet a horizontal line drawn from any point on the y-axis. *Where the two lines meet, a **point** is formed on the grid.*

- A point can be formed at every place where a vertical line from the x-axis crosses a horizontal line extending from the y-axis,.

Coordinates

Every point has two **coordinates** that describe where, on the graph, each point is:

- The first coordinate shows the measurement on the x-axis.
- The second shows the measurement on the y-axis.
- Point coordinates are listed inside brackets.

Example: Look back to Graph 1. Point A on the graph is listed as Point A (4, 3).

1. If you look down to the x axis below Point A, you will see the point is directly above 4 on the axis.
 - 4 is the x coordinate.
2. If you look across to the Y axis you will see that Point G is directly across from the 3.
 - 3 is the y coordinate.

Point A (4,3) is the point where 4 is the x co-ordinate and 5 is the y co-ordinate.

Plotting Points on a Grid

In general, the information that is used to form the points on a grid comes from a table such as Table 1, 2 or 3. The table will list two sets of numbers or measurements in a way that shows the connection between them.

Look at Table 4 which lists values for x and y. This table does not give an equation or formula, it only lists a series of values. We will look at how this table can be used to plot the points on a grid.

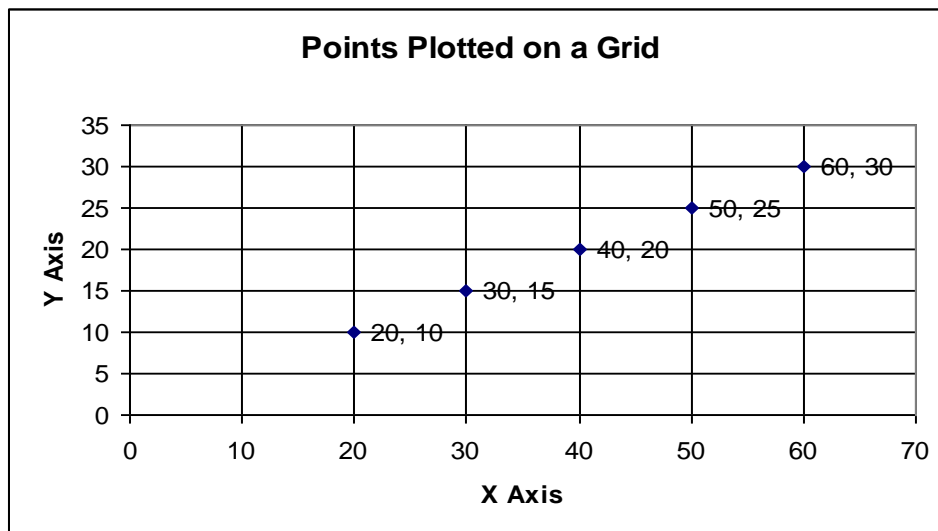
TABLE 4: List of x and y values

| x value | y value |
|---------|---------|
| 20 | 10 |
| 30 | 15 |
| 40 | 20 |
| 50 | 25 |
| 60 | 30 |

These points can be plotted on a grid with an x and y-axis, as shown on Graph 1.

To make the data in Table 4 into a graph:

1. We assign each axis numbers that are in the range of the information on the table.
 - X values in the table start at 20 end at 60,
 - So the x axis will start at 0 but it must include that range of values.
 - Y values range between 10 and 30
 - The Y axis also starts at 0 and must include that range of values.
2. We put a point on the grid each place where the y value and the x value meet.
 - We put a point on the grid where the x value, 20, meets the y value, 10.
 - The next point goes where $x = 30, y = 15$.
 - Another point goes where $x = 40, y = 20$, and so on.
3. The result is Graph 2, below.



GRAPH 2 Points Plotted on a Grid

The grid in Graph 2 shows the horizontal x-axis and the vertical y-axis. The x and y scales are simple number intervals without any units such as centimeters or feet.

- Notice that the y scale is smaller or closer together than the x scale on the graph.
- Also note that only positive values are shown in this graph.

The points are plotted on the grid like this: Look at the third point on Graph 2. It has an x coordinate of 40 and a y coordinate of 20. It is named as point A (40, 20).

Finding Points on a Grid

To locate a point on a grid, use the x and y numbers listed in the co-ordinate pair for that point.

- The order of the numbers is important.
- The first number gives the x coordinate. The second number is the y coordinate.

To find the point (30,45) on Graph 2:

1. Draw a vertical line extending up from the number 30 on the x-axis.
2. Then draw a horizontal line extending over from the number 45 on the y-axis.
3. The point (30,45), is where the two lines meet.
4. Note that this point has no relation to the other points on the graph.

GRAPHING A LINEAR RELATIONSHIP

Graphs are often used to show a relationship between two quantities, such as the relationship between the numbers of hours required to charge a battery and the voltage required. If a line is drawn connecting the points, the graph is called a ***line graph***.

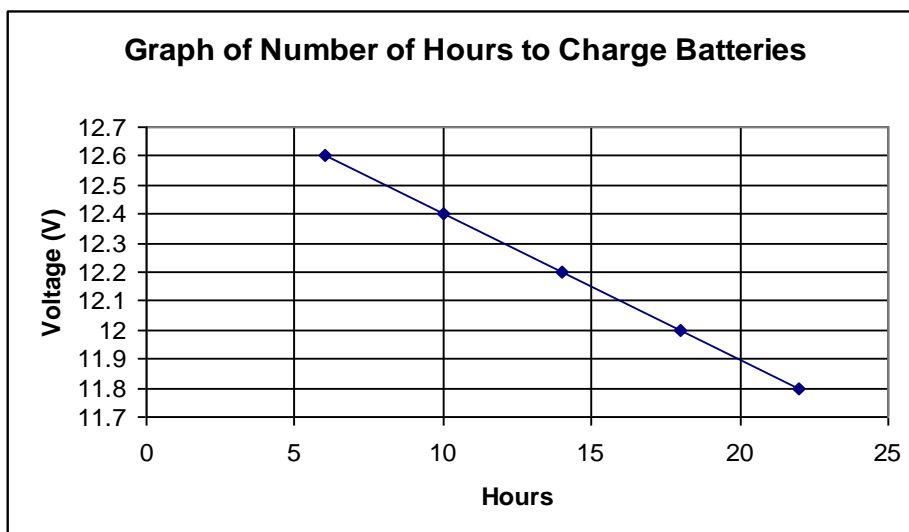
To graph the relationship between the open circuit voltage of a battery and the hours required to charge it using a 10 amp charger, follow these steps:

1. List the voltage of different batteries and the hours required to recharge each battery.
 - Table 5 lists the measurements.

TABLE 5: Number of hours to charge batteries with a 10 amp charger

| hours to charge | open circuit voltage |
|-----------------|----------------------|
| 6 | 12.6 V |
| 10 | 12.4 V |
| 14 | 12.2 V |
| 18 | 12.0 V |
| 22 | 11.8 V |

2. The number of hours forms the scale on the x-axis.
 - The time scale goes from 6 to 22 hours.
3. The voltage measurements form the scale on the y-axis.
 - The voltage scale goes from 11.8 V to 12.6 V.
4. Use the quantities listed in the table to form the points on Graph 3.
5. Draw a line to link the points.



GRAPH 3: Number of Hours to Charge Batteries

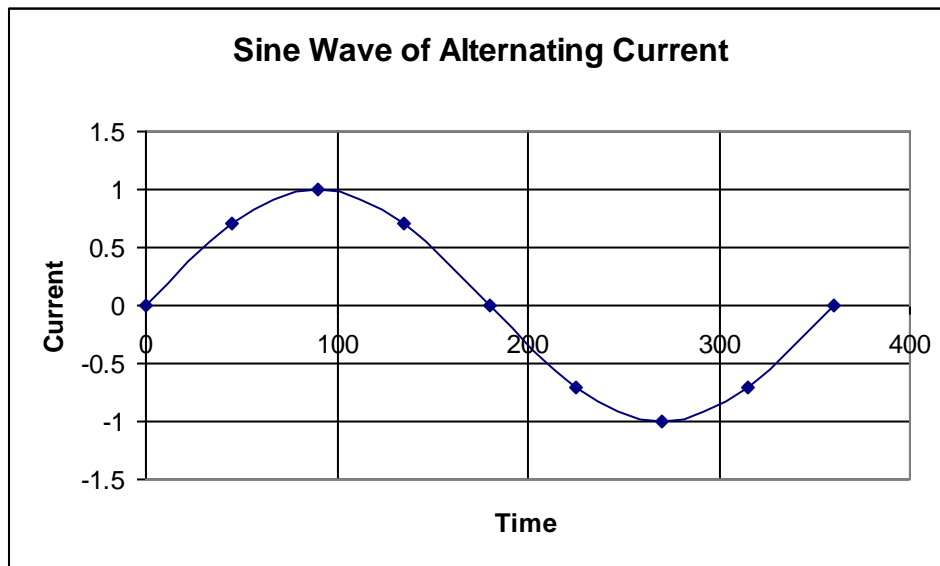
When you look at Graph 3, you can see that the higher the voltage of a battery, the less time that is required to charge it, using a 10 amp charger.

When the points are joined, a straight line is formed. If a straight line results when the points plotted on a graph are joined, a **linear relationship** exists between the two quantities.

Graphs with Curved Lines

Some of the graphs in your work will not be straight lines. Instead the graph will form a curved line.

Example: Below is the graph of a sine wave. The current in power lines is supplied as an alternating current. This current moves in one direction, stops and then moves in the opposite direction. Graph 4 shows a sine wave of one cycle of an alternating current.



GRAPH 4: Sine Wave of Alternating Current

In Graph 4:

- The scale of the x-axis is the time it takes to make a complete cycle.
- The scale of the y-axis is the amount of current.
 - When the current is moving forward, the scale is positive.
 - When the current is moving in the reverse direction, the scale is negative.
- The graph does not show exact numbers for the current and time scale:
 - but it does show how the amount, and direction of the current changes through one cycle.

BLUEPRINT DRAWINGS AS POINTS ON A GRID

You can think of a two dimensional blueprint or working drawing as a series of points drawn on a grid.

Example: If you were making a rectangular box, you might have a blueprint showing three rectangular pieces:

- the top,
- the long side,
- and the short side.

Any of the rectangular pieces can be drawn as a series of points on a grid.

Graphs are created by drawing programs to generate working drawings and blueprints. The actual program for creating computer designed blueprints is, of course, very complex. However, it is based on the same grid system that simple graphs use.

Example: A CADD (computer-aided drafting and design) drawing is based on these principles.

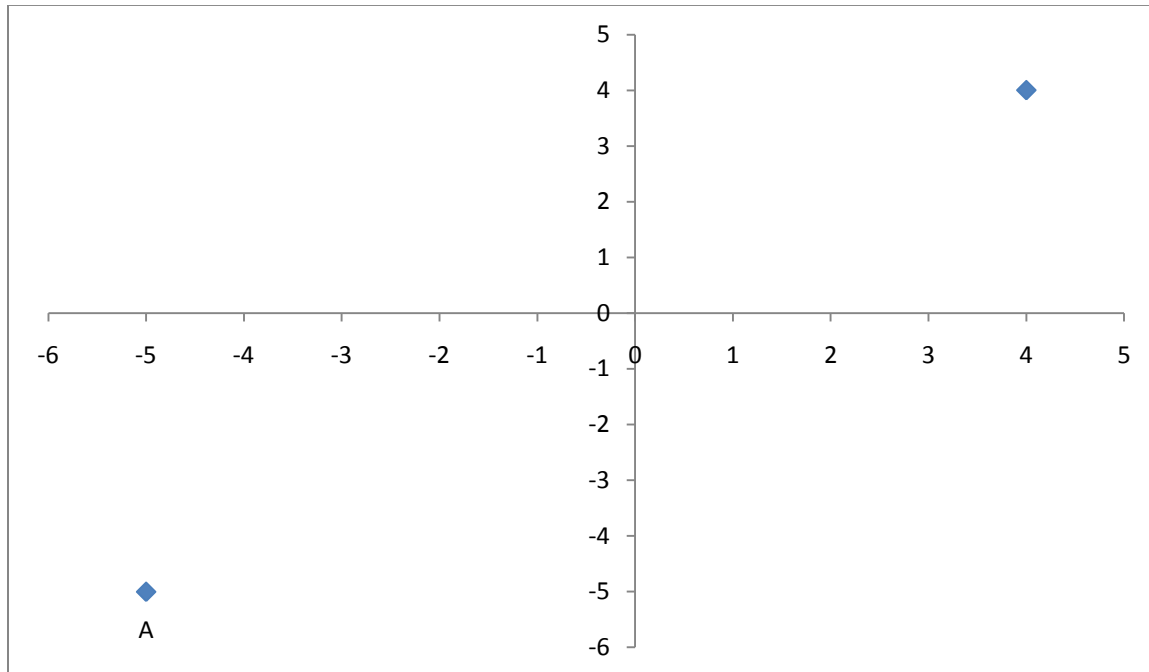
- Every line on the drawing is defined by the relevant points.
- If the computer is given the needed information, such as the center point of a circle and the radius, it can draw that circle.

Example: CADD can also generate three dimensional diagrams using a third axis, the z-axis.

- The z-axis extends upwards and downwards at right angles from the point of origin.
- A point in a three dimensional diagram is named by the x, y, and z coordinates.

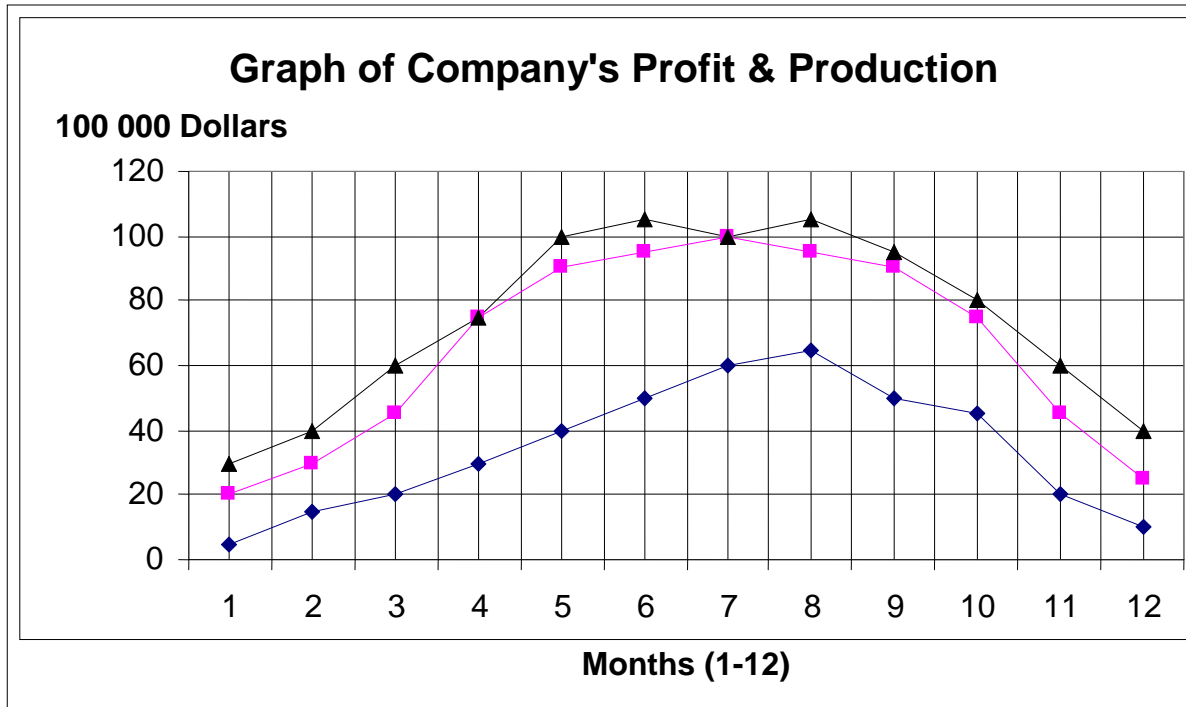
Answer the questions based on the graphs shown. Answers are on the last page.

1. a) Give the coordinates of Point A, located on the Graph below.
b) Find the location and draw point C (3, -2) on the Graph below.



2. In the graph below:

- A company's projected production is represented by the line which has the symbol (Δ) at each of its points.
- Actual production is represented by the graph line with \blacksquare at its point.
- Net profit is represented by the line with the symbol (\diamond) for each month of one year.

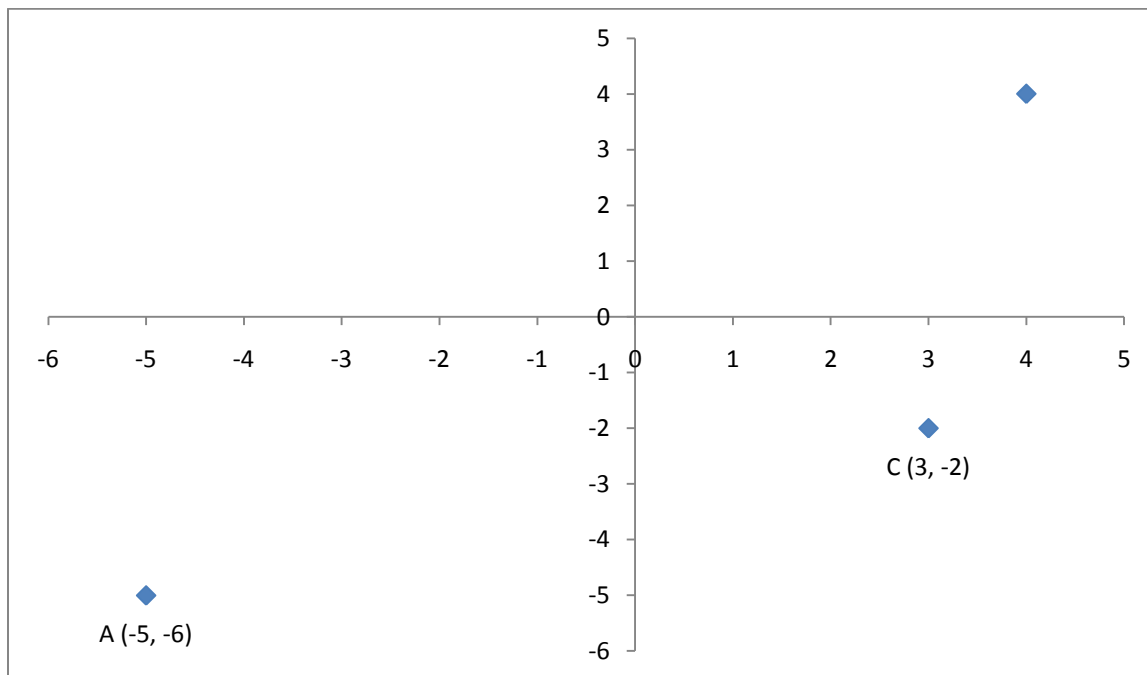


- What month was the value of units produced the highest?
- What months was the actual production the same as the projected production?
- What month were profits the greatest?
- Would you say that the company was fairly accurate in its prediction for the year?

ANSWER PAGE

1. a) The x co-ordinate of point A is -5 and the y co-ordinate is -6.
The location of Point A is (-5, -6).

b) The location of Point C (3, -2) is shown below.



2. a) The value of units produced was the greatest in June.

b) The projected value was the same as the actual production in March, July and September.

c) Profits were greatest in August.

d) This question requires you to make a value judgement. The graph lines for the value of actual units produced and the projected value are close so it would be fair to say the company made an accurate prediction for the year.