EVALUATING
ACADEMIC READINESS
FOR APPRENTICESHIP TRAINING
Revised for
ACCESS TO APPRENTICESHIP

MATHEMATICS SKILL
STATISTICS

AN ACADEMIC SKILLS MANUAL
for
The Precision Machining And Tooling Trades

This trade group includes the following trades:
General Machinist, Tool & Die Maker,
Mould Maker, Pattern Maker, and
Machine-Tool Builder Integrator

Workplace Support Services Branch
Ontario Ministry of Training, Colleges and Universities

Revised 2011

In preparing these Academic Skills Manuals we have used passages, diagrams and questions similar to those an apprentice might find in a text, guide or trade manual.

This trade related material is not intended to instruct you in your trade. It is used only to demonstrate how understanding an academic skill will help you find and use the information you need.

MATHEMATICS SKILL
STATISTICS

An academic skill required for the study of the Precision Machining and Tooling Trades

INTRODUCTION

Statistics is the collecting and analyzing of information or data. The data collected and used by a precision machinist is usually in the form of numerical measurements. This information can be used in various ways.

> **Example:** You have the measurements of a group of rods that have been machined to a certain length. The rods vary slightly in length. You could find the average (or mean) of the lengths of the rods. You might also want to know how much an individual measurement varies from the average length. In this case, you would use a statistical formula called the variance formula to find the answer.

Before you can use this formula to analyze the precision of the work you have done, you need to learn a few background terms. This skills manual covers the following topics concerned with statistical analysis:

- Finding the average or mean
- Finding variance

FINDING THE AVERAGE OR MEAN

An average gives you a calculation of the most likely middle position of a group of numbers. Three methods are used to determine an average: *arithmetic mean*, *median*, and *mode*.

Arithmetic Mean

Finding the **arithmetic mean** is the most common way is to find average.

To find the mean of a group of numbers:

1. Add all the numbers in the group; then
2. Divide the sum by the number of items in the group.

Example: Find the mean of 6, 9, 5, and 12.

6 + 9 + 5 + 12 = 32	add the numbers
32 ÷ 4 = 8	divide by the number of items

Example: You have four machined pins and want to find the mean of their lengths. When measured to a precision of two decimal places, they are found to measure 2.4 cm, 2.5 cm, 2.6 cm and 2.5 cm.

Here are the steps:

- Add the numbers: 2.4 cm + 2.5 cm + 2.6 cm+ 2.5 cm = 10.0 cm
- Determine the number of measurements. You already know there are four. If you did not know the number of items, you would have to count to find out how many there are.
- Divide the sum 10.0 cm by the number of measurements, which is 4.
- 10.0÷ 4 = 2.5 cm
- The mean or average of this group of measurements is 2.5 cm.

Example: Find the mean of the following numbers: 5, 1, 7, 5, 8, 4, 6, 4

Add to find the total:

5 +1 + 7 + 5 + 8 + 4 + 6 + 4 = 40

Count the number of items in the group. There are 8 numbers in the group. Divide 40 by 8:

40 ÷ 8 = 5

The mean of the group of numbers is 5.

Example: A group of pins is machined and measured to a precision of one decimal place. The measurements are as follows: 4.2", 4.1", 4.3", 4.2", 4.3", and 4.1". What is the mean of this group of measurements?

Add to find the total:

4.2 + 4.1 + 4.3 + 4.2 + 4.3 + 4.1 = 25.2"

Count the number of measurements. There are 6.
Divide the total by the number of measurements:

25.2 ÷ 6 = 4.2"

The mean of the measurements is 4.2 inches.

Example: During a five day period, a machine was down 2.5 hours the first day, .5 hours the second day, 0 hours the third, 1 hour the fourth and .25 hours the fifth. What is the mean downtime per day of the machine that week?

Add to find the total hours of downtime:

2.5 + .5 + 0 + 1 + .25 = 4.25 hours

There are 5 days. Divide the sum 4.25 hours by 5 days.

4.25 ÷ 5 = .85 hours per day

The mean downtime is .85 hours per day during that week.

Median
*The **median** is the middle number of a group of numbers arranged in order of size.*

Example: Find the median of 6, 4, 9, 5, and 12.

4, 5, 6, 9, 12, arrange the numbers by size

6 is the middle number.

The is median 6.

Mode
*The **mode** is the number in a group which occurs most frequently.*

Example: You have 4 rods. They measure 6.892 cm, 6.890 cm, 6.891 cm, and 6.892 cm. Find the mode.

6.890, 6.891, 6.892, 6.892 arranged by size

6.892 appears twice so it is the mode

FINDING VARIANCE

***Variance** is the measure of differences between two or more things.* This measurement can be calculated by a mathematical formula.

Example: When you machine a series of products to a certain measurement, you expect the result to be close to the ideal, or exact, measurement. You want as little variation (difference) in lengths as possible. When you are finished the job, you can find out how close the lengths of the items you machined are to the mean of those lengths.

To calculate this difference we use the variance formula.

$$V = \frac{\sum(X - x)^2}{n}$$

Where:

- **V** is variance, the measure of difference in a group of measurements.
- $\sum$ is the sum of the following amounts (this symbol is called ***sigma***)
- **X** is an individual measurement.
  - In the first example on page 2, the individual measurements are 2.4 cm, 2.5 cm, 2.7 cm and 2.5 cm.
- **x** is the mean. The mean is the average of all of the measurements, found by adding the measurements and then dividing the sum by the number of items.
  - The mean in the first example is 2.5 cm.
- **n** is for the number of measurements.

Look at the formula again.

$$V = \frac{\sum(X - x)^2}{n}$$

We read this formula as:
Variance** equals **the sum of all of the values for $(X - x)^2$** divided **by the number of values** (or measurements) **used.

This means that we follow these steps:

Step 1. Calculate $(X\text{-}x)^2$, for each measurement we have.
Step 2. Add those calculations together.
Step 3. Substitute into the formula:
- the calculation for $\Sigma\ (X\text{-}x)^2$, and
- the value for n (the number of values we calculated).

Step 4. Complete the calculation:
- divide $\Sigma(X - x)^2$ by n.

Example:

Step 1. If we have 8 pieces we calculate $(X\text{-}x)^2$ 8 times.
Step 2. We add the results of those 8 calculations together
Step 3. We substitute that value for $\Sigma(X - x)^2$ *and* the value for n
Step 4. Divide.

The trick now will be to keep track of all of these calculations.

- We can keep track by assigning a marker to each calculation.
- For X of the first piece we assign the marker $_1$
  - We call the first piece X_1
  - The second piece will be X_2
  - and so on for X_3, X_4, X_5 …

Example: If four machined pins measure 2.4cm, 2.5cm, 2.7cm, and 2.5m and the mean is 2.5, find the variance.

$$V = \frac{\sum(X - x)^2}{n}$$

We know the following:
$X_1 = 2.4 X_2 = 25\ X_3 = 2.7$ $X_4 = 2.5.$

$x = 2.5$

$n = 4$

For X_1: $(2.4 - 2.5)^2 = 0.01$ cm
For X_2: $(2.5 - 2.5)^2 = 0.00$ cm
For X_3: $(2.7 - 2.5)^2 = 0.04$ cm
For X_4: $(2.5 - 2.5)^2 = 0.00$ cm

Step 1. Do each calculations for $(X - x)^2$:

∑ in front of the brackets tells us to add together the answers for all of the measurements.

$\sum (X - x)^2 = 0.01 + 0.00 + .040 + 0.00$
$= 0.05$ cm

Step 2. Calculate $\sum (X - x)^2$

To find the variance of these measurements, finish the calculation.

- Write the answer to $\sum (X - x)^2$ above the fraction line.
- Substitute for n, the number of measurements, below the fraction line.
- Then divide.

$$V = \frac{\sum(X-x)^2}{n}$$

We will use the figures from the example above.

$V = \frac{0.05}{4}$

Step 3. Substitute $\sum (X - x)^2 = 0.05$
n=4 measurements

$V = 0.0125$ cm

Step 4. Divide

Variance = 0.0125 cm

Example: After a number of pins are machined, they are measured to a precision of one decimal place. The measurements are as follows: 4.2", 4.1", 4.3", 4.2", 4.3", and 4.1". The mean was calculated to be 4.2 in (from the example on page 2). What is the variance (V) of the measurements of these pins?

Use the formula:

$$V = \frac{\sum(X-x)^2}{n}$$

We know:
X = the measurements
$X_1 = 4.2$ in., $X_2 = 4.1$ in., $X_3 = 4.3$ in., $X_4 = 4.2$ in., $X_5 = 4.3$ in., and $X_2 = 4.1$ in.
x = 4.2 in
n = 6 pins

For X_1 $(4.2 - 4.2)^2 = 0.00$ in
For X_2 $(4.1 - 4.2)^2 = 0.01$ in
X_3 $(4.3 - 4.2)^2 = 0.01$ in
X_4 $(4.2 - 4.2)^2 = 0.00$ in
X_5 $(4.3 - 4.2)^2 = 0.01$ in
X_6 $(4.1 - 4.2)^2 = 0.01$ in

1. Calculate $(X-x)^2$ for each value of X

0.0 + 0.01 + 0.01 + 0.00 + 0.01 +0.01 = .04 in

2. Calculate $\sum (X - x)^2$:

$$V = \frac{\sum(X-x)^2}{n}$$

3. Substitute into the formula:

$V = \frac{.04 \text{ in}}{6}$

$V = \frac{.04 \text{ in}}{6}$

$V = .0067 \text{ in}$

Variance is .0067 inches

$\sum (X - x)^2 = .04 \text{ in}$
$n=6$

4. Divide

Answer the following question on finding the mean and the variance of a group of measurements. Answers are on the next page.

1. Find the mean of the following groups of numbers:

a) 8, 12, 7, 10, 7, 9, 8, 11

b) 5 cm, 6 cm, 6 cm, 8 cm, 5 cm, 6 cm

c) 10.1 inches, 10.4 inches, 10.0 inches, 10.3 inches,

d) .051 m, .048 m, .052 m, .050 m, .049 m

2. Find the variance *for each group of numbers in Question 1* above. Use the value of the mean calculated for a) to d) in Question 1. The variance formula is:

$$V = \frac{\sum(X - x)^2}{n}$$

ANSWER PAGE

1. a) 8 + 12 + 7 + 10 + 7 + 9 + 8 + 11 = 72
72 ÷ 8 = 9
n =9

b) 5 cm + 6 cm + 6 cm + 8 cm + 5 cm + 6 cm = 36 cm
36 ÷÷6 = 6 cm
n = 6 cm

c) 10.1 inches + 10.4 inches + 10.0 inches + 10.3 inches = 40.8
40.8 ÷ 4 = 10.2 inches
n= 10.2 in

d) .051 m + .048 m + .052 m + .050 m + .049 m = .250
.250 ÷ 5 = .050 m
n =.050 m

2. a) $V = \frac{\sum(X-x)^2}{n}$

We know:
The value of X_1 is 8, X_2 is12, X_3 is 7, X_4 is 10, X_5 is 7, X_6 is 9, X_7 is 8, and X_8 is 11.
The value of the mean, x, is 9.
n=8

Do the calculations for $(X - x)^2$

For X_1 $(8 - 9)^2 = 1$
For X_2 $(12 - 9)^2 = 9$
For X_3 $(7 - 9)^2 = 4$
For X_4 $(10 - 9)^2 = 1$
For X_5 $(7 - 9)^2 = 4$
For X_6 $(9 - 9)^2 = 0$
For X_7 $(8 - 9)^2 = 1$
For X_8 $(11 - 9)^2 = 4$

$\sum (X - x)^2 = 24$ Add the results above to find $\sum (X - x)^2$.

Substitute in the formula

$V = \frac{24}{8}$ $\sum (X - x)^2 = 24$
n=8

V = 3

2. b) $V = \frac{\sum(X-x)^2}{n}$

x, = 6 cm calculated in 1b).
n= 6

For X_1 $(5 - 6)^2 = 1$
For X_2 $(6 - 6)^2 = 0$
For X_3 $(6 - 6)^2 = 0$
For X_4 $(8 - 6)^2 = 4$
For X_5 $(5 - 6)^2 = 1$
For X_6 $(6 - 6)^2 = 0$

$\sum (X - x)^2 = 6$

$V = \frac{6}{6}$ $\sum (X - x)^2 = 6$
$n = 6$

V = 1 cm

c) $V = \frac{\sum(X-x)^2}{n}$

x =10.2
n = 4

For X_1 $(10.1 - 10.2)^2 = .01$
For X_2 $(10.4 - 10.2)^2 = .04$
For X_3 $(10.0 - 10.2)^2 = .04$
For X_4 $(10.3 - 10.2)^2 = .01$

$V = \frac{.10}{4}$ $\sum (X - x)^2 = .10$
$n = 4$

V = .025 inches

d) $V = \frac{\sum(X-x)^2}{n}$

x = .050 cm
n = 5

For X_1 $(.051 - .050)^2 = .000001$
For X_2 $(.048 - .050)^2 = .000004$
For X_3 $(.052 - .050)^2 = .000004$
For X_4 $(.050 - .050)^2 = 0$
For X_5 $(.049 - .050)^2 = .000001$

$V = \frac{.000010}{5}$ $\sum (X - x)^2 = .000010$
n= 5

V = = .000002 m