

**EVALUATING  
ACADEMIC READINESS  
FOR APPRENTICESHIP TRAINING**  
Revised for  
**ACCESS TO APPRENTICESHIP**

**MATHEMATICS SKILLS  
PROPERTIES OF ANGLES**

**AN ACADEMIC SKILLS MANUAL  
for**

**The Motive Power Service Trades**

This trade group includes the following trades:  
Automotive Service Technician, Heavy Duty Equipment Mechanic,  
Motive Power Parts Person, Transmission Mechanic,  
Truck & Coach Technician, and Truck & Trailer Service Technician

*Workplace Support Services Branch  
Ontario Ministry of Training, Colleges and Universities*

*Revised 2011*

In preparing these Academic Skill Manuals we have used passages, diagrams and questions similar to those an apprentice might find in a text, guide or trade manual.

**This trade related material is not intended to instruct you in your trade. It is used only to demonstrate how understanding an academic skill will help you find and use the information you need.**

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# MATHEMATICS SKILLS

## PROPERTIES OF ANGLES

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*An academic skill required for the study of the  
Motive Power Service Trades*

### **INTRODUCTION**

Any motive power service technician is expected to have a basic knowledge of angles in order to successfully complete certain projects on the job. An engine won't work properly if its components are put in the wrong places. Every part must be in exactly the right position. One of the ways that a technician checks the position of a part is to measure the angle between it and another component.

Angles are involved in many of the jobs you do. When you check steering, adjust caster and camber angles in wheel alignments or adjust the brakes, you are working with angles. When you are welding, you have to hold the torch at the correct angle to the surface of the steel to get the right size bead. The angle is usually  $45^\circ$  for oxy-acetylene welding,  $20^\circ$  for arc welding and  $35^\circ$  when welding a butt joint. You need to have a working sense of the size of these angles.

Often a technical manual will instruct you to carry out a procedure at a certain angle. For example, when grinding the valve seat, the manual will tell you that an angle stone of  $15^\circ$  will remove material at the top of the seat while angle stone of  $45^\circ$  will remove material at the bottom of the seat. Although you will use guides to direct your cuts, you should be familiar with what these angles mean.

This skills manual on angles covers the following topics:

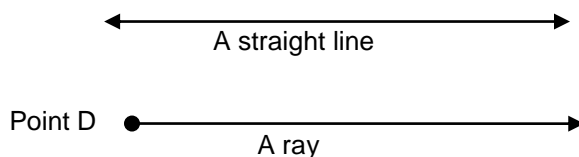
- ◆ Basic terms used to describe angles
- ◆ Definition of an angle
- ◆ Different types of angles, including
- ◆ Angles in geometric figures
- ◆ Angles formed by a pair of intersecting lines
- ◆ Angles formed when two parallel lines are intersected by another line

### **BASIC GEOMETRIC TERMS**

Specific terms are used to describe the parts of an angle and the different types of angles. A **line** is described as a set of points. A **straight line** is the shortest distance between two points. When we talk about a line, we assume it is a straight line. Otherwise it is called a **curved line**.

**Example:** A highway consists of a series of connected straight and curved lines.

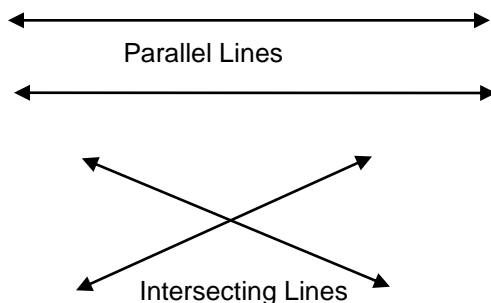
A line can extend indefinitely or it can be limited by one or two **endpoints**. Points on a line and endpoints are named by a letter such as point A or point X. A **ray** is a line that has one endpoint.



**FIGURE 1: A Straight Line and a Ray**

**Parallel lines** are lines that run side by side and never intersect or meet. When a truck is moving in a straight line, the front wheels are parallel to each other.

**Intersecting lines** are lines that cross each other.

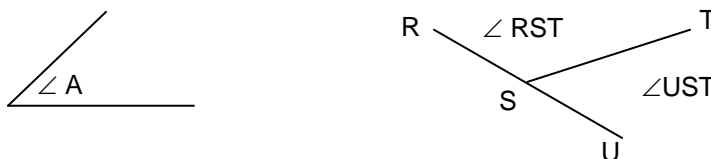


**FIGURE 2: Parallel lines and Intersecting Lines**

### ***DEFINITION OF AN ANGLE***

An **angle** is formed by two rays having the same endpoint. The endpoint of an angle is also called the **vertex**. When we name an angle, we use the word “angle” or the symbol for angle,  $\angle$ .

Angles can be identified by one letter, usually a capital, written at the vertex. See Figure 3, Angle A.



**FIGURE 3: Two Ways Of Naming Angles**

Angles can also be identified by three letters, one for a point on each ray and one for the vertex. Look at Figure 3 and  $\angle RST$  and  $\angle TSU$ .

The vertex is always named second if three letters are used. If two angles share the same vertex, three letters must be used to name the angles so there is no confusion as to which angle is being referred to.

When instructed to use a certain angle while working, you don't always have two distinct lines to consider.

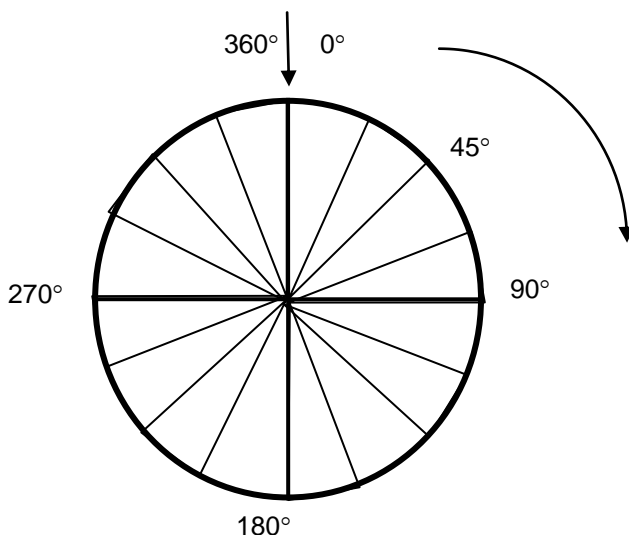
**Example:** If you are told to hold the welding torch at an angle of  $45^\circ$ , you need to know what actually forms the angle. In this case, the steel surface forms the bottom line. The torch in your hand forms the other line. If one end of the torch is almost touching the surface, you can move the other end up or down to change the angle the torch makes with the surface. To make a  $45^\circ$  angle, you want to hold the torch so it is halfway between flat and straight up and down.

### *Measuring Angles*

Angles are divided into units of measurement called **degrees** ( $^\circ$ ). To get a picture of the size of a degree, think of the outside of a circle divided into 360 equal parts with each division marked.

- ◆ Lines drawn from the centre of the circle to each of the 360 dividing marks form 360 equal angles, each measuring 1 degree.
- ◆ To determine the size of any angle, say a  $45^\circ$  angle, you could draw a line from the centre of the circle to one of these degree marks and then counted 45 more marks. If you draw another line from the centre of the circle to this second mark, the two lines would be  $45^\circ$  apart.
- ◆ We use a circular compass that is divided into degree marks to draw an angle of a certain size.
- ◆ The angle is drawn with two rays that meet at a common endpoint. The length of the rays does not affect the size of the angle.

In Figure 4, a circle is divided into angles each measuring 15 degrees. One complete rotation around the circle measures 360 degrees. One quarter of a circle measures  $90^\circ$ . One half of a circle measures  $180^\circ$ . Three quarters of a circle measures  $270^\circ$ .



**FIGURE 4: Degrees in a Circle**  
Some of the angles formed by dividing a circle into  $360^\circ$

**Example:** When the driver on a belt drive rotates through  $360^\circ$ , it makes one full turn or rotation. If you followed one point on the edge of the driver, you would see the mark turn a complete circle or  $360^\circ$  for one rotation. Two complete rotations are  $720^\circ$ .

Interpreting the typical four stroke diesel engine valve timing requires an understanding of angles. The intake and exhaust valves in the cylinder of an engine are opened and closed by the action of the rotating camshaft. Each valve should stay open a specific amount of time.

The timing of the opening and closing of the valves is described by the degrees of crankshaft rotation. On the intake stroke, the valves open when the camshaft rotates to  $16^\circ$  and close at  $36^\circ$ , while on the exhaust stroke, they open at  $60^\circ$  and close at  $16^\circ$ .

### **TYPES OF ANGLES**

**Right angle:** A  $90^\circ$  angle is called a **right angle**. Angle A in Figure 5 is a right angle. The term  $90^\circ$  is often used to describe the position of an object.



**Figure 5: Indicating The Number Of Degrees In An Angle.**

**Example:** When working on with air pressured equipment, you might be told that a  $90^\circ$  elbow in a service line restricts air flow the same amount as 7 feet of straight line. You should have a clear picture of what a  $90^\circ$  bend looks like.

A line drawn through the centre of a  $90^\circ$  angle forms two  $45^\circ$  angles. A  $45^\circ$  angle, Angle JKL, is shown in Figure 5.

**Straight angle:** The  $180^\circ$  angle formed by one half of a complete rotation of a circle is called a *straight angle*. It is an important angle because it forms a straight line.



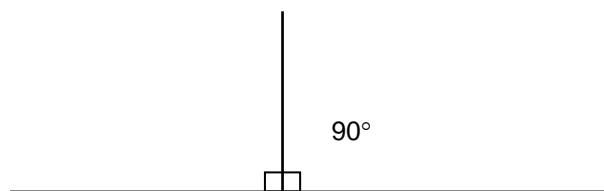
**Figure 6: A Straight Angle**

Other common angles you should be familiar with include the  $30^\circ$  angle and the  $60^\circ$  angle.



**FIGURE 7: A  $30^\circ$  and a  $60^\circ$  Angle**

**Perpendicular lines:** When one straight line extends out from another straight line in such a way that two right angles ( $90^\circ$ ) are formed, the two lines are *perpendicular*.

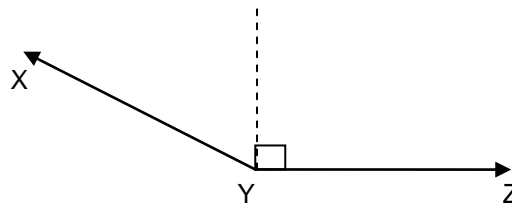


**FIGURE 8: Perpendicular Lines**

Some angles are named by their size relative to right and straight angles.

**Acute angle:** An acute angle is greater than  $0^\circ$  but less than  $90^\circ$ . See Figure 5,  $\angle$  JKL, and Figure 3,  $\angle$  A.

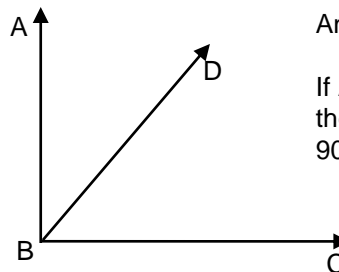
**Obtuse angle:** An obtuse angle is greater than  $90^\circ$  but less than  $180^\circ$ . See  $\angle XYZ$  in Figure 9 and  $\angle ABD$  in Figure 11.



**Figure 9: Obtuse Angle**

### **Complementary Angles**

If the sum of the measurements of two angles is  $90^\circ$ , the angles are called **complementary angles**. If there are two angles within a right angle and you know the measurement of one of them, you can find the measurement of the unknown angle. You do this by subtracting the value of the known angle from  $90^\circ$ .



Angle ABC is a right angle.

If Angle ABD =  $35^\circ$   
then Angle DBC equals  
 $90^\circ - 35^\circ = 55^\circ$

**Figure 10: Finding An Unknown Complementary Angle When The Other One Is Known.**

**Example:** If you have two complementary angles and one angle measures  $40^\circ$ , what is the measure of the other angle?

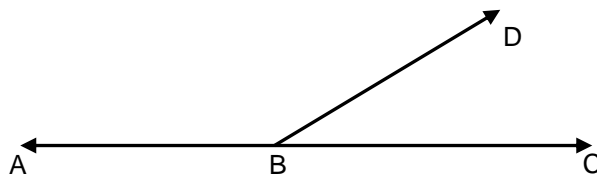
Complementary angles add up to  $90^\circ$ .

$$90^\circ - 40^\circ = 50^\circ$$

The other angle measures  $50^\circ$ .

### **Supplementary Angles**

If the sum of the measurements of two angles adds up to  $180^\circ$ , they are **supplementary angles** (Figure 11). If you have two angles contained within a straight angle, you can find one angle if you know the other. Subtract the known angle from  $180^\circ$ .



Angle ABC is a straight angle equal to  $180^\circ$ .  
If  $\angle ABD$  is equal to  $130^\circ$  then  
 $\angle CBD$  is equal to  $180^\circ - 130^\circ$   
 $180^\circ - 130^\circ = 50^\circ$   
 $\angle CBD = 50^\circ$

**FIGURE 11: Finding An Unknown Supplementary Angle When The Other Angle Is Known**

**Example:**  $\angle RST$  and  $\angle TSU$  are supplementary angles. If  $\angle RST$  is  $120^\circ$ , what is  $\angle TSU$ ?

Since supplementary angles add up to  $180^\circ$ , subtract the known  $\angle RST$  from  $180^\circ$  to find  $\angle TSU$ .

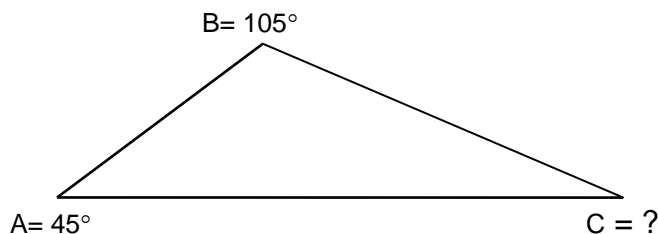
$$180^\circ - 120^\circ = 60^\circ$$
$$\angle TSU \text{ is } 60^\circ.$$

### ANGLES FOUND IN GEOMETRIC FIGURES

A closed geometric figure is made up of straight lines that are joined at distinct endpoints. Where two lines meet at an endpoint, angles are formed. For this reason, descriptions of common figures such as triangles and rectangles include information about the angles at the endpoints of the figures. (We actually use the word *angle* in the names of these figures.) We will look at the angles in rectangles and triangles.

**Rectangles:** A rectangle is a four-sided figure with equal opposite sides that are parallel. The sum of the angles of any four-sided figure is equal to  $360^\circ$ . If you draw a rectangle and measure the four angles in the corners, you will find that each angle in a rectangle is a right angle measuring  $90^\circ$  and the sum of the four angles equals  $360^\circ$ .

**Triangles:** A triangle is a closed, three-sided figure formed by three connected line segments. At each of the three endpoints (or vertices), an angle is formed by the lines of the adjacent (next to each other) sides. See Figure 12.



**FIGURE 12: The Sum Of The Angles Of A Triangle Is  $180^\circ$**

If you draw any triangle and measure its three angles, the value of the three angles adds up to  $180^\circ$ . Because of this relationship, if you know the value of two angles in a triangle, you can find

the value of the third. If you add the values of the two, known angles and subtract the answer from  $180^\circ$  you will have the value of the third angle.

**Example:** If two of the angles in a triangle are  $45^\circ$  and  $105^\circ$ , what is the value of the third angle?

Find the sum of angles A + B.

$$45^\circ + 105^\circ = 150^\circ$$

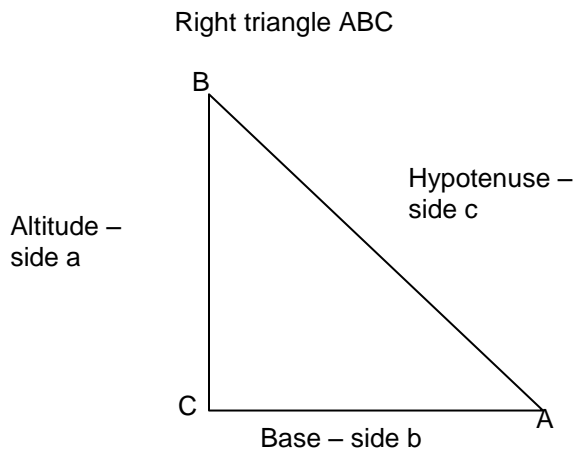
Subtract that sum from  $180^\circ$ .

$$180^\circ - 150^\circ = 30^\circ$$

The third angle, C, is equal to  $30^\circ$ .

### **Right Angle Triangles**

A **right triangle** has one right, or  $90^\circ$ , angle and two acute angles. The side opposite to the right angle is called the **hypotenuse**. The hypotenuse is always the longest side. The side that the triangle rests on is called the **base**. The vertical side is called the **altitude** or **height**. There are some special relationships between the angles and sides of a right triangle.



**FIGURE 13: Parts of a Right Triangle**

### **Pythagoras' Theorem**

You can find the length of an unknown side of a right triangle by using Pythagoras' theorem. The theorem states that in a right triangle:

*The square of the hypotenuse is equal to the sum of the squares of the other two sides.*

The formula expressing this theorem for  $\Delta ABC$  is:

$$c^2 = a^2 + b^2 \quad \text{where } c \text{ is the hypotenuse, } a \text{ is the altitude and } b \text{ is the base.}$$

To calculate  $c$ , find the square root of both sides.

$$c^2 = \sqrt{a^2 + b^2}$$

Using a calculator lets you find the square root of a number easily and accurately.

**Example:** Find the hypotenuse of a right triangle if the altitude is 4 cm and the base is 3 cm.

$$a = 4 \text{ cm}$$

$$b = 3 \text{ cm}$$

$$c = ?$$

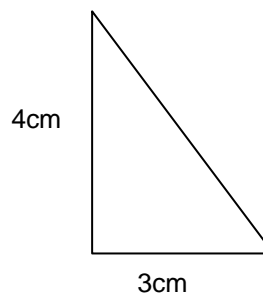
$$c^2 = a^2 + b^2$$

$$c = \sqrt{a^2 + b^2} =$$

$$c = \sqrt{4^2 + 3^2}$$

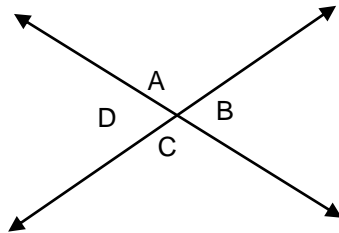
$$c = \sqrt{25}$$

$$c = 5 \text{ cm}$$



### ANGLES FORMED BY A PAIR OF INTERSECTING LINES

When two lines intersect, four angles are formed. The following diagram, Figure 14, shows two intersecting lines forming four angles. A pair of intersecting lines provides us with some interesting information about the angles they form.



**FIGURE 14:** Four Angles  $\angle A$ ,  $\angle B$ ,  $\angle C$ , and  $\angle D$  Are Formed By Intersecting Lines

**Supplementary angles formed by intersecting lines:** *Supplementary angles* are two angles, that together will form a  $180^\circ$  angle, (a straight line).

There are several interesting relationships between the angles in Figure 14. The intersecting lines are straight lines. We know that lines form straight angles measuring  $180^\circ$ . Therefore, there are four sets of angles that are supplementary angles whose measurements add up to  $180^\circ$ .

- angles  $A + B = 180^\circ$ ,
- angles  $B + C = 180^\circ$ ,
- angles  $C + D = 180^\circ$ , and
- angles  $A + D = 180^\circ$ ,

**Opposite angles formed by intersecting lines:** When two straight lines intersect, the **opposite angles are equal**. We can see this important relationship in Figure 14. The pairs of angles that are opposite each other are equal.

In Figure 14, the opposite angles B and D are equal; the opposite angles A and C are also equal.

The relationships between supplementary and opposite angles formed by intersecting lines enables us to find the value of unknown angles.

**Example:** In Figure 14, if  $\angle A$  is  $110^\circ$  what is the value of  $\angle D$ ?

Angles A and D are supplementary angles whose value adds up to  $180^\circ$ .

$$\begin{aligned}\angle A + \angle D &= 180^\circ \\ \angle D &= 180^\circ - \angle A \\ \angle D &= 180^\circ - 110^\circ \\ \angle D &= 70^\circ\end{aligned}$$

**Example:** In Figure 14, if  $\angle A$  is equal to  $110^\circ$ , what is the value of  $\angle C$ ?

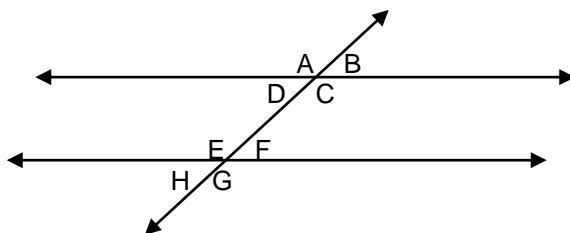
$\angle A$  and  $\angle C$  are opposite so they are equal.  
Both measure  $110^\circ$ .

**Example:** In Figure 14, if  $\angle A$  and  $\angle C$  both measure  $110^\circ$  and  $\angle D$  equals  $70^\circ$ , what does  $\angle B$  equal?

$\angle B$  has the same value as  $\angle D$  because they are opposite angles.  
 $\angle B$  equals  $70^\circ$ .

### ANGLES FORMED BY TWO PARALLEL LINES INTERSECTED BY ANOTHER LINE

Two straight lines are parallel if the distance between them always remains the same. If another line cuts across the parallel lines, eight angles are formed as in Figure 15. The two sets of four angles formed by the intersecting line are identical. Angles A, B, C, and D form one set. Angles E, F, G, and H form the second set.



**FIGURE 15: Sets Of Angles Formed By A Line Intersecting Two Parallel Lines**

**Remember:** any two angles next to each other in the diagram are supplementary angles and their combined value is  $180^\circ$ . Also, angles opposite each other are equal.

**Corresponding angles:** In Figure 15 the two lines that have been intersected are parallel. Some special relationships exist between angles that are in corresponding (similar) positions in each of the two sets. Look at  $\angle A$  and  $\angle E$ . Each is on top of its intersected line and on the left side of the intersecting line.  $\angle A$  and  $\angle E$  are in similar positions and are identical to each other. They are called **corresponding angles**.

Look at Figure 15 as you read the following:

1.  $\angle A$  and  $\angle C$  are opposite and equal, as are  $\angle E$  and  $\angle G$ .
2. The four angles,  $\angle A$ ,  $\angle E$ ,  $\angle C$  and  $\angle G$ , all have the same value.
3. Since  $\angle C$  and  $\angle G$  are equal corresponding angles:
  - a. their supplementary angles,  $\angle B$  and  $\angle F$ , are also equal to each other.
  - b.  $\angle D$  and  $\angle H$  are opposite and equal to  $\angle B$  and  $\angle F$ .
4. Therefore,  $\angle B$ ,  $\angle D$ ,  $\angle F$  and  $\angle H$  all have the same value.

*You only need to know the value of **one** angle out of the **eight** angles formed by a line intersecting two parallel lines in order to find the value of the rest of the angles.*

As long as you know one angle and you keep straight what the corresponding and supplementary angles are, you can find all the other angles.

**Note:** All the information above applies no matter in what directions the parallel lines and the intersecting lines run. Figure 14 shows the parallel lines running horizontally but you could get questions where they run in any direction. In this case, first determine which are the two parallel lines and which is the intersecting line.

**Example:** Use Figure 15. If  $\angle A$  is  $120^\circ$ , what is the measurement of  $\angle B$ ? What are the values of the other angles?

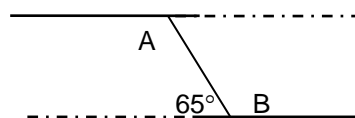
Angles A and B are supplementary. Therefore,  
 $\angle B = 180^\circ - 120^\circ = 60^\circ$

$\angle A$  equals  $\angle E$ ,  $\angle C$  and  $\angle G$ . So, they all equal  $120^\circ$ .  
 $\angle E = 120^\circ$   
 $\angle C = 120^\circ$  and,  
 $\angle G = 120^\circ$

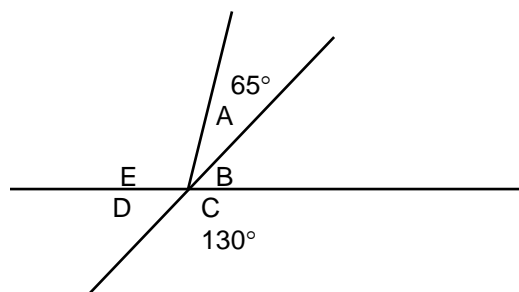
$\angle B$  equals  $\angle D$ ,  $\angle H$  and  $\angle F$ . They all equal  $60^\circ$ .  
 $\angle D = 60^\circ$   
 $\angle H = 60^\circ$  and,  
 $\angle F = 60^\circ$

**Example:** Use the diagram below for this question. If  $\angle A$  equals  $\angle B$ , what is the value of  $\angle A$ ?

$$\begin{aligned} \angle B + 65^\circ &= 180^\circ \quad (\text{supplementary angles}) \\ \angle B &= 180^\circ - 65^\circ = 115^\circ \\ \angle B &= \angle A = 115^\circ \end{aligned}$$



**Example:** Use the diagram below for this problem. In the drawing,  $\angle A$  and  $\angle E$  added together form an opposite angle to angle C. If  $\angle A = 25^\circ$ , what is the value of  $\angle E$ ?



$$\begin{aligned} \angle A + \angle E &= \angle C \\ \text{so } \angle E &= \angle C - \angle A \\ \text{if } \angle C &= 130^\circ \\ \text{and } \angle A &= 25^\circ \\ \text{then } \angle E &= 130^\circ - 25^\circ \\ \angle E &= 105^\circ \end{aligned}$$

Here is another way to solve this problem:

$$\begin{aligned} \text{For } \angle B \\ \angle D &= \angle B \\ \angle D + \angle C &= 180^\circ \\ \angle D &= 180^\circ - 130^\circ \\ \angle D &= 50^\circ \\ \angle B &= 50^\circ \\ \text{and for } \angle E \\ \angle A + \angle B &= 25^\circ + 50^\circ \\ &= 75^\circ \\ \angle A + \angle E + \angle B &= 180^\circ \\ \angle E &= 180^\circ - 75^\circ \\ &= 105^\circ \end{aligned}$$

### In Brief

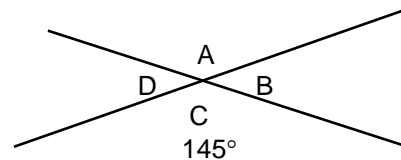
Here are some angle facts to remember:

- The sum of a pair of complementary angles is  $90^\circ$ .
- The sum of a pair of supplementary angles is  $180^\circ$ .
- The sum of the angles in a triangle is  $180^\circ$ .
- The sum of the angles in a rectangle is  $360^\circ$ .
- When two straight lines intersect, the opposite angles are equal.
- Corresponding angles formed when a line intersects two parallel lines are equal.

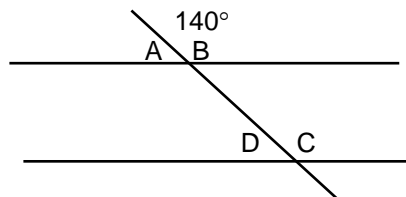
**Answer these questions on angles and their relationships.** Look back at the examples for guidance. **Answers are on the last page.**

1. A complete rotation of the circumference of a circle is equal to \_\_\_\_\_ degrees.
2. A right angle measures \_\_\_\_\_ degrees.
3. A straight angle measures \_\_\_\_\_ degrees.
4. The sum of the three angles in a triangle adds up to \_\_\_\_\_ degrees.
5. The sum of the angles in a four-sided figure adds up to \_\_\_\_\_ degrees.
6. If two angles in a triangle are equal to  $55^\circ$  and  $80^\circ$ , what is the value of the third angle? \_\_\_\_\_
7. If a line extends at a right angle from another line, the two lines are \_\_\_\_\_ to each other.
8. When two straight lines intersect, the \_\_\_\_\_ angles are equal.

9. In this drawing  $\angle C$  is  $145^\circ$ :  
Answer the following:  
 $\angle A$  is equal to \_\_\_\_\_.  
 $\angle B$  is equal to \_\_\_\_\_.  
 $\angle D$  is equal to \_\_\_\_\_



10. If two parallel lines are intersected by another line, two sets of four angles are formed. In each set, opposite angles are \_\_\_\_\_ and the value of any two angles next to each other adds up to \_\_\_\_\_ degrees.
11. Use the drawing below. If  $\angle B$  is equal to  $140^\circ$ , what is the value of  $\angle A$ ? \_\_\_\_\_  
What is the value of  $\angle D$ ? \_\_\_\_\_



12. Use the drawing opposite to answer these questions.

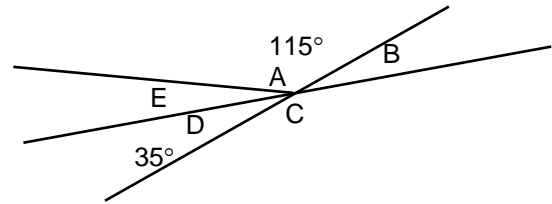
$$\angle A = 115^\circ \text{ and } \angle D = 35^\circ$$

What is the value of  $\angle E$ ?

$$(\angle A + \angle D + \angle E = 180^\circ.) \quad \underline{\hspace{2cm}}$$

What is the value of  $\angle B$ ?  $\underline{\hspace{2cm}}$

What is the value of  $\angle C$ ? ( $\angle C$  is equal to  $\angle A + \angle E$ )  $\underline{\hspace{2cm}}$



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**ANSWER PAGE**

1.  $360^\circ$
2.  $90^\circ$
3.  $180^\circ$
4.  $180^\circ$
5.  $360^\circ$
6. angles in a triangle =  $180^\circ$   
 $\angle 1 + \angle 2 = 55^\circ + 80^\circ = 135^\circ$   
 $\angle 3 = 180^\circ - 135^\circ = 45^\circ$
7. perpendicular
8. opposite
9.  $145^\circ$   $\angle A$  is opposite angle to  $145^\circ$   
 $35^\circ$   $\angle B$  is supplementary to  $\angle A$   $180^\circ - 145^\circ = 35^\circ$   
 $35^\circ$   $\angle D$  is opposite to  $\angle B$ , and is supplementary to  $\angle A$
10. equal,  $180^\circ$
11.  $40^\circ$   $\angle A$  is supplementary to  $B$   
 $40^\circ$   $\angle D$  is corresponding angle to  $\angle A$
12.  $30^\circ$   $\angle$ 's  $A$ ,  $E$ , and  $D$  are supplementary.  
 $35^\circ$   $\angle$ 's  $D$  and  $B$  are complementary.  
 $145^\circ$