EVALUATING
ACADEMIC READINESS
FOR APPRENTICESHIP TRAINING
Revised for
ACCESS TO APPRENTICESHIP

MATHEMATICS SKILLS
RATIOS AND PROPORTIONS

AN ACADEMIC SKILLS MANUAL
for
The Small Motor Service Trades

This trade group includes the following trades
Marine & Small Powered Equipment Mechanic
Motorcycle Mechanic, and Small Motor Mechanic

Workplace Support Services Branch
Ontario Ministry of Training, Colleges and Universities

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In preparing these Academic Skills Manuals we have used passages, diagrams and questions similar to those an apprentice might find in a text, guide or trade manual.

This trade related material is not intended to instruct you in your trade. It is used only to demonstrate how understanding an academic skill will help you find and use the information you need.

MATHEMATICS SKILLS
RATIO AND PROPORTION

An academic skill required for the study of the Small Motor Service Trades

INTRODUCTION

Comparing Numbers

Numbers can be compared in a variety of ways. We can compare numbers by noting their difference.

> **Example:** If one car sells for $36,000 and another car sells for $18,000, we say the first car costs $18,000 more than the second. In this comparison, we subtract to find the difference.

We can also compare the cost of the two cars by division.

> **Example:** If we divide the cost of the first car by the cost of the second ($36 000 ÷ $18 000 = 2), we find that the first car costs twice as much as the second.

We can compare by using a ***ratio***. *A **ratio** compares numbers in a form that indicates division.* Usually the numbers in a ratio are reduced to lowest terms but not actually divided.

Ratios give useful information about the relationship between numbers. Ratios can be used to describe such things as the relationship between teeth on gears and speed ratios on pulleys, chains and belts. They are also important in interpreting measuring tools.

Examples:

To calculate a gear ratio, we can divide the rpm of the driving gear by the rpm of the driven gear.

- A gear ratio of 2 to 1 (or 2:1) means the driving gear is rotating twice as fast as the driven gear.

A gear ratio can also be found by dividing the number of teeth on the driving gear by the number of teeth on the driven gear.

- In the gears mentioned above, the driving gear has twice as many teeth as the driven gear.

*Two equal ratios can be set up as an equation called a **proportion***. Proportions can be used to find an unknown, fourth quantity if the other three ratio quantities are known.

Example:
A portable engine might require a mix of 1 part of oil with 40 parts of gasoline. We have to be sure to put 40 times more gas than oil into the mixture. We use a proportion to figure out how much gas and how much oil to use. A proportion tells us that if we use 20 liters of gas, we must use a 1/2 liter of oil.

This skills manual looks at the following topics concerning ***ratio, proportion, and scale***:

- Ratio, including
  - finding ratios from given information
  - rates
- Proportions, including
  - direct and indirect (inverse) proportions
  - solving a proportion when three out of four terms are known
  - solving problems using proportions

RATIO

Ratio as a Comparison
Comparing two numbers by writing a ratio: If one measuring tape sells for $12 and another is $36, we say the second costs three times as much. We divide the second number by the first. *The comparison of two numbers by division is called a **ratio**.*

You don't always have to know exact amounts for a ratio to be useful.

Example: A survey finds that 6 people out of 10 prefer an engine made by Brand X over the one made by Brand Y. We don't know the actual number of people who prefer each brand but we know the relationship between the two numbers. If we ask 10 people about their preference, likely 6 of them will like Brand X better. The ratio of preference is 6 to 10.

There are several ways to indicate this ratio:

- **By comparing one amount to another**, as when we say 6 out of 10.
- **By putting a colon between the numbers.** The ratio is written 6 : 10. We read this as "*the ratio of six to ten*".
- **By writing the ratio as a fraction.** The first number being compared becomes the numerator, which is placed over the second number, the denominator. The fraction is usually written in lowest terms. So 6 out 10 becomes 6/10 and can be reduced to 3/5.

When you write a ratio, you don't actually do the division unless you want one of the terms of the ratio to be 1.

Lowest terms: The ratio 3:4 is already in lowest terms. The ratio 8 to 32 is not in lowest terms. When this ratio is reduced to lowest terms, it is written as 1 to 4. A ratio, like a fraction, is usually, but not always, written in lowest terms.

To reduce a fraction or a ratio to lowest terms:

1. Look for a number (a common factor) that will divide evenly into the numerator and denominator of the fraction or the terms of the ratio.
2. Divide the common factor into the numerator and the denominator or into each term.
3. Continue dividing until there are no more common factors.
4. The last division answers form the fraction or ratio in lowest terms.

Notice a ratio has no units. If we are comparing the number of teeth on one gear to the number of teeth on the second gear, so the units, which are teeth, cancel out. When the two numbers being compared have the same unit of measurement, there are no units in the ratio.

Ratios with 1: The ratio 2:1 has the number 1 as one of its terms. The ratio 3:4 does not. Sometimes a ratio like 3:4 is more useful if one of the terms is 1. You could divide both terms by 4 and then express the ratio as .75 to 1, or you could divide both terms by 3 and express the ratio as 1 to 1.33.

Equivalent ratios: Reducing a fraction to lowest terms does not change the value of the fraction, nor will it change the value of a ratio. The fractions 2/8 and 4/16 can each be reduced to 1/4. 1/4, 2/8, and 4/16 are ***equivalent fractions***. They each represent the same amount.

In the same way, ratios representing the same amount are called ***equivalent ratios***. The ratio 3 to 4 and the ratio .75 to 1 represent the same comparison and are equivalent ratios.

Finding a Ratio from Given Information

Before using ratios to solve problems, we will look at setting up ratios from given information.

Questions that ask you to set up ratios are generally worded in one of two ways.

1. You might need to compare part of an amount to the total amount; or
2. You might be asked to compare two parts to each other.

Situation one: You are asked to compare part of the amount to the total amount. If the total amount isn't given, you first have to find it.

Example: A class of apprentices consisted of 6 women and 24 men. What is the ratio of women to the whole class and the ratio of men to the whole class?

First you have to find the total number of students.
Adding 6 + 24 gives a total of 30 apprentices in the class.

Now find the ratios:

a) Ratio of women to the whole class is 6 out of 30, reduced to 1 out of 5, 1/5 or 1:5.
b) Ratio of men to the whole class is 24 out of 30, reduced to 4 out of 5, 4/5 or 4:5.

Situation two: The question asks you to compare one amount to another. This time you don't need to know the total.

Example: Using the class of 6 women and 24 men, what is the ratio of women to men and men to women?

Ratio of women to men is 6 to 24, reduced to 1 to 4, 1/4 or 1:4.

Ratio of men to women is 24 to 6, reduced to 4 to 1, 4/1 (or 4:1.
Note: if the denominator is 1 when writing a ratio, you must show it)

General Rules For Reading And Writing Ratios

Rule 1: *When you read or write ratios, compare the terms in the same order as they are given, unless they are part of a table or formula.*

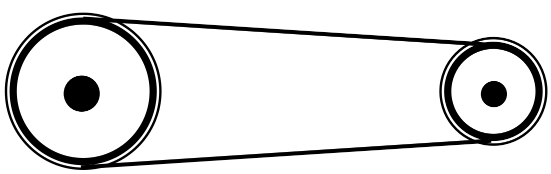
Example: To compare the number of women to the class total, the number of women is stated before the class total.

Ratio of women to class = 6:30
This is reduced to 1:5.

To compare the number of men to women, the number of men is written before the number of women.

Ratio of men to women = 24:6
This is reduced to 4:1.

Example: What is the speed ratio of pulley A to pulley B in Figure 1?



PULLEY A
20 cm in diameter

PULLEY B
10 cm in diameter

FIGURE 1:Finding The Speed Ratio Of Two Pulleys

Look at Figure 1.

- Pulley B (20 cm) is twice as large as pulley A (10 cm).
- In one turn of pulley A, the belt will move a distance equal to the circumference of pulley A.
- But one turn of pulley A will cause pulley B to make only one half a turn because it is twice as large.
- Therefore, pulley A is turning twice as fast as pulley B.
- The speed ratio of A to B is 2:1.

In meshed gears, the speed ratio of the driven gear to the driving gear is found by dividing the speed of the driven gear in rpm by the speed of the driving gear.

Example: What is the speed ratio if the speed of the driven gear is 60 rpm and the speed of the driving gear is 160 rpm?

Divide 60 by 160 to get .375.
The speed ratio is .375:1.

Rule 2: *If the units of each term in the ratio are the same, they cancel each other out and are not written in the ratio.*

The ratio of 25 centimeters to 1 meter is not 25:1. The ratio has to be written as 25 cm to 1 m or 25cm:1m.

Usually it is easier to work with ratios if there are no units, so make the units the same. If you convert 1 meter to 100 centimeters, the units will be the same. You can then cancel them out. The ratio is then written as 25:100 without any units.

If you can't write the ratio with the same unit for all terms, the units must remain in the ratio.

Example: The compression ratio of an engine is found by comparing the cylinder volume when the piston is at the bottom of the cylinder to when the piston is at the top of the cylinder. What is the compression ratio if the volume at the bottom of the cylinder is 126 cubic inches and the volume at the top is 9 cubic inches?

Divide 129 cubic inches by 9 cubic inches to get 14.
The units, cubic inches, are the same, so they cancel.
The compression ratio is 14:1.

Rule 3: *Ratios without units are usually expressed in lowest terms.*

Example: Find the gear ratio of two meshed gears if gear A has 88 teeth and gear B has 32 teeth.

Divide 88 teeth by 32 teeth to get 2.75.
The units cancel.
The division answer is the first term; the second term is 1.
The gear ratio is 2.75:1.

Rates

When units of the quantities in a ratio are the same, they cancel out and so are not shown. When units in a ratio are different, or there is only one unit, the units must be included in the ratio.

Ratios can be used to compare quantities of different types, such as kilometers per hour or cost per kilowatt-hour. These comparisons are called *rates*. *A **rate** is a quantity or amount of something measured per unit of something else.* A rate includes the word "per".

Usually the ratio is divided so the amount of the unit following the word “per” is 1. If a rate involves two different kinds of units, they must be included in the ratio.

Example: Driving speed is a rate. Say you drove 300 km in 3 hours. The ratio 300 km/3 hr is reduced to 100 km/1 hr or 100 km/hr. Your rate of speed is 100 km per hr.

Rates that involve a cost per unit, such as the rate we pay for electricity, include the dollar sign. Your electrical bill might say that your electrical rate is $.30/kw-hr. For every kilowatt-hour of electricity you use, you pay $.30.

Answer the following questions on ratios. Express the ratios using a colon between the two quantities. Reduce to lowest terms. If the quantities being compared are not in the same units, convert where possible. **Check your answers, which are on the last page, as you proceed.**

1. Write as ratios using the colon form. Don’t forget to reduce.

   a) 1 to 4 b) 5 to 8 c) 6 m to 3 m d) 20 in to 45 in

   e) 15 L to 9 L f) 3 m to 90 cm g) 5 in to 1 ft h) 2 kg to 125 g

   i) 1 m to 50 cm j) 15 min to 1 hr k) 5 ft to 6 ft 6 in l) one nickel to a quarter

2. Find the ratio of two meshed gears if one gear has 72 teeth and the other gear has 36 teeth.

3. What is the rate of speed if you travel 400 km in 4 hr?

4. What is the cost of gas per liter if you pay $5.90 for 10 L?

5. What is the speed ratio of a chain drive if the driven sprocket has 60 teeth and the drive sprocket has 20 teeth?

PROPORTIONS

Two equivalent ratios express the same relationship but are written using different but related terms or numbers. For example, 1/4 and 2/8 are equivalent ratios and they represent the same amount. We can say that 1/4 equals 2/8. We can write this statement as:

1/4 = 2/8

Equivalent ratios written in fraction form with an equal sign between them form a ***proportion****.*

- A proportion has four *terms*, or parts.
  - The terms of the proportion above are 1, 4, 2 and 8.
  - When we read the proportion, we name all four terms. 1/4 = 2/8 is read as "1 to 4 equals 2 to 8."

Direct and Indirect (or Inverse) Proportions

There are two basic types of proportions: direct proportions and indirect (sometimes called inverse) proportions.

In a ***direct proportion****, as one quantity increases, the corresponding quantity also increases.* Similarly, as one quantity decreases, the other one also decreases.

Example: The relationship between the size of drill bit you choose and the size of hole you drill is a direct proportion.

- The larger the bit, the larger the hole.
- The smaller the bit, the smaller the hole.

This is a direct proportion because as the bit changes in size, the hole changes in size *in the same way*.

In an ***inverse, or indirect, proportion****, as one quantity increases, the corresponding quantity decreases.* As one quantity decreases, the other one increases.

Example: The relationship between the number of teeth on a gear and the speed of the gear is an indirect proportion. The number of teeth in a gear determines the amount of torque or turning force.

- But the more teeth on a gear, the less speed there is available.
- As one quantity (the number of teeth or the torque) increases, the other quantity (speed) decreases.
- You cannot have an increase in both torque and speed in one gear.
- Torque is inversely proportional to speed.

Solving a Proportion When Three of Four Terms Are Known

Proportions such as 1/4 = 2/8 in the example above don't tell us much. We already know that two ratios or fractions that represent the same amount equal each other. However, if we only know three of the four terms, a proportion can be used to find the fourth term.

The following are the general steps of finding an unknown amount using a proportion. Here are the steps to find the fourth, unknown term:

Here are the steps to find the fourth, unknown term:

1. **Set up a proportion using a letter to represent the unknown amount** in one of the ratios. The letter can be manipulated (moved around) in an equation just like a number. Write the ratios with an equal sign between them, forming an equation.

   Example: Write the proportion using the two ratios n:10 and 8:20.

   $$\frac{n}{10} = \frac{8}{20}$$

2. ***Cross-multiply to get rid of the denominators on both sides.*** To cross-multiply, multiply the diagonal numbers across the equal sign. In other words, multiply the numerator of one ratio by the denominator of the other ratio.

   If an unknown term is represented by a letter, cross multiply in the same way.

   Example: Cross-multiply in the equation below to get rid of the denominators.

   $$\frac{n}{10} = \frac{8}{20}$$

   Notice that n represents the unknown term.

   Multiply n by 20 and 10 by 8. Keep the equal sign.

   20n = 10(8)

   20n = 80

3. ***Isolate the unknown term*** (get it alone on one side of the equal sign). To do this divide both sides by the number in front of the unknown term.

   Example: Isolate n in the following equation.

   20n = 80

   $$\frac{20n}{20} = \frac{80}{20}$$

   Divide both sides by 20.

   n = 4

Here are some other manipulations that can help isolate the letter representing the unknown term.

A. If the letter representing the unknown term is on the right side, reverse the equation before dividing. You can reverse an equation without changing its value.

Example: You can reverse:

3(15) = 5n to 5n = 3(15).

Both equations have the same value.

B. You can invert (turn all of the terms upside down) both sides of the equation without changing its value.

Example: You can invert

4/s = 5/6 to s/4 = 6/5.

Both equations have the same value.

Note: *If you invert one side of an equation, you must invert the other side to keep the equation equal.*

Now let's look at some examples of finding an unknown term in a proportion using these steps.

Example: Solve for n in the following proportion.

$\frac{4}{5} = \frac{n}{15}$ Set up the proportion

4(15) = 5n cross-multiply

60 = 5n

5n = 60 Reverse the equation so that n is on the left side of the equal sign.

5n ÷ 5 = 60 ÷ 5 Divide both sides of the equation by the number in front of the unknown term.

n = 12 The letter is isolated on the left hand side of the equation.
The answer is on the right hand side

Substitute 12 for n to write the complete proportion.

$\frac{4}{5} = \frac{12}{15}$

Example: Find the value of n when:

$$\frac{n}{12} = \frac{5}{15}$$

15n = 5(12)	cross multiply
5n = 60	divide by 15 to isolate n

$$\frac{15n}{15} = \frac{60}{15}$$

n = 4	
4/12 = 5/15	Substitute 4 for n to write the complete proportion.

Example: Find the value of n when:

$$\frac{n}{8} = \frac{10}{16}$$

16n = 10(80)	cross multiply
16n = 80	divide both sides by 16
n = 5	

Example: Find the value of s.

$$\frac{3}{4} = \frac{9}{s}$$

3s = 9(4)	cross multiply
3s = 36	
3s ÷ 3 = 36 ÷ 3	divide by 3
s = 12	

Solving Problems Using Proportions

Proportions can be used to solve problems. You have to figure out what goes with what and then set up your proportion to find the unknown quantity. Notice that when you first set up your ratios, you do not usually reduce to lowest terms.

Method 1: These suggestions are one method to set up a proportion.

a) Set up the ratios (or fractions) so the same units are over each other.
    a. Set up minutes over minutes, kilometers over kilometers, or meters over meters.
b) The units of the two given quantities that form one fraction will cancel out.
    a. The unit of the third known quantity will be the unit of the unknown quantity
c) Set up the smaller unit over the larger unit. The proportion will look like this:

$$\frac{\text{small}}{\text{large}} = \frac{\text{small}}{\text{large}}$$

Example: If it takes 50 minutes to travel 25 kilometers, how far will you travel in 100 minutes at the same speed?

Setting up the proportion:
This is a direct proportion.
As the amount of time increases, the distance gets longer.
The unknown distance will be larger than 25 kilometers and will go underneath.
Let n represent the unknown distance.
Put the same units together and the smaller quantities over the larger.

The proportion looks like this:

$$\frac{50\ \cancel{\text{min}}}{100\ \cancel{\text{min}}} = \frac{25\text{ km}}{n}$$ minutes cancel

Solve the proportion to find the value of n.

$$\frac{50}{100} = \frac{25\text{ km}}{n}$$

$50n = 25(100)$ Cross multiply.
$50n = 2500$

$n = 2500 \div 50$ Divide both sides by 50.
$n = 50$ km

You will travel 50 kilometers in 100 min.

Method 2: You can also set up the two ratios so each is given as a rate. When the ratios are set up as rates, in each ratio, one unit is over the other, different, unit. The following example shows how to set up the proportion.

Example: The strength of a piece of wire is directly proportional to the square of its diameter. If a .25 inch wire will support a load of 4 kg, what load will a .75 in wire support?

We know this is a direct proportion because the question says the quantities are "directly proportional." Set up the proportion so the smaller load is over the thinner wire and the larger load is over the thicker wire.
Let X represent the larger load.

$$\frac{4}{.25} = \frac{x}{.75}$$

.75(4) = .25(x)

3.00 = .25x

12 = x

A 75 in wire will support a load of 12 kg.

Example: Find the speed in rpm of a driven gear with 80 teeth if the driving gear has 40 teeth and is rotating at 120 rpm.

We will use the first method. This is an indirect proportion. The smaller number of teeth is related to the larger speed. The smaller gear has a speed of 120 rpm; therefore, the larger gear with 80 teeth must have a slower speed than 120 rpm. Set up the proportion with the smaller gear over the larger gear and the slower, unknown speed over the faster speed:

$$\frac{\text{smaller number of teeth}}{\text{larger number of teeth}} = \frac{\text{slower speed}}{\text{faster speed}}$$

Let s represent the unknown speed.

$\frac{40 \text{ ~~teeth~~}}{80 \text{ ~~teeth~~}} = \frac{s}{120 \text{ rpm}}$	teeth cancel
40 x 120 = 80s	Cross multiply.
80s = 4800	Reverse the equation.
s = 4800/80	Divide by 80.
s = 60 rpm	

The speed of the driven gear is 60 rpm.

Example: If an engine requires an oil to gas mixture of 1 : 18, and you are putting 3 L of gas into the engine, how much oil do you need to add?

This is a direct proportion. The more gas you put in, the more oil you need. Since there is more gas than oil in the mix, the amount of gas goes on the bottom because it is the larger quantity.

$$\frac{1}{18} = \frac{n}{3}$$

$3 = 18n$	cross multiply
$18n = 3$	reverse the equation
$n = 3/18$	divide by 18
$n = 1/6$ L	

You need to add 1/6 L oil to 3 L gas for an engine that requires a 1:18 ratio.

Here are some questions on proportions. Answers are on the last page.

6. Solve for the unknown quantity.

a) $\frac{n}{24} = \frac{1}{2}$ b) $\frac{2}{x} = \frac{10}{10}$ c) $\frac{16}{2} = \frac{s}{3}$

d) $\frac{5}{10} = \frac{12}{n}$ e) $\frac{n}{7} = \frac{3}{21}$ f) $\frac{2}{6} = \frac{n}{7.35}$

7. If it takes 70 minutes to travel 35 km, how long will it take to travel 85 km at the same speed?

8. A gear with a diameter of 20 cm is turning at 400 rpm. Find the rpm of a gear with a diameter of 80 cm that is turning with it. (This is an inverse proportion.)

9. If an engine requires a 1:20 oil to gas mixture, how much oil has to be added to 70 L of gas?

10. If a quart of paint covers 12 sq ft, how much paint is needed to cover 72 sq ft?

11. Six mechanics can rebuild a motorcycle in 3 days. How long would it take 3 workers to do the same job?

12. Bolts costs $32.50 for 100. How much would 360 bolts cost?

ANSWER PAGE

RATIOS

1. a) 1:4
   b) 5:8
   c) 2:1
   d) 4:9
   e) 5:3
   f) 10:3 (change m to cm)
   g) 5:12 (change 1 ft to 12 in.)
   h) 16:1 (change kg to g)
   i) 2:1 (change m to cm)
   j) 1:4 (change hr. to min.)
   k) 10:13 (change 5' to 60" and 6' 6" to 78")
   l) 1:5

2. 2:1
3. 100 km/hr
4. $.59/L
5. 3:1

PROPORTIONS

6. a) 12
   b) 8
   c) 24
   d) 24
   e) 1
   f) 2.45

7. 70/n = 35/8
   70 x 85 = 35n
   35n = 70 x 85
   35n = 5950
   n = 5950 ÷ 35
   n = 170 min

8. x/400 = 20/80
   x = 400(20/80)
   x = 100 rpm

9. 1/x = 20/70
   20x = 70
   x = 3.5 L

10. 1/x = 12/72
    12x = 72
    x = 6 qt

11. 3/6 = 3/x
    x = 6 days

12. 100/360 = 32.50/x
    100x = 360(32.50)
    100x = 11700
    x = $117.00