

**EVALUATING
ACADEMIC READINESS
FOR APPRENTICESHIP TRAINING**
Revised for
ACCESS TO APPRENTICESHIP

**MATHEMATICS SKILLS
CALCULATION OF AREA AND VOLUME OF
GEOMETRIC FIGURES**

**AN ACADEMIC SKILLS MANUAL
for**

The Small Motor Service Trades

This trade group includes the following trades
Marine & Small Powered Equipment Mechanic
Motorcycle Mechanic, and Small Motor Mechanic

*Workplace Support Services Branch
Ontario Ministry of Training, Colleges and Universities*

Revised 2011

In preparing these Academic Skills Manuals we have used passages, diagrams and questions similar to those an apprentice might find in a text, guide or trade manual.

This trade related material is not intended to instruct you in your trade. It is used only to demonstrate how understanding an academic skill will help you find and use the information you need.

MATHEMATICS SKILLS: CALCULATION OF AREA AND VOLUME OF GEOMETRIC FIGURES

*An academic skill required for the study of the
Small Motor Service Trades*

INTRODUCTION

To make sure that an engine is working properly, you have to constantly make measurements. You might have to measure the width of the engine cylinder or you might have to find the perimeter of a gasket. You might also need to calculate the volume of a gas tank.

This skill sheet reviews the steps in finding the perimeter, area and volume of simple two and three dimensional geometric figures, including:

- ◆ Two dimensional figures
- ◆ Finding the perimeter
- ◆ Finding the area
- ◆ Calculating the cost of covering an area
- ◆ Three dimensional figures
- ◆ Finding the surface area
- ◆ Finding the volume

TWO DIMENSIONAL GEOMETRIC FIGURES

A simple, closed, two dimensional (flat) figure with three or more straight sides is called a **polygon**.

- Triangles, squares, rectangles, and parallelograms (figures with 2 pair of opposite sides parallel) are all examples of polygons.

A **circle** is also a flat, closed figure but it is a curve, consisting of points that are all the same distance from the center.

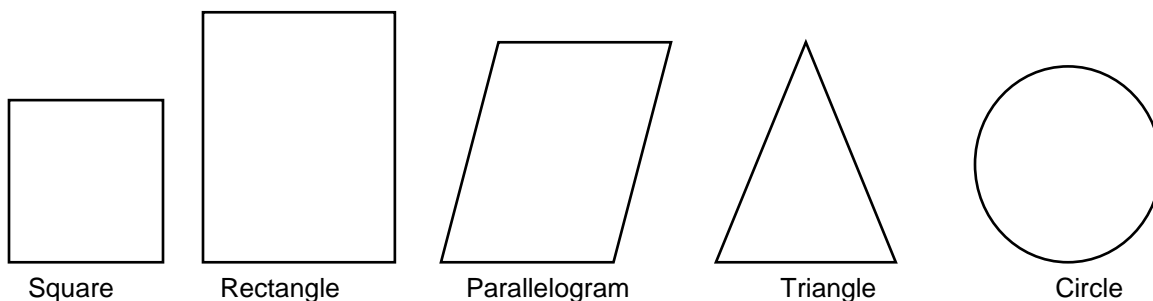


FIGURE 1: Some simple geometric shapes

These figures can be measured in different ways.

- Whenever we use measurements to make calculations with geometric figures, all measurements must be in the same linear units.
 - The units might be meters or centimeters, but they can't be a mix of meters and centimeters.

FINDING THE PERIMETER

The **perimeter** (P) of any polygon is the distance around its boundary. Perimeter is found by adding together the lengths of the sides.

Perimeter of a Rectangle

A **rectangle** is a polygon with four 90° angles (right angles) and with each pair of parallel sides the same length. (see Figure 1)

- This means that we can find the perimeter of a rectangle by adding the lengths of the two long side to the lengths of the two shorter side.

The perimeter of a rectangle equals twice the length (l) added to twice the width (w). The formula is written in two forms:

$$P = 2l + 2w \quad \text{where } P \text{ is the perimeter, } l \text{ is the length and } w \text{ is the width of the rectangle.}$$

$$\text{or } P = 2(l + w)$$

Remember: When finding perimeter, all units must be the same. If the length is measured in feet and the width in yards, one unit must be changed to that of the other.

Example: Find the perimeter of a garage that is 30 m long and 16 m wide.

$$\begin{aligned} p &= 2l + 2w \\ &= 2(30 \text{ m}) + 2(16 \text{ m}) \\ &= 60 \text{ m} + 32 \text{ m} \\ &= 92 \text{ m} \end{aligned}$$

The perimeter is 92 m.

Example: Find the amount of fencing required to close in a space that is 400 yd wide and 1500 ft long.

Known:

$$l = 1500 \text{ ft}$$

$$w = 400 \text{ yd} = 1200 \text{ ft} \qquad 400 \text{ yd} \times 3 = 1200 \text{ ft}$$

Find perimeter (P)

$$\begin{aligned} P &= 2(l + w) \\ &= 2(1500 \text{ ft} + 1200 \text{ ft}) \\ &= 2(2700 \text{ ft}) \\ &= 5400 \text{ ft} \end{aligned}$$

The space will require 5400 ft of fencing.

Perimeter of a Square

A square is a rectangle with all four sides the same length.

To find the perimeter of a square, multiply the length by 4.

$$\text{Perimeter of a square} = 4l$$

Finding the Length of an Unknown Side When the Perimeter Is Known

If you know the perimeter of a rectangle and the length of one side, you can find the other side.

1. Manipulate (or rearrange) the variables in the formula for perimeter so the letter for length or width is by itself on the left side.
2. Solve to find the unknown side.
3. *Remember, whatever you do to one side of the formula, you need to do to the numbers and letters on the other side.*

Example: The perimeter of the bed of a truck is 20 m. The length of the truck is 8 m. What is the width?

Known

$$P = 20 \text{ m}$$

$$l = 8 \text{ m}$$

Find w

$$\begin{aligned} P &= 2l + 2w \\ 20 &= 2(8) + 2w \\ 20 &= 16 + 2w \\ 16 + 2w &= 20 && \text{Reverse the equation.} \end{aligned}$$

$$16 - 16 + 2w = 20 - 16 \quad \text{Subtract 16 from both sides to isolate } w.$$

$$2w = 4 \quad \text{Divide both sides by 2.}$$

$$w = 2\text{m} \quad \text{Write in the units, meters.}$$

The width is 2 m.

FINDING THE AREA

The **area** of a polygon is the measure of the surface inside the boundary. The units of area are squared units.

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Area of a Rectangle

The area of a rectangle is the amount of surface enclosed within its boundaries of **length** and **width**.

Example: The area of a room is the amount of floor space it has.

Area is calculated by multiplying the length of the rectangle times its width.

The formula for area is:

$$A = lw$$

Remember: When finding the area of a rectangle, the units used to measure the length and the width must be the same. If the length is in meters, the width must also be in meters. If the units are different, one must be converted to the other before you can multiply.

Example: Find the area of a rectangle that is 52 cm long and 44 cm wide.
(The units are the same so we don't have to convert.)

Draw the rectangle

Known:
 $l = 52 \text{ cm}$
 $w = 44 \text{ cm}$

Find:
Area
Use $A = lw$

$$\begin{aligned} A &= lw \\ &= 52 \text{ cm} \times 44 \text{ cm} \\ &= 2288 \text{ cm}^2 \end{aligned}$$



Note: When two of the same units are multiplied together, such as the centimeters in our example, they become square units. Instead of writing square centimeters, you can use the short form of cm^2 or sq cm. (Sq is the short form for square.) Four square feet is written 4 sq ft or 4 ft^2 .

Example: Find the area of a space with length 5 m and width 142 cm.

First we must convert one of the units so both are the same. We will change centimeters to meters by dividing by 100 or by moving the decimal point two spaces to the left. 142 cm becomes 1.42 m.

$$\begin{aligned}A &= lw \\A &= 5 \text{ m} \times 1.42 \text{ m} \\A &= 7.1 \text{ m}^2\end{aligned}$$

Example: Find the floor space of a shop that measures 60 feet long by 12 feet wide by 10 feet high.

(The information on height is not needed to answer this question.)

Known
 $l = 60 \text{ ft}$
 $w = 12 \text{ ft}$

Find
Area
Use $A = lw$

$$\begin{aligned}A &= lw \\&= 60 \times 12 \\&= 720 \text{ sq ft}\end{aligned}$$

Example: Find the floor space of a garage that measures 10 m long by 5 m. wide.

$$\begin{aligned}A &= lw \\&= 10 \times 5 \\&= 50 \text{ m}^2\end{aligned}$$

Area of a Square

The four sides of a square are all the same length. To find the area of a square, square the length. (To square a number, multiply it by itself. Three squared is $3 \times 3 = 9$.)

Example: Find the area of a square with sides 5 ft long.

Known: $l = 5 \text{ ft}$
 $w = 5 \text{ ft}$

Find A

$$\begin{aligned}A &= lw \text{ or } l^2 \\A &= 5 \text{ ft} \times 5 \text{ ft} \\A &= 25 \text{ sq ft}\end{aligned}$$

Area of a Parallelogram and a Trapezoid

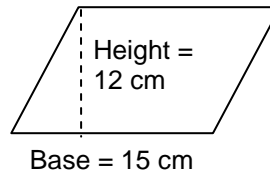
Parallelogram: The area of a parallelogram is equal to the altitude or height times the base. The formula is:

$$A = ab \text{ or } bh$$

Example: Find the area of a parallelogram with a height of 12 cm and a base of 15 cm.

Draw and label a parallelogram

Known: $b = 15 \text{ cm}$
 $h = 12 \text{ cm}$



Find A

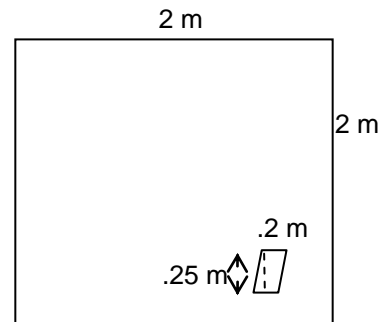
$$\begin{aligned} A &= bh \\ &= 15 \text{ cm} \times 12 \text{ cm} \\ &= 180 \text{ cm}^2 \end{aligned}$$

Example: A parallelogram with a base of 20 cm and a height of 25 cm is cut from a square sheet of metal that is 2 m on each side. How much of the sheet will be left over?

Draw and label the shapes.

Known:
sheet of metal sides = 2m
parallelogram $b = 20 \text{ cm} = .20 \text{ m}$
 $h = 25 \text{ cm} = .25 \text{ m}$

Find: Area of the metal sheet
Area of the parallelogram
Amount of sheet metal that will be left
(Area of sheet – Area of parallelogram)



The area of the sheet is:

$$\begin{aligned} A &= l^2 \\ &= 2^2 \\ &= 4 \text{ m}^2 \end{aligned}$$

Now find the area of the parallelogram. The cm should be changed to m.

$$\begin{aligned} 20 \text{ cm} &= .2 \text{ m} \\ 25 \text{ cm} &= .25 \text{ m} \\ A &= bh \\ &= .2 \times .25 \\ &= .05 \text{ m}^2 \end{aligned}$$

The area of the parallelogram is subtracted from the area of the sheet of metal.

$$4 \text{ m}^2 - .05 \text{ m}^2 = 3.95 \text{ m}^2$$

There are 3.95 m^2 left over.

Finding the Length of an Unknown Side When the Area Is Known

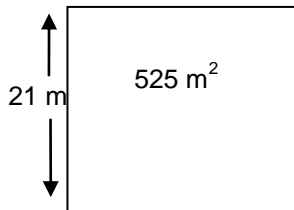
If you know the area of a rectangle and the width of one side, you can find the length of the other side. Manipulate the variables in the formula for area so the letter for length ends up by itself on the left side. Substitute the known measurements for area and width.

You can substitute the given measurements either before or after you manipulate the formula.

Example: If the area of a rectangle is 525 m^2 and the width is 21 m, what is the length?

Known:
 $w = 21 \text{ m}$
 $A = 525 \text{ m}^2$

Find l



$$A = lw$$

$$525\text{m}^2 = l \times 21\text{m}$$

$$l \times 21 \text{ m} = 525\text{m}^2$$

Reverse the equation.

$$l \times \frac{21\text{m}}{21\text{m}} = \frac{525\text{m}^2}{21\text{m}}$$
$$l = 25 \text{ m}$$

Divide both sides by 21 m to isolate l on the left.

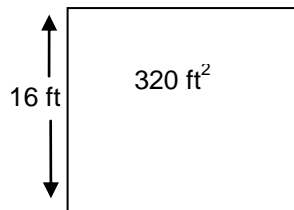
When you divide a squared unit by a linear unit, such as square meters by meters, the meters on the bottom cancel one of the units on the top, leaving meters in the answer.

The length is 25 meters.

Example: Find the length of a room that has an area of 320 sq ft and a width of 16 ft.

Known:
 $w = 16 \text{ ft}$
 $A = 320 \text{ ft}^2$

Find l



First rearrange the letters of the formula so l is by itself on the left.

$$A = lw$$

$$lw = A$$

Reverse the equation.

$$l = A/w$$

Divide both sides by w .

$$l = \frac{320 \text{ ft (ft)}}{16 \text{ ft}}$$

Fill in the given amounts. Divide.
Cancel the units where possible.

$$l = 20 \text{ ft}$$

The length is 20 ft.

Finding the Cost of Covering an Area

To find the cost of covering an area like a roof with a material such as steel roofing, first find the area and then multiply it by the cost per unit area. The cost of material is usually given as a rate, such as \$32.95 per sq m.

To find the cost of covering an area:

1. First calculate the area to be covered.
2. Then multiply the area by the cost per unit area.

Example: Find the cost of installing a new floor that measures 4.6 m long and 3.5 m wide. The flooring costs \$24.99 per square meter.

Known:

Top $l = 4.6$ m

$w = 3.5$ m

Cost of flooring 24.99 per m^2

Find:

Area of the top ($A = lw$)

Cost of flooring ($\text{Area} \times \$24.99/m^2$)

First find the area:

$$\begin{aligned} A &= lw \\ &= 4.6 \text{ m} \times 3.5 \text{ m} \\ &= 16.1 \text{ sq m} \end{aligned}$$

Find the cost of the flooring:

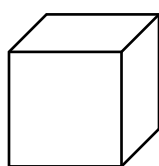
$$16.1 \text{ sq m} \times \$24.99/\text{sq m} = \$402.34$$

(Notice that the square meters cancel.)

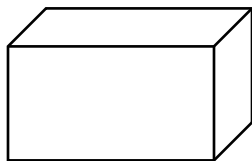
The cost to cover the floor is \$402.34.

THREE DIMENSIONAL FIGURES

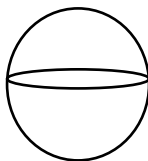
A closed, solid geometric figure has three dimensions. It has length, width and height or depth. Some solid figures are the cube, the rectangular solid, the cylinder, the cone and the sphere.



Cube



Rectangular solid



Sphere



Cone

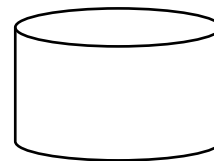


FIGURE2: Solid geometric figures

SURFACE AREA OF THREE DIMENSIONAL FIGURES

The surface area of a three dimensional figure is the combined areas of all the outside surfaces or faces of the figure. When finding the surface area, all measurements must be in the same linear units. The answer will be in square units.

Finding the surface area of a rectangular solid

To find the total area of the outside surface of a rectangular solid, we have to find the areas of each face of the figure.

1. First find the area of the front surface by multiplying the length times the height.
 - The back surface is the same area, so multiply that answer by 2.
2. Next find the area of one side by multiplying the width times the height.
 - Since the opposite side is the same, multiply the answer by 2.
3. Now find the base by multiplying the length times the width.
 - The top is the same as the base, so multiply that answer by 2 also.

The formula is:

$$A = 2lh + 2wh + 2lw$$

or $A = 2(lh + wh + lw)$

or $A = 2(lh + wh + lw)$

Example: Find the total area of the outside surface of a rectangular solid 5 cm long, 3 cm wide and 6 cm high.

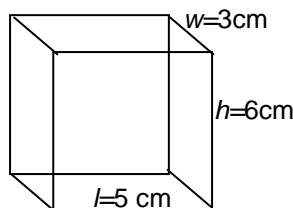
Draw and label the solid

Known:

$$l = 5 \text{ cm}$$

$$w = 3 \text{ cm}$$

$$h = 6 \text{ cm}$$



Find:

Outside surface area of the solid $A = 2(lh + wh + lw)$

$$\begin{aligned} A &= 2(lh + wh + lw) \\ &= 2(5\text{cm} \times 6\text{cm} + 3\text{cm} \times 6\text{cm} + \\ &\quad 5\text{cm} \times 3\text{cm}) \\ &= 2(30 \text{ cm}^2 + 18 \text{ cm}^2 + 15 \text{ cm}^2) \\ &= 2(63 \text{ cm}^2) \\ &= 126 \text{ cm}^2 \end{aligned}$$

Finding the surface area of a cube

A cube is made of six identical squares. Each edge is the same length, each side has the same area.

To find the area of a cube:

1. Find the area of one side (l^2) and multiply it by 6.

The formula is:

$$A = 6(l^2)$$

Example: Find the total surface area of a cube whose edges measure 10 in.

Known:

Edges of cube = 10 in

Find:

Surface area of cube $A = 6(l^2)$

$$\begin{aligned} A &= 6(l^2) \\ &= 6(10^2) \\ &= 6(100) \\ &= 600 \text{ sq in.} \end{aligned}$$

Finding the surface area of a cylinder

The surface area of a cylinder consists of the outside curved surface, which is actually a rectangle if it is straightened, and the circular areas at the top and bottom.

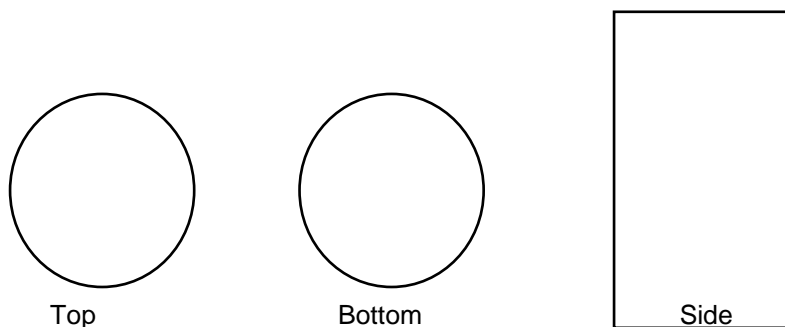


FIGURE 3: Finding the surface area of a cylinder

To find the surface area of a cylinder:

1. Find the area of each of the top and bottom circles.
2. Find the area of the rectangular side:
3. Add the areas together.

1. To find the area of the top and bottom: Use the formula $A = \pi r^2$. A cylinder has two circles (the top and the bottom), so we need to find the two areas, $2\pi r^2$.

Remember: $\pi = 3.14$

2. To find the area of the side of the cylinder (a rectangle): Multiply the length times the width.

The formula is: $A = lw$.

This rectangle has a width equal to the height of the cylinder so substitute height (h) for the width.

The formula is now: $A = 2lh$.

The length of the rectangle is the same as the perimeters of the circles at the top and bottom. We find the perimeter of a circle using the formula $P = 2\pi r$. Substitute this formula for the length of the rectangle.

The formula becomes $A = 2\pi rh$.

- 3. To find the area of the cylinder** add the areas of the top and bottom ($2\pi r^2$) to the area of the rectangle ($2\pi rh$).

$$A = 2\pi r^2 + 2\pi rh.$$

- 4. The formula is rearranged** to become:

$$A = 2\pi r(r + h)$$

Example: Find the surface area of a cylinder when its radius is 8 ft and its height is 20 ft.

Known:

r of cylinder = 8 ft

h of cylinder = 20 ft

Find the surface area of the cylinder

$$\begin{aligned} A &= 2\pi r(r + h) \\ &= (2 \times 3.14 \times 8)(8 + 20) \\ &= (50.24)(28) \\ &= 1406.72 \text{ sq ft} \end{aligned}$$

The surface area of a sphere

A sphere is a ball. The surface area of a sphere is equal to 4 times π times the radius squared. The formula is:

$$A = 4\pi r^2$$

Example: Find the surface area of a sphere with a radius of 5 cm.

Known

r = 5 cm

$$\begin{aligned} A &= 4\pi r^2 \\ &= 4 \times 3.14 \times 5^2 \\ &= 314 \text{ cm}^2 \end{aligned}$$

Using surface area to find the cost of covering an object

To find the cost of covering the outside surface of an object, find the surface area and multiply it by the cost per unit area.

Example: Find the cost of painting a metal cube 10 feet by 10 feet by 10 feet. A can of paint costs \$24.99 and covers about 100 square feet.

Known

l of sides of the cube = 10 ft

1 can of paint covers 100 ft²

Find

1. Surface area of the cube ($A = lw$)
2. How many cans of paint will be needed ($A \div 100 \text{ ft}^2$)
3. The cost of the paint (number of cans \times \$24.99)

$$A = 6(l^2)$$

$$A = 6(10^2)$$

$$A = 6(100)$$

$$A = 600 \text{ sq ft}$$

1 can covers 100 square feet.

600 square feet will need $600 \div 100 = 6$ cans of paint.

The price per can is \$24.99. Cost to cover the box will be:

$$\$24.99 \times 6 = \$149.94$$

Example: It takes 3 cans of spray paint to cover the outside of a metal box. Each can covers approximately 10 sq ft. What is the approximate surface area of the box? What will it cost to cover the box if each can costs \$4.99.

Known:

1 can covers 10 ft²

It takes 3 cans to cover the box

1 can paint costs \$4.99

Find:

The approximate surface area of the box.

The cost to cover the box.

If one can of paint covers 10 ft², 3 cans will cover three times as much.

$$3 \times 10 = 30 \text{ ft}^2$$

The surface area of the box is about 30 sq ft.

If 1 can costs \$4.99, 3 cans will cost:

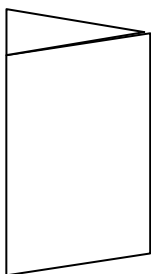
$$3 \times \$4.99 = \$14.97$$

VOLUME OF THREE DIMENSIONAL FIGURES

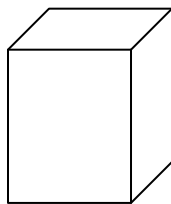
The **volume** or **capacity** of a solid figure is the amount of space contained within its boundaries.

Regular solids

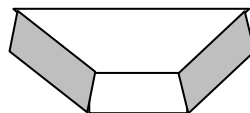
A regular solid is a three dimensional object with straight sides. A solid is named for the shape of its base.



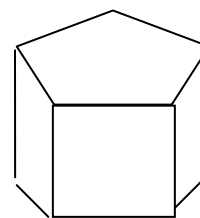
Triangular solid



Rectangular solid



Trapezoidal solid



Pentagonal solid

Volume of a Rectangular Solid

The volume of a rectangular solid equals the length times the width times the height. The formula is:

$$V = lwh.$$

Example: Find the volume of a rectangular solid 9 cm long, 4 cm wide and 3 cm high.

$$\begin{aligned} V &= lwh \\ &= 9 (4) (3) \\ &= 108 \text{ cm}^3 \end{aligned}$$

Volume of a Cube

The volume of a cube equals the length of one edge cubed. The formula is:

$$V = l^3$$

Example: Find the volume of a cube whose length measures 2 m.

$$\begin{aligned} V &= l^3 \\ &= 2^3 \\ &= 8 \text{ m}^3 \end{aligned}$$

Volume of a Cylinder

The volume of a cylinder equals π times the square of the radius of the base times the height. The formula is:

$$V = \pi r^2 h$$

Example: Find the volume of a cylinder with a radius of 12 ft and a height of 72 in.

$$72 \text{ in} \div 12 = 6 \text{ ft}$$

Change the units of height to feet by dividing by 12.

Now use the formula.

$$\begin{aligned} V &= \pi r^2 h \\ &= 3.14 (12^2) (6) \\ &= 2713 \text{ cu ft} \end{aligned}$$

Note: A volume of 1000 cm^3 of liquid is equivalent to 1 liter of the liquid. If you know the volume, you can calculate the number of liters of liquid.

Volume of a Sphere

The volume of a sphere equals $\frac{4}{3}$ times π times the cube of the radius. The formula is:

$$V = \frac{4\pi r^3}{3}$$

Example: Find the volume of a sphere with a radius of 10 inches.

$$V = \frac{4\pi r^3}{3}$$

$$V = \frac{4(3.14)(10^3)}{3}$$

$$= 4186.67 \text{ cu in}$$

Answer the following questions about geometric figures. Answers are on the last page.

1. Find the number of meters of trim needed to go around the perimeter of a room that is 4.5 m by 6 m.
2. Find the area of a rectangle that is 18 meters long and 400 centimeters wide.
3. Find the area of a parallelogram with a base of 7.2 m and a height of 4.7 m.
4. Find the area of a square with sides that are 16 yards long.
5. Find the amount of fencing needed to fence in a garden that is 30 feet by 10 yards.
6. Find the cost of installing metal roofing on a roof with two equal sides if one side measures 18.4 m long by 10.1 m wide. The roofing costs \$35.95 per square meter.
7. Find the cost of painting a warehouse floor that is 30 m by 40 m if a can of paint covers 600 m² and each can costs \$19.95.
8. If a sheet of metal measures 4 ft by 8 ft, find the amount required to make a rectangular metal box that has a height of 48 in, a width of 48 in and a length of 60 in. (First change the inches to feet.)
9. Find the volume of air in a room that measures 14 ft by 12 ft with 8 ft ceilings.
10. Find the volume of a cube whose sides measure 8 yards.
11. What is the surface area of a cylinder with a radius of 100 cm and a height of 150 cm?

ANSWER PAGE

- $p = 2l + 2w$
 $= 2(4.5 \text{ m}) + 2(6 \text{ m})$
 $= 9 \text{ m} + 12 \text{ m} = 21 \text{ m}$
- Change cm to m. $400 \text{ cm} = 4 \text{ m}$
 $18 \times 4 = 72 \text{ m}^2$
- $A = 7.2 \text{ m} \times 4.7 \text{ m}$
 $A = 33.84 \text{ m}^2$
- 256 sq yd
- Change feet to yards. $30 \text{ ft} = 10 \text{ yd}$
 $P = 2(10 \text{ yd.}) + 2(10 \text{ yd.})$
 $= 40 \text{ yd}$
- $A = 18.4 \text{ m} \times 10.1 \text{ m}$
 $= 185.84 \text{ m}^2$ (area of one side of roof)
 $185.84 \text{ m}^2 \times 2 = 371.68 \text{ m}^2$ (area of both sides of roof)
Cost = $371.68 \text{ m}^2 \times \$35.95/\text{m}^2$
 $= \$13,361.90$
- $A = 30 \text{ m} \times 40 \text{ m}$
 $= 1200 \text{ m}^2$
One can of paint covers 600 m^2 .
Number of cans needed = $1200 \text{ m}^2 \div 600 \text{ m}^2 = 2$ cans
One can costs \$19.95. Two cans cost $2 \times \$19.95 = \39.90
- First change inches to feet. $48 \text{ in} = 4 \text{ ft}$, $60 \text{ in} = 5 \text{ ft}$
Area of top and bottom = $2 \times 4 \times 5 = 40 \text{ sq ft}$
Area of long sides = $2 \times 4 \times 5 = 40 \text{ sq ft}$
Area of short sides = $2 \times 4 \times 4 = 32 \text{ sq ft}$
Find the total area and divide that by the area of 1 sheet of metal.
Total surface area of the box = $40 + 40 + 32 = 112 \text{ sq ft}$
Area of a sheet of metal = $4 \times 8 = 32 \text{ sq ft}$
Number of sheets needed = $112 \div 32 = 3.5$ sheets
- $V = lwh$
 $= 14 \text{ ft} \times 12 \text{ ft} \times 8 \text{ ft}$
 $= 1344 \text{ cu ft}$
- $V = l^3$
 $= 8^3$
 $= 8 \times 8 \times 8$
 $= 512 \text{ cu yd}$

$$\begin{aligned} 11. A &= 2\pi r (r + h) \\ &= (2 \times 3.14 \times 100)(100 + 150) \\ &= (628)(250) \\ &= 157\,000 \text{ cc (cubic centimeters)} \end{aligned}$$