EVALUATING
ACADEMIC READINESS
FOR APPRENTICESHIP TRAINING
Revised for
ACCESS TO APPRENTICESHIP

MATHEMATICS SKILLS
RATIO AND PROPORTION

AN ACADEMIC SKILLS MANUAL
for
The Food Preparation Trades

This trade group includes the following trades:
Baker & Cook, and
Retail Meat Cutter

Workplace Support Services Branch
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In preparing these Academic Skills Manuals we have used passages, diagrams and questions similar to those an apprentice might find in a text, guide or trade manual.

This trade related material is not intended to instruct you in your trade. It is used only to demonstrate how understanding an academic skill will help you find and use the information you need.

MATHEMATICS SKILLS
RATIO AND PROPORTION

An academic skill required for the study of the Food Preparation Trades

INTRODUCTION

Comparing Numbers
When we prepare food, we organize the ingredients in a recipe and it is important for the success of the final product that we use the ingredients in the amounts stated. To do this we compare the amounts we need.

Example: A simple chocolate cake recipe, requires three cups of flour, one cup of sugar and ½ cup of cocoa. If we need to make the cake larger, or smaller, we use that comparison of the volume of 3 c flour to 1 c sugar to ½ c cocoa. The recipe will only work if we keep that same balance of volume for the three ingredients. If we double the recipe, we have to double each of those ingredients.

Example: The amount of fat is compared by weight to the amount of muscle in ground meats when the ground meat is given a designation lean ground beef. The American standard for lean ground beef requires that there are between 15 and 20 pounds fat and between 80 and 85 pounds of meat without fat in 100 pounds of lean ground beef. If you buy 100 pounds of lean ground beef you know it will only have that amount of fat in it.

(The Canadian standard may differ slightly but the system of comparison will work the same way)

We use comparisons of amounts like this to ensure a consistency of a final product, like a chocolate cake, and to ensure that the products we buy, like lean ground beef, meet standards we expect for nutrition.

Numbers are used to make comparison in a variety of ways. One way to compare numbers is to note the difference between them.

Examples:
To compare the volume of flour to the volume of sugar in the cake recipe, we say that the recipe uses three times more flour than sugar.

Or we could compare them the other way around the volume of sugar is 1/3 the amount of flour.

Comparing numbers like this is known as using a ***ratio***. Ratios give useful information about the relationship between numbers. Ratios can be used to describe such things as the relationship between the volumes of ingredient in recipes, the amounts of nutrients in foods.

If we say that the ratio of flour to sugar in the cake is 3 to 1, it means that there are 3 parts of flour for every 1 part of sugar in the cake. The ratio doesn't tell us what the measure is (a cup) but we now could use that ratio of 3 to 1 with a metric measure (such as a litre) if we wanted. As long as we maintain that ratio of ingredients, the cake recipe should work.

Ratios are also used to solve problems by proportion.

> **Example:** We can use the ratio of 3 to 1 to make the cake in a different size. If we use 6 cups of flour, we can calculate the amount of sugar we need – as long as we keep the same ratio. If we double the flour, we have to double the sugar.
> 3 c flour to 1 c sugar = 6 c flour to 2 c sugar.

This skills manual looks at the following topics concerning ***ratio, and proportion.***

- Ratio, including
  - finding ratios from given information
  - rates
- Proportions, including
  - direct and indirect (inverse) proportions
  - solving a proportion when three out of four terms are known
  - solving problems using proportions

RATIO

Comparing two numbers by writing a ratio: Say one cutting board is costs 6 dollars and another costs 10. You can compare the two costs by writing them as a ratio.

There are several ways to indicate this ratio:

- **By comparing one amount to another**, as when we say 6 to 10 (or 6 out of 10).
- **By putting a colon between the numbers.** The ratio is written 6 : 10. We read this as "*the ratio of six to ten*".
- **By writing the ratio as a fraction.** The first number being compared becomes the numerator, which is placed over the second number, the denominator. The fraction is usually written in lowest terms. So 6 out 10 becomes 6/10 and can be reduced to 3/5.

When you write a ratio, you don't actually do the division unless you want one of the terms of the ratio to be 1.

Lowest terms: The ratio 3:4 is already in lowest terms. The ratio 8 to 32 is not in lowest terms. When this ratio is reduced to lowest terms, it is written as 1 to 4. A ratio, like a fraction, is usually, but not always, written in lowest terms.

To reduce a fraction or a ratio to lowest terms:

1. Look for a number (a common factor) that will divide evenly into the numerator and denominator of the fraction or the terms of the ratio.
2. Divide the common factor into each of the terms, or the numerator and the denominator, of the ratio.
3. Continue dividing until there are no more common factors.
4. The last division answers form the fraction or ratio in lowest terms.

Example: You are going to prepare individual stir fries on demand for a buffet lunch. The amounts cut up by the night staff were 18 cups of red pepper and 54 cups of other vegetables.

The ratio is 18:54.
This ratio is not in lowest terms.
The common factor is 18.
Divide the terms of the ratio by 18.
The ratio reduced to lowest terms is 1:3.

There should be one cup of red pepper for every 3 cups of other ones.

Notice there are no units in this ratio. Because we are comparing sliced vegetables to sliced vegetables the units cancel out. When the numbers being compared have the same unit of measurement, there are no units in the ratio.

Ratios with 1: The ratio 2:1 has the number 1 as one of its terms. The ratio 3:4 does not. Sometimes a ratio like 3:4 is more useful if one of the terms is 1. You could divide both terms by 4 and then express the ratio as .75 to 1, or you could divide both terms by 3 and express the ratio as 1 to 1.33.

Equivalent ratios: Reducing a fraction to lowest terms does not change the value of the fraction, nor will it change the value of a ratio. The fractions 2/8 and 4/16 can each be reduced to 1/4. 1/4, 2/8, and 4/16 are ***equivalent fractions***. They each represent the same amount.

In the same way, ratios representing the same amount are called ***equivalent ratios***. The ratio 3 to 4 and the ratio .75 to 1 represent the same comparison and are equivalent ratios.

Finding Ratios from Given Information

Before using ratios to solve problems, we will look at setting up ratios from given information.

Questions that ask you to set up ratios are generally worded in one of two ways.

1. You might need to compare part of an amount to the total amount; or
2. You might be asked to compare two parts to each other.

Situation one: You are asked to compare part of the amount to the total amount. If the total amount isn't given, you first have to find it.

Example: A class of apprentices consisted of 6 women and 24 men. What is the ratio of women to the whole class and the ratio of men to the whole class?

First you have to find the total number of students.
Adding 6 + 24 gives a total of 30 apprentices in the class.

Now find the ratios:

a) Ratio of women to the whole class is 6 out of 30, reduced to 1 out of 5, 1/5 or 1:5.
b) Ratio of men to the whole class is 24 out of 30, reduced to 4 out of 5, 4/5 or 4:5.

Situation two: The question asks you to compare one amount to another. This time you don't need to know the total.

Example: Using the class of 6 women and 24 men, what is the ratio of women to men and men to women?

Ratio of women to men is 6 to 24, reduced to 1 to 4, 1/4 or 1:4.

Ratio of men to women is 24 to 6, reduced to 4 to 1, 4/1 (or 4:1.
Note: if the denominator is 1 when writing a ratio, you must show it)

General Rules For Reading And Writing Ratios

Rule 1: *When you read or write ratios, compare the parts in the same order in language and in numbers, unless they are part of a table or formula*.

To compare the number of women to the class total, the number of women is stated before the class total.

Ratio of women to class = 6:30
This is reduced to 1:5.

To compare the number of men to women, the number of men is written before the number of women.

Ratio of men to women = 24:6
This is reduced to 4:1.

Rule 2: *If the units in each term of the ratio are the same, they will cancel each other out. If the units cancel out, you don't need to include them in the ratio.* (Sometimes, however, you want to keep the units in the ratio or they don't cancel out. We will look at them later.)

The ratio of 25 grams to 1 kilogram is not 25:1. The ratio has to be written as 25 g to 1 kg, or 25g:1kg.

Usually it is easier to work with ratios if there are no units, so make the units the same.

- If you convert 1 meter to 100 centimeters, the units will be the same. You can then cancel them out. The ratio is then written as 25:100 without any units.

If you can't write the ratio with the same unit for all terms, the units must remain in the ratio.

Rule 3: *Ratios without units are usually expressed in lowest terms.*

Example: Write the ratio of part time to full time employees in a catering company with 25 part time and 15 full time workers.

Answer: The units, which are employees, are the same. Since the question lists part time before full time, that is how the numbers are listed. The ratio is 25:15

Reduce the ratio to lowest terms.
Five is a common factor that divides into 25 and 15, giving the answers 5 and 3.
The ratio 25:15 reduced to lowest terms is 5:3.
The ratio of part time to full time workers is 5:3.

Example: A hotel kitchen took a survey and found that 8 salads out of every 60 set out on the buffet showed signs of wilting after one hour. Write the ratio of wilted salads to the total number of salads set out.

The units, which are salads, are the same. They cancel out and are not written in the ratio.

The ratio is 8:60.

Reduce the ratio to lowest terms.
Four is a common factor that divides into 8 and 60, giving the answers 2 and 15.

The ratio 8:60 reduced to lowest terms is 2:15.

Example: What is the slope of the roof of a dining tent if the rise (the vertical distance) is 2 meters and the run (the horizontal distance) is 4 meters.

Use the ratio:

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$= \frac{2}{4} \quad \text{reduce to lowest terms}$$

$$= \frac{1}{2}$$

The slope of roof is 1 to 2.

Rates

When the units of the quantities in a ratio are the same, they cancel out and so are not shown. When the units in a ratio are different, or there is only one unit, the units must be included in the ratio.

Ratios can be used to compare quantities of different types, such as kilometers per hour or cost per kilowatt-hour. These comparisons are called **rates**.

> *A **rate** is a quantity or amount of something measured per unit of something else.* A rate includes the word "per" which is indicated by the fraction line.

Usually a ratio is divided so the amount of the unit following the word "per" is 1. If a rate involves two different kinds of units, they must be included in the ratio.

> **Example:** Cost per serving is a rate. A dinner might be catered at a rate of $ 30.00 per plate

> **Example:** Driving speed is a rate. Say you drove 300 km in 3 hours. The ratio 300 km/3 hr is reduced to 100 km/1 hr or 100 km/hr. Your rate of speed is 100 km per hr.

Rates that involve a cost per unit, such as the rate you pay for electricity, include the dollar sign. Your electrical bill might say that your electrical rate is $.30/kw-hr. For every kilowatt-hour of electricity you use, you pay $.30.

Answer the following questions on ratios. Answers are at the end of this skills manual.

1. Write as ratios using a colon between the two quantities. Convert quantities to the same unit where possible (that is, if the units are cm and m, convert so both quantities are either cm or m). Reduce to lowest terms.

   a) 1 to 4   b) 5   to 8   c) 5 cm to 1 m   d) 2 kg to 125 g

   e) 1 m to 50 cm   f) 15 min to 1 hr   g) 5 ft   to 6 ft 6 in   h) one nickel to a quarter

2. Write as ratios using the fraction form. Reduce to lowest terms.

   a) 6 mg to 3 mg   b) 20 in to 45 in   c) 15 L to 9 L   d) 3 m to 90 cm

3. What is your rate of speed if you travel 400 km in 4 hr?

4. What is the cost per kilo of pork if you pay $65.90 for 10 kilograms? In other words, express the two numbers as a rate.

5. If it cost $6.30 for enough spice rub to cover 9 kilos of beef, what is the cost of covering one kilo?

6. The directions on a bottle of flavouring extract say to mix 1.5 liters with 45 liters of water. What is the ratio of extract to water?

PROPORTIONS

Two equivalent ratios express the same relationship but are written using different but related terms or numbers. For example, 1/4 and 2/8 are equivalent ratios and they represent the same amount. We can say that 1/4 equals 2/8. We can write this statement as:

1/4 = 2/8

Equivalent ratios written in fraction form with an equal sign between them form a ***proportion***. A proportion has four *terms*, or parts. The terms of the proportion above are 1, 4, 2 and 8. When we read the proportion, we name all four terms. 1/4 = 2/8 is read as "1 to 4 equals 2 to 8."

Direct and Indirect (or Inverse) Proportions

There are two basic types of proportions: direct proportions and indirect (sometimes called inverse) proportions.

In a ***direct proportion****, as one quantity increases, the corresponding quantity also increases.* Similarly, as one quantity decreases, the other one also decreases.

Example: The relationship between the amount of nutrients consumed and the amount of growth in young animals or humans is a direct relationship.

- The larger the amount of nutrients consumed, the more growth that will be observed.
- The smaller the amount of nutrients consumed, the less the growth.

This is a direct proportion because as the nutrient amounts change, growth changes *in the same way*.

In an ***inverse, or indirect, proportion****, as one quantity increases, the corresponding quantity decreases.* As one quantity decreases, the other one increases.

Example: The relationship between the number of minutes of boiling time and the volume of a soup stock is an inverse relationship.

- The *more time* you boil it, the *less stock* you have.

They change in the opposite way to each other. This is an inverse proportion.

Solving a Proportion When Three of Four Terms Are Known

Proportions such as 1/4 = 2/8 in the example above don't tell us much. We already know that two ratios or fractions that represent the same amount are equal to each other.

But what if you need find out how many boxes of top sirloin elk steaks you need order to feed a banquet of 125 people. If you know that 1 box contains 10 steaks, you can calculate the number of boxes you will need.

We will find the solution to this problem later, but first we will look at a simpler version of it.

We can use proportion to find the fourth term in a proportion if we know three of the four terms.

Here are the steps to find the fourth, unknown term:

1. **Set up a proportion using a letter to represent the unknown amount** in one of the ratios. The letter can be manipulated (moved around) in an equation just like a number. Write the ratios with an equal sign between them, forming an equation.

   Example: Write the proportion using the two ratios n:10 and 8:20.

   $$\frac{n}{10} = \frac{8}{20}$$

2. ***Cross-multiply to get rid of the denominators on both sides.*** To cross-multiply, multiply the diagonal numbers across the equal sign. In other words, multiply the numerator of one ratio by the denominator of the other ratio.

   If an unknown term is represented by a letter, cross multiply in the same way.

   Example: Cross-multiply in the equation below to get rid of the denominators.

   $$\frac{n}{10} = \frac{8}{20}$$

   Notice that n represents the unknown term.

   Multiply n by 20 and 10 by 8. Keep the equal sign.

   $20n = 10(8)$

   $20n = 80$

3. ***Isolate the unknown term*** (get it alone on one side of the equal sign). To do this divide both sides by the number in front of the unknown term.

   Example: Isolate n in the following equation.

   $20n = 80$

   $$\frac{20n}{20} = \frac{80}{20}$$

   Divide both sides by 20.

   $n = 4$

Here are some other manipulations that can help isolate the letter representing the unknown term.

A. If the letter representing the unknown term is on the right side, reverse the equation before dividing. You can reverse an equation without changing its value.

Example: You can reverse:

3(15) = 5n to 5n = 3(15).

Both equations have the same value.

B. You can invert (turn all of the terms upside down) both sides of the equation without changing its value.

Example: You can invert

4/s = 5/6 to s/4 = 6/5.

Both equations have the same value.

Note: *If you invert one side of an equation, you must invert the other side to keep the equation equal.*

Now let's look at some examples of finding an unknown term in a proportion using these steps.

Example: Solve for n in the following proportion.

$\frac{4}{5} = \frac{n}{15}$ Set up the proportion

4(15) = 5n cross-multiply

60 = 5n

5n = 60 Reverse the equation so that n is on the left side of the equal sign.

5n ÷ 5 = 60 ÷ 5 Divide both sides of the equation by the number in front of the unknown term.

n = 12 The letter is isolated on the left hand side of the equation.
The answer is on the right hand side

Substitute 12 for n to write the complete proportion.

$\frac{4}{5} = \frac{12}{15}$

Example: Find the value of n when:

$$\frac{n}{12} = \frac{5}{15}$$

15n = 5(12) cross multiply

5n = 60 divide by 15 to isolate n

$$\frac{15n}{15} = \frac{60}{15}$$

n = 4

4/12 = 5/15 Substitute 4 for n to write the complete proportion.

Example: Find the value of n when:

$$\frac{n}{8} = \frac{10}{16}$$

16n = 10(80) cross multiply

16n = 80 divide both sides by 16

n = 5

Example: Find the value of s.

$$\frac{3}{4} = \frac{9}{s}$$

3s = 9(4) cross multiply
3s = 36
3s ÷ 3 = 36 ÷ 3 divide by 3
s = 12

Solving Problems Using Proportions

Proportions can be used to solve problems. You have to figure out what goes with what and then set up your proportion to find the unknown quantity. Notice that when you first set up your ratios, you do not usually reduce to lowest terms.

Method 1: These suggestions are one method to set up a proportion.

a) Set up the ratios (or fractions) so the same units are over each other.
    a. Set up minutes over minutes, kilometers over kilometers, or meters over meters.
b) The units of the two given quantities that form one fraction will cancel out.
    a. The unit of the third known quantity will be the unit of the unknown quantity

c) Set up the smaller unit over the larger unit. The proportion will look like this:

$$\frac{\text{small}}{\text{large}} = \frac{\text{small}}{\text{large}}$$

Example: How many boxes of top sirloin elk steaks do you need to order for a banquet serving 125 people if one box contains 10 sirloin steaks?

Set up your proportion.
Put steaks over steaks and boxes over boxes, with the smaller amount of each quantity on the top.
Let h equal the unknown number of boxes needed.

The proportion looks like this:

$$\frac{10 \text{ steaks}}{125 \text{ steaks}} = \frac{1 \text{ box}}{h}$$

Put steaks over steaks and boxes over boxes
Let h equal the unknown number of boxes.

This looks like the proportions we already know how to solve. Find the answer by solving for h:

$$\frac{10 \text{ ~~steaks~~}}{125 \text{ ~~steaks~~}} = \frac{1 \text{ box}}{h}$$ steaks cancel out

10 h = 125 x 1 cross-multiply

10 h = 125 divide both sides by 10

h = 12.5 boxes

You need 12.5 boxes of sirloin elk steaks.

Method 2: You can also set up the two ratios so each is given as a rate. When the ratios are set up as rates, in each ratio, one unit is over the other, different, unit. The following example shows how to set up the proportion.

Example: You travel 25 km in 50 minutes. How long will it take to travel 75 km at that speed?

The first ratio or rate is 50 min/25 km.
The second ratio is *unknown minutes*/75 km.
Set up the proportion by writing the two ratios.
Let m represent the unknown time.

$$\frac{50 \text{ min}}{25 \text{ ~~km~~}} = \frac{m}{75 \text{ ~~km~~}}$$

km cancel
you can leave out the other unit, minutes, until the end

50(75) = 25m cross-multiply

25m = 3750 min reverse the equation
m = 150 min divide both sides by 25 and put in the unit min
It will take 150 minutes to travel 75 km.

Example: The commercial dishwasher in a large restaurant runs on average about 15 cycles per week. The specifications for the detergent suggest that .25 liters be used per wash cycle. How many liters of detergent are used on a weekly basis?

We will use the second method. The first ratio is .25 liters/cycle. The second ratio is unknown liters/15cycles. Let t represent the unknown number of liters. Set up the proportion.

$\frac{.25\text{ L}}{1\text{cycles}} = \frac{t}{15\text{cycles}}$ cross multiply

3.75L = 1t reverse the equation and divide both sides by 1 cycle

t= 3.75 L

You will use 3.75 liters of detergent in the dishwasher on a weekly basis.

Here are some questions on proportions. Answers are at the end of this skills manual.

7. Solve for the unknown quantity.

a) $\frac{n}{24} = \frac{1}{2}$ b) $\frac{2}{x} = \frac{10}{10}$ c) $\frac{16}{2} = \frac{s}{3}$

d) $\frac{5}{10} = \frac{12}{n}$ e) $\frac{n}{7} = \frac{3}{21}$ f) $\frac{2}{6} = \frac{n}{7.35}$

8. If it takes 70 minutes to travel 35 km, how long will it take to travel 85 km at the same speed?

9. Potatoes wholesale for $63.50 per 10 kilograms. How much would 36 kilograms cost?

10. If a marinade requires a 1:3 lime juice to olive oil mixture, how much juice has to be added to 6 cups of oil?

11. If a liter of syrup covers 7 dozen cup cakes, how much is needed to cover 84 dozen?

12. If a .2 kilogram salmon steak loses 30 grams of water during cooking, how much water will a whole salmon weighing 4 kilograms lose?

13. If there is 35 parts wastage for every 100 parts of turkey bought, how much meat will be obtained from a 30 pound turkey? (Hint: the amount of meat is the difference between 100 and 35)

14. To prepare a sterilizing solution to wipe down cutting surfaces a meat cutter must mix 1 ml of concentrated sterilizing solution with 1 liter of water. If you need 5 liters of solution, how much concentrate should you add?

ANSWER PAGE

RATIOS

1. a) 1 : 4
   b) 5 : 8
   c) 1 : 20 (change 1 m to 100 cm, reduce)
   d) 16 : 1 (change kg to g)
   e) 2 : 1 (change m to cm)
   f) 1 : 4 (change hr to min)
   g) 10 : 13 (change 5' to 60" and 6 ‘ 6" to 78")
   h) 1 : 5 (change nickels and quarters to cents)

2. a) 2/1
   b) 4/9
   c) 5/3
   d) 10/3 (change m to cm)

3. 100 km/hr

4. $6.59/kg

5. Divide each by 9 to find cost per kg.
   $.70/kg

6. 1.5 : 45 reduced to 1 : 30

PROPORTIONS

7. a) 12
   b) 8
   c) 24
   d) 24
   e) 1

Note: *You may set up your proportions differently than we have. It doesn't matter which way you set up your proportion as long as you get the correct answer.*

8. $\frac{70 \text{ min}}{35 \text{ km}} = \frac{m}{85 \text{ km}}$
   70(85) = 35m
   35m = 5950
   m = 170 min

9. $\frac{\$63.50}{10} = \frac{k}{36}$

   10k = $63.50 x 36
   10k = $2286.
   k = $228.60

10. $\frac{1}{3} = \frac{t}{6}$

   3t = 6
   t = 2 cups of juice

11. $\frac{1 \text{ liter}}{7 \text{ dozen}} = \frac{s}{84 \text{ dozen}}$

   7s = 84 liters
   s = 12 liters

12. $\frac{30 \text{ g}}{.2 \text{ kg}} = \frac{n}{4 \text{ kg}}$

   n = 600 g

13. 100 – 35 = 65 parts

   $\frac{65}{100} = \frac{m}{30 \text{ lb}}$

   m = 19.5 lb

14. $\frac{1 \text{ ml}}{1 \text{ liter}} = \frac{x}{5 \text{ liters}}$

   x = 5 ml