

**EVALUATING
ACADEMIC READINESS
FOR APPRENTICESHIP TRAINING**
Revised for
ACCESS TO APPRENTICESHIP

**MATHEMATICS SKILLS
WHOLE NUMBER OPERATIONS**

**AN ACADEMIC SKILLS MANUAL
for
The Industrial Maintenance Mechanic Trades**

This trade group includes the following trades:
Boiler Maker,
Facilities Maintenance Mechanic & Technician, and
Industrial Mechanic (Millwright)

*Workplace Support Services Branch
Ontario Ministry of Education and Training*

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In preparing these Academic Skills Manuals we have used passages, diagrams and questions similar to those an apprentice might find in a text, guide or trade manual.

This trade related material is not intended to instruct you in your trade. It is used only to demonstrate how understanding an academic skill will help you find and use the information you need.

MATHEMATICS SKILLS:

WHOLE NUMBER OPERATIONS

*An academic skill required for the study of the
Industrial Maintenance Mechanic Trades*

INTRODUCTION

To be successful in your trade, it is essential that you have an understanding of the basic mathematical operations. You need a foundation in addition, subtraction, multiplication and division in order to estimate how long a job will take, to order parts from a manufacturer, to choose the proper materials, or to accurately fabricate a part.

These situations might require you to make mathematical calculations using fractions, decimals, measurements and equations. But, you can't work out the answers to more complex problems until you are sure of the basics.

Being competent in the basic mathematical skills will give you the confidence to make decisions, based on accurate mathematical information that can make your business a success.

This skill sheet reviews the operations of addition, subtraction, multiplication and division using step by step explanations, including the following sections

- ◆ Place Value
- ◆ Addition
- ◆ Subtraction
- ◆ Multiplication
- ◆ Division
- ◆ Rounding off numbers
- ◆ Using a calculator

If you only need a quick review, read the explanations and do the questions. Check your answers at the end of each section. If you have any difficulties, go back and study the examples carefully.

You might want to work through the entire skills manual. Because this skills manual is long, work on one or two sections at a time. Read the explanation for each section and follow the steps in each example. Look at the examples as you read the explanations. ***Go back and solve the sample questions on your own. Glance at the solutions and explanations when you need to.***

This is the best way to make sure that you understand how to do the questions. Once you understand the concept, do the section questions. It is a good idea to work in pencil and have an eraser handy, so you can easily make changes as you go. Check your answers often as you work through the questions.

PLACE VALUE

Whole numbers are the numbers used for counting. If you have 245 nails, the number of nails is, of course, two hundred and forty-five. The individual numbers that make up the number 245 are called **digits**. The digits are the numbers from 0 to 9. Each digit has a **place value** that depends on where it occurs in the number. See Table 1.

In the number 245:

- The place value of the digit 5 on the right hand side is called the **ones place**; it represents 5 ones.
- The digit 4 to the left of the ones place is in the **tens place**; it represents 4 tens, or forty.
- The digit 2 is in the **hundreds place**; it represents 2 hundreds, or 200.
- The number 245 is made of 2 hundreds, 4 tens and 5 ones.

In a larger number:

- The **thousands place** is to the left of the hundreds place; a digit here represents the number of thousands.
- The **ten thousands** place is to the left of the thousands place.
- The **hundred thousands** place is to the left of the ten thousands place.
- The **millions** place is to the left of the hundred thousands place.

Table 1 shows the place value of each digit in the seven digit number 4 782 951.

TABLE 1: PLACE VALUES FROM ONES TO MILLIONS

millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones
4	7	8	2	9	5	1

We read this number as four million seven hundred eighty-two thousand nine hundred fifty-one.

We sometimes refer to the **place value column** that a digit occurs in.

- In 4 782 951, we say that 5 is in the tens column, the second column from the right side of the number.
- The “tens column” means the same as the “tens place value”.

Place Value and Reading Large Numbers

We read and say the parts of a number from left to right, naming the digit in the largest place value first. When we write large numbers, we group the digits together in threes and separate each group by either a comma or a space.

Look again at Table 1 to see how this grouping helps us to read numbers. To read a number such as 4 782 951 do it in the following order

1. First we say or read (or write) the number of millions, which is *four million*;
2. Then we group all the thousands together as *seven hundred and eighty-two thousand*;
3. Next we say the number of hundreds, which is *nine hundred*; and,
4. Lastly, we group the tens and ones together as *fifty-one*.

So, we write the number 4 782 951 and it means 4 million, 782 thousand, 951.

If there were more millions place values, group them the same way as the thousands:

- millions (**4** 000 000),
- ten millions (**14** 000 000) and
- hundred millions (**614** 000 000).
- Billions, trillions, and so on follow the same pattern.

Place Value and Doing Calculations

To add, subtract or multiply larger numbers, start on the right side, *in the ones column* and work across to the left through the tens column, the hundreds column etc. Long division starts on the *left*.

ADDITION OF WHOLE NUMBERS

Addition starts with memorizing the basic addition facts. Use a chart like Table 2.

TABLE 2: ADDITION FACTS

0	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

Note: Rows go across the table horizontally. Columns go down the table vertically.

Add the numbers from the left hand column to the numbers from the top row. *The answer appears where the row of the first number intersects with the column of the second.*

Example: To add 6 and 7, look to where row 6 and column 7 meet. The answer 13 is there.

Review the addition facts by adding the numbers from the top row to each of the numbers in the first column, covering the answers with a file card. Uncover the answers to check as you go. Redo the answers you get wrong. Make up your own practice table and fill it in.

Adding larger numbers

Once the basic addition facts have been memorized, you can start adding larger numbers. The following example shows how.

Example: $395 + 834$

Line up the two numbers so the ones digits are in the same column, the tens digits are in the same column, etc. Start by adding the digits in the ones column:

$$\begin{array}{r} 395 \\ 834 \\ \hline 9 \end{array} \quad 5 + 4 = 9, \text{ write the } \mathbf{ones} \text{ in the answer line}$$

Now add the digits in the tens column.

$$\begin{array}{r} 395 \\ 834 \\ \hline 29 \end{array} \quad 9 + 3 = 12, \text{ write the } \mathbf{tens} \text{ in the answer line}$$

We have to do something with the 1 in the tens place from the answer 12.

Important: Any digit or digits in your individual answer larger than the ones place is **carried to**, or written over the top of, the next column. In this case it is carried to the hundreds column. It is then added with the other digits in that column.

$$\begin{array}{r} 1 \\ 395 \\ 834 \\ \hline 1229 \end{array} \quad \text{add the } \mathbf{hundreds} \text{ column including the carried digit} \quad 1 + 3 + 8 = 12$$

Write the complete answer 12 in the answer line, as there are no more columns to add.

To add more than two whole numbers, follow the same procedure:

1. Write the numbers in columns, lining up the ones, tens, etc.
2. Add the digits in the ones column.
3. Write the ones digit from your first answer below the line in the ones column. Carry any extra digits above the next column.
4. Add the digits in the tens column, including any carried digits. Write the ones digit from this answer on the answer line below that column. Carry any other digits to the next column if necessary.

5. Continue adding the hundreds digits, the thousands digits, etc, in the same way, writing the ones digit from each answer in its column and carrying other digits to the next column.

Example: Add: $774 + 893 + 1342 + 56$

$$\begin{array}{r} 221 \\ 774 \\ 893 \\ 1342 \\ \underline{56} \\ 3065 \end{array}$$

Problem solving: Sometimes you are given a list of numbers to add. At other times you may be given information in a problem and be expected to figure out what numbers to add on your own. You may have to figure out credit card charges, mileage, or overtime hours.

Example: You are paid mileage for the driving done while on the job. Last week you drove 45 kilometres on Monday while getting to different jobs, 56 on Tuesday, 21 on Wednesday, 9 on Thursday, and 33 on Friday. To record your total mileage for the week, you have to add all of the mileage numbers together to get 164 kilometres.

To solve an addition problem:

1. Read through the question first.
2. Decide what needs to be found.
3. Set up the addition question using the facts and information given.
4. Complete the addition to find the answer.

The questions that follow test how well you remember the basic addition facts. The problems test how well you can find the answer by first figuring out what to do with given information. Check your answers often as you work to see if you are on the right track. If you get wrong answers, make sure your basic addition facts are correct and then look back at the examples.

TO ADD WHOLE NUMBERS

1. Memorize the basic addition facts.
2. Put the numbers in columns, lining up ones under ones, tens under tens and so on.
3. Add up each column, starting with the ones column.
4. Write the ones digit from each individual answer under the column just added.
5. If the sum of any column is ten or more, carry the other digit(s) to the next column.
6. Add in the carried digit(s) when you add the next column.
7. Continue until all the columns are added.

PRACTICE WITH ADDITION: Answers are on the last page of the skills manual.

1. Add the following numbers.

$$\begin{array}{r} \text{a) } 38 \\ + 46 \\ \hline \end{array}$$

$$\begin{array}{r} \text{b) } 275 \\ + 98 \\ \hline \end{array}$$

$$\begin{array}{r} \text{c) } 429 \\ \quad 32 \\ + 8 \\ \hline \end{array}$$

$$\text{d) } 5 + 81 + 3357 + 274$$

$$\text{e) } 6723 + 466 + 28 + 93$$

$$\text{f) } 976 + 40883 + 54$$

$$\begin{array}{r} \text{g) } 5853 \\ \quad 724 \\ \quad 1275 \\ + 651 \\ \hline \end{array}$$

$$\begin{array}{r} \text{h) } 9075 \\ \quad 8731 \\ + 553 \\ \hline \end{array}$$

$$\begin{array}{r} \text{i) } 39 \\ \quad 492 \\ \quad 78 \\ + 6 \\ \hline \end{array}$$

$$\begin{array}{r} \text{j) } 450 \\ \quad 3000 \\ \quad 90 \\ \quad 120 \\ + 8010 \\ \hline \end{array}$$

Solve the following problems:

2. John worked the following hours in one week: 8 hours, 10 hours, 9 hours, 6 hours, 7 hours. How many hours did he work?
3. Mark counted the following numbers of bearings in 4 different tool boxes: 48, 86, 32 and 21. What was the total number of bearings he counted?
4. Joan keeps track of her expense claims for driving to remote work sites. In the last 4 months she drove 254 kilometers the first month, 310 kilometers the second month, 108 kilometers the third, 567 kilometers the fourth and 398 kilometers the last month. How many kilometers did she drive altogether?

SUBTRACTION OF WHOLE NUMBERS

You subtract to find the difference between two numbers. Subtraction is closely related to addition. You can always add the answer to a subtraction question with the number you subtracted to get the number you subtracted from.

Example: $8 - 3 = 5$

$5 + 3 = 8$ Add the answer 5 to the amount you subtracted, 3.

You get the number you subtracted from, 8.

Example: $23 - 9 = 14$

$14 + 9 = 23$ Add the answer to the amount you subtracted.

When you add, it doesn't matter what order you write the numbers, but when you subtract, the order is important. When you subtract whole numbers, the larger number is placed first.

Basic subtraction facts

You can use the addition table (Table 2) to memorize the basic subtraction facts. Start with answers to the addition questions this time. Use the table to subtract 5 from 12 (12-5):

1. Locate 5 on the **first row** (rows go across horizontally) of the addition table.
2. Move down the **column** (columns go up and down vertically) **below the 5** until you find 12.
3. Move across the **row that 12 is in** back to the farthest left hand column. The number in that space is 7, which is the answer to 12 - 5.
So $12 - 5 = 7$.

Practice by writing out all the basic subtraction facts from 1 - 1 to 18 - 9. Use the table to check your answers until you get all the answers correct.

Important fact: *Any number subtracted from itself equals 0.*

Example: $5 - 5 = 0$ The addition table doesn't show this.

Subtracting larger numbers:

Once you know the basic subtraction facts, you can use them to subtract larger numbers.

To subtract larger numbers:

1. Write the numbers in columns so the corresponding place values are lined up.
2. Use the subtraction facts to subtract the bottom digit from the top digit, starting at the ones column, until all the lined up digits have been subtracted.

Example: $8687 - 7153$

First write the numbers in columns with the corresponding place values lined up:

$$\begin{array}{r} 8687 \\ -7153 \\ \hline \end{array}$$

Start subtracting.

$$\begin{array}{r} 8687 \\ -7153 \\ \hline 4 \end{array}$$

Beginning in the ones column $7 - 3 = 4$.

Write the 4 in the answer line below the ones.

Next:

$$\begin{array}{r} 8687 \\ -7153 \\ \hline 34 \end{array}$$

In the tens column: $8 - 5 = 3$.

Write this answer in the answer line below the tens.

Then:

$$\begin{array}{r} 8687 \\ -7153 \\ \hline 534 \end{array}$$

In the hundreds column: $6 - 1 = 5$.

Write 5 in the answer line under the hundreds.

Finally:

$$\begin{array}{r} 8687 \\ -7153 \\ \hline 1534 \end{array}$$

In the thousands column: $8 - 7 = 1$.

Write this answer in the thousands place.

1534 is the final answer

Borrowing in Subtraction

In the example above, each digit in the top number was larger than the digit on the bottom. If you are subtracting with larger numbers, you will often find that the value of the digit you are subtracting from is smaller than the amount you have to subtract from it. You need to borrow. *All borrowing takes place in the number you are subtracting from.*

Here are the basic steps when borrowing in subtraction.

1. **Always borrow from the digit in next largest place value** – the place value to the left of the digit you are borrowing for.
2. **Borrow 1 from that digit**
 - The value of that 1 will be the same as 10 for the place value column you are borrowing for.

3. Keep track of the borrowing.

- *Mark the digit you borrowed from.* Cross out the digit you have borrowed from and replace it with a digit one less than it originally was.
 - If you borrow 1 from 7 cross out the 7 and write in the digit 6 like this: ${}^6\bar{7}$
- *Mark the digit you borrowed for.* Write a small 1 in front of the digit you borrowed for to represent the 10 you have borrowed.
 - If you borrowed for a 5 put a small 1 in front of to represent 15, like this: 15 .

4. Subtract using the new value for the digit you are subtracting from.

- Remember, after you have borrowed 15 represents 15

5. Write the answer in the answer line and continue subtracting.

6. Remember, the value of the next digit will be the value you gave it when you borrowed.

- When you borrowed from 7 you marked it ${}^6\bar{7}$.
- You will subtract from 6 *not* 7.

7. Complete the subtraction question.

- You will continue to borrow as needed.
- Each time you must borrow, follow steps 1 through 5

Example: Find $637 - 484$.

$$\begin{array}{r} 637 \\ -484 \\ \hline 3 \end{array}$$

First write the numbers in columns.
Start subtracting in the ones column. $7 - 4 = 3$.
Write 3 in the answer line below the ones.

In the tens place, you can't subtract $3 - 8$. Borrow 1 from the 6, the next digit to the left, which is in the hundreds column.

$$\begin{array}{r} {}^5\bar{6}137 \\ -484 \\ \hline 53 \end{array}$$

Write the 1 in front of the 3 to make it 13.
Cross out the 6 and rewrite it above as 5.
Subtract $13 - 8$ in the tens column.

$$\begin{array}{r} {}^5\bar{6}137 \\ -484 \\ \hline 953 \end{array}$$

Subtract $5 - 4$ in the hundreds column.

Here is another example:

$$\begin{array}{r} 476 \\ -382 \\ \hline 4 \end{array}$$

You can subtract $6 - 2 = 4$ but $7 - 8$ won't work.

Borrow from the 4 in the hundreds column so 7 becomes 17.

$$\begin{array}{r} 34^{1}76 \\ -382 \\ \hline 94 \end{array}$$

Now subtract. $17 - 8 = 9$
The 4 you borrowed from in the hundreds column becomes 3.
Now subtract: $3 - 3 = 0$. Since the zero comes at the beginning of the number, you do not need to write it down. 94 is the answer.

Here is an example of borrowing from zero:

$$\begin{array}{r} 601 \\ -382 \\ \hline \end{array}$$

You can't subtract $1 - 2$ so you borrow from the tens column.
You can't borrow from 0 so look to the hundreds column.

$$\begin{array}{r} 59 \\ 6^{+}0^{1}1 \\ -382 \\ \hline \end{array}$$

Borrow 1 from 6. The 6 becomes 5 and the 0 becomes 10.
Now borrow from that 10, making it 9

Now, subtract. Write each answer in its column as you proceed.

$$\begin{array}{r} 59 \\ 6^{+}0^{1}1 \\ -382 \\ \hline 219 \end{array}$$

Begin in the ones column. $11 - 2 = 9$
Subtract in the tens column. $9 - 8 = 1$. Then the hundreds. $5 - 3 = 2$

Here is an example where you have to borrow several times:

$$\begin{array}{r} 453301 \\ -370459 \\ \hline \end{array}$$

You can't subtract $1 - 9$. Borrow from the 0 in the tens column. That can't be done, so borrow 1 from the 3 in the hundreds column. The 3 becomes a 2. The 0 becomes 10.

Now borrow 1 from the 10 so that the 1 in the ones column becomes 11. The 10 becomes a 9.

$$\begin{array}{r} 29 \\ 45330^{1}1 \\ -370459 \\ \hline 42 \end{array} \quad \begin{array}{l} 11 - 9 = 2 \\ 9 - 5 = 4 \end{array}$$

You can't subtract $2 - 4$ in the hundreds column so we have to borrow again. The 2 becomes a 12 and the 3 in the thousands column becomes a 2. Continue to subtract. The question now looks like this:

$$\begin{array}{r} 2^{1}29 \\ 45330^{1}1 \\ -370459 \\ \hline 2842 \end{array} \quad \begin{array}{l} 12 - 4 = 8 \\ 2 - 0 = 2 \end{array}$$

In the ten thousands column, you can't subtract $5 - 7$. You have to borrow again. The 5 becomes 15 and the 4 in the hundred-thousands column becomes one less or 3.

$$\begin{array}{r} 32^{1}29 \\ 4^{1}5330^{1}1 \\ -370459 \\ \hline 82842 \end{array} \quad \begin{array}{l} 15 - 7 = 8 \\ 3 - 3 = 0 \text{ don't write this final } 0 \\ \text{This is the final answer.} \end{array}$$

Here is an example with several zeros:

$$\begin{array}{r} 7000 \\ -215 \\ \hline \end{array}$$

When there are several zeros and you have to borrow.

Borrow from the first column with a digit larger than zero. The first zero to the right of that becomes 10 so borrow 1 from it, making it 9.

The other zeros become 9's in the same way until you reach the digit where you started borrowing, on the right. Now you can subtract.

$$\begin{array}{r} 6 \ 9 \ 9 \\ \cancel{7}^1 \ \cancel{0}^1 \ \cancel{0}^1 \ 0 \\ - \ 2 \ 1 \ 5 \\ \hline 6 \ 7 \ 8 \ 5 \end{array} \quad \begin{array}{l} 10 - 5 = 5 \\ 9 - 1 = 8 \\ 9 - 2 = 7 \\ 6 - 0 = 6 \end{array}$$

Example:

$$\begin{array}{r} 104 \\ -76 \\ \hline \end{array}$$

You need to borrow but the tens digit is a 0.

Cross out until you come to a digit greater than zero.

Make that digit one less and make all the zeros you crossed out into 9's.

Remember, if the first digit greater than zero is 1, it becomes one less or 0, so the 9 is the last digit on the top.

$$\begin{array}{r} 9 \\ \cancel{1}^9 \ \cancel{0}^1 \ 4 \\ - \ 76 \\ \hline 28 \end{array} \quad \begin{array}{l} 14 - 6 = 8 \\ 9 - 7 = 2 \end{array}$$

TO SUBTRACT WHOLE NUMBERS

1. Memorize the basic subtraction facts.
2. Line up the numbers you are subtracting, placing ones under ones, tens under tens, etc.
3. Start subtracting in the ones column, and work to the left.
4. If any digit on the top is smaller than the digit on the bottom, borrow 1 from the number in the next column, reducing the digit in that column by 1, and place the borrowed 1 in front of the digit you are subtracting from. For example, 1 placed in front of 6 will change it into 16.
5. If the number that is to be borrowed from is 0, move to the left until you reach a digit that is not zero. Reduce that digit by one and change the zeros to 9's.

PRACTICE WITH SUBTRACTION: Answers are at the end of the skills manual. Check your answers as you work.

5. Subtract.

a) $56 - 23$

b) $345 - 34$

c) 84 subtract 46

d) $450 - 226$

e) $255 - 37$

f) 634 minus 369

g) $200 - 124$

h) $6841 - 2346$

i) $\begin{array}{r} 708 \\ -523 \\ \hline \end{array}$

j) $\begin{array}{r} 1250 \\ -389 \\ \hline \end{array}$

k) $\begin{array}{r} 500 \\ -347 \\ \hline \end{array}$

l) $\begin{array}{r} 1004 \\ -657 \\ \hline \end{array}$

6. There are 500 m of wire on a new roll. If 348 m were used, how many metres are left on the roll?
7. If the width of one sheet of metal is 168 cm, and the width of another sheet is 145 cm, how much wider is the first sheet than the second sheet?
8. A warehouse filled orders for 54, 26, 43 and 12 large bolts from a stock of 235. How many bolts were left in stock after filling the orders?

MULTIPLICATION OF WHOLE NUMBERS

Before doing longer multiplication questions, you have to know the basic multiplication facts. A multiplication table can help you memorize them. To fill in the table below, multiply each number in the top row by each number in the left hand column. The answer goes in the space where the lines extending from both numbers intersect. Fill in Table 3 below, checking your answers with the completed table on the last page of the skills manual. Practice any facts you aren't sure of by writing them out.

TABLE 3: MULTIPLICATION FACTS

x	1	2	3	4	5	6	7	8	9
1	1								
2									
3									
4									
5									
6									
7									
8									
9									

Important Fact: *Zero multiplied by any number always equals zero.*

Multiplying larger numbers

Once you know the basic multiplication facts, you can use them to multiply larger numbers.

1. Start by writing the two numbers to be multiplied, one under the other, with the place value columns lined up.
2. Usually we write the larger number on top, although it doesn't matter to the answer.
3. The bottom number is called the **multiplier**.

Important Facts:

1. If there is a tens digit in the answer to any individual step, you must carry it.
2. To carry in multiplication: place the tens digit from the answer above the digit to the left of the digit you have just multiplied. In the example, the 1 from the 18 is carried above the 7, the digit next in place value to the digit that was just multiplied.

Example:

$$\begin{array}{r} 76 \\ \times 3 \\ \hline \end{array}$$

Start by multiplying the ones digit in the number on top by 3.

$$\begin{array}{r} ^176 \\ \times 3 \\ \hline 8 \end{array}$$

6 times 3 = 18.

Write only the 8, in the ones place in the answer line.
Carry the 1.

$$\begin{array}{r} ^176 \\ \times 3 \\ \hline 228 \end{array}$$

Next, multiply the 7 times 3. ($7 \times 3 = 21$)

Add the carried 1 to get 22. ($21 + 1 = 22$)

Write 22 in the answer line to the left of the 8.

Multiplying with more than one digit in the multiplier

When there are more than two digits in the multiplier, continue to add another answer line for each digit

Important fact: When we multiply by a number such as 345, we multiply by each of the digits in the number. For each digit we will start a new answer line, using zero as a placeholder in each line. Once we complete all the multiplication, we add the answers from all of the answer lines to get the final answer.

To multiply with more than one digit in the multiplier, use zero as a placeholder in each answer line as you multiply with the tens column, the hundreds columns etc.

1. Multiply by the digit in the ones column.
2. Multiply by the digit in the tens column:
 - Start the second answer line
 - Put a zero in the ones column of the second answer line.
 - Multiply by the tens digit, placing the answers to the left of the zero.
3. Multiply by the digit in the hundreds column:
 - Put a zero in the ones column and a zero in the tens column of the third answer line
 - Multiply by the hundreds digit, placing the answers to the left of the zeros.
4. Multiply by the digit in the thousands column.
 - Put a zero in the ones column, the tens column, and hundreds column of the fourth answer line.
 - Multiply.
5. The second answer line has one zero; the third answer line will have two zeros as place holders. The fourth has three zeros, and so on.
6. To complete the question, add all the answer lines together.

Note: Erase or cross out the carrying from the each line as you complete it so you don't get mixed up. When you are more confident, keep track of the carrying in your head.

Here is an example with two digits in the multiplier

$$\begin{array}{r} ^489 \\ \times 35 \\ \hline 445 \end{array}$$

Multiply each digit in the top number, 89, by the ones digit, which is 5.

Multiplying $9 \times 5 = 45$. Write 5 in the answer line and carry the 4 above the 8.

Now multiply $8 \times 5 = 40$. Add the carried 4 to 40 and write 44 to the left of the 5 already on the answer line to get 445.

This completes the first answer line.

$$\begin{array}{r} ^4 89 \\ \times 35 \\ \hline 445 \\ 0 \end{array}$$

Now, continue multiplying the example using the steps above.

First write a 0 in the ones column as a place holder.

Multiply 89 by 3, the next digit in the multiplier.

$$\begin{array}{r} ^2 89 \\ \times 35 \\ \hline 445 \\ 70 \end{array}$$

Start by multiplying $9 \times 3 = 27$

Carry the 2 (from the 27) and put it above the next digit to be multiplied

Then write the 7 next to the 0.

Now multiply 8×3 .

$$\begin{array}{r} ^2 89 \\ \times 35 \\ \hline 445 \\ 2670 \end{array}$$

$8 \times 3 = 24$.

Add the carried 2 to get the answer 26.

Put the 26 to the left of the 70 to get 2670 as your second answer line.

Add the two answer lines together:

$$\begin{array}{r} ^2 89 \\ \times 35 \\ \hline 445 \\ 2670 \\ \hline 3115 \end{array}$$

Here is an example with three digits in the multiplier:

$$\begin{array}{r} 654 \\ \times 821 \\ \hline 654 \\ 13080 \\ 523200 \\ \hline 536934 \end{array}$$

Start with the ones column.

Multiply each digit in the top number by each digit in the multiplier.

Make a new answer line each time you multiply by another digit in the multiplier.

Use zeros as place holders in the second, third, etc. answer lines.

Add all the answer lines together.

Here is an example in which the multiplier has zeros in it:

$$\begin{array}{r} 1952 \\ \times 3040 \\ \hline 0000 \\ 78080 \\ 000000 \\ 5856000 \\ \hline 5934080 \end{array}$$

zero times any number is 0

use 1 zero as place holder

use 2 zeros as place holders

use 3 zeros as place holders

add all the answer lines

TO MULTIPLY WHOLE NUMBERS

1. Write the two numbers so they are lined up one under the other. Usually the larger number is on top, although it doesn't make any difference in the final answer.
2. If the multiplier has one digit, multiply each digit in the top number by the multiplier, starting at the right side. Write the answer to each individual multiplication step on the answer line, also starting at the right. If an answer to any individual step has two digits, write only the ones digit on the answer line and carry the second (tens) digit above the next digit to be multiplied. After multiplying the next digit, add the carried number to the answer before writing it down.
3. If there is more than one digit in the multiplier, multiply the top number by the second digit in the multiplier in the same way. Write the answers to the individual multiplication steps in a second answer line below the first. Before writing the answer, put a zero on the right side of the answer line as a place holder.
4. Continue multiplying the top number by each digit in the multiplier in the same way, putting one additional zero in each subsequent answer line.
5. Add all the answer lines together to get your final answer.

PRACTICE WITH MULTIPLICATION

Answers are on the last page of the skills manual. Check your answers as you work.

9. Multiply.

a) 364×8

b) 405×9

c) 21×15

d) 700×35

e)
$$\begin{array}{r} 62 \\ \times 43 \\ \hline \end{array}$$

f)
$$\begin{array}{r} 831 \\ \times 29 \\ \hline \end{array}$$

g)
$$\begin{array}{r} 8007 \\ \times 44 \\ \hline \end{array}$$

h)
$$\begin{array}{r} 379 \\ \times 60 \\ \hline \end{array}$$

i)
$$\begin{array}{r} 400 \\ \times 600 \\ \hline \end{array}$$

j)
$$\begin{array}{r} 4751 \\ \times 65 \\ \hline \end{array}$$

k)
$$\begin{array}{r} 5463 \\ \times 720 \\ \hline \end{array}$$

l)
$$\begin{array}{r} 405 \\ \times 906 \\ \hline \end{array}$$

10. A box holds 18 lag screws. There are 15 full boxes on the shelf. How many lag screws are there in total?

11. If Joseph drives 23 km to work *one* way, how many kilometers does he drive each week if he works five days a week?

12. An electric motor turns 850 revolutions per minute. How many revolutions will it turn in 14 minutes?

DIVISION OF WHOLE NUMBERS

Division is related to multiplication. You need to know the basic multiplication facts to do division. (*If you are unsure of the multiplication facts, make your own table on a file card, like the one on the answer page. Keep it for reference.*)

You can think of a division question as the answer to a multiplication question and one of the multipliers. To solve the division question, you have to find the other multiplier that was used to get that answer.

Example: To find the answer to 27 divided by 9, we regard 27 as the multiplication answer and 9 as one of the multipliers. We want to find what 9 was multiplied by to get 27. Since $9 \times 3 = 27$, we know that 3 is the other multiplier. The answer to 27 divided by 9 is 3.

Example:

18 divided by 2 = ?

means

2 times ? = 18 .

Reading and Writing Division Questions

Division questions can be written several different ways. Here are some examples:

Divide 45 by 9.

$628 \div 64$ Read as 628 divided by 64.

$30/6$ Read as 30 divided by 6.

$8 \overline{)72}$ Read as 72 divided by 8. Notice that the number in the box is read first.

This last form, which uses a division box, is the usual way to write a division question in order to solve it.

When you use a calculator to divide, it is important to key in the numbers in the correct order. *You must key in the numbers in the same order as the four examples above.*

Note: *Division questions can also be worded, “7 divided into 56” or, “How many times does 7 go into 56?”. To divide with a calculator when the question is worded in this way, change the order of the two numbers so that they read “56 divided by 7.” before keying them in.*

To talk about division, it is helpful to know two terms. In the expression $45 \div 9$, the first number, 45, is called the **dividend**. It is the number that you are *dividing into* and it goes inside the box when you write the question like this:

$9 \overline{)45}$

The number before the division sign $\overline{)$ is called the **divisor**. It is the number that you are *dividing by*. The divisor, 9, goes to the left of the box.

To divide by a number with one digit, as in our example $45 \div 9$, you ask yourself what you multiply 9 by to get 45. To get the answer 45 when one of the multipliers is 9, the other multiplier has to be 5. So, by reversing $9 \times 5 = 45$, we get $45 \div 9 = 5$.

Important division facts!

- 1. Any number divided by itself is 1.**

$$9 \div 9 = 1$$

- 2. Any number divided by 1 is that number.**

$$5 \div 1 = 5$$

- 3. Zero divided by any number is 0.**

$$0 \div 7 = 0$$

- 4. A number can't be divided by 0.**

$8 \div 0$ doesn't have an answer

Answer these division questions to become familiar with the basic division facts. Use Table 3 (Multiplication Facts) to check your answers. Set up the question $72 \div 8$ by saying 72 divided by 8. However, for each individual step of a question using a division box, you can say to yourself, "How many times does 8 go into 72?"

$$7 \overline{)42} \quad 9 \overline{)63} \quad 8 \overline{)24} \quad 7 \overline{)14} \quad 6 \overline{)54} \quad 3 \overline{)18} \quad 5 \overline{)15}$$

$$6 \overline{)24} \quad 8 \overline{)32} \quad 4 \overline{)12} \quad 7 \overline{)63} \quad 9 \overline{)27} \quad 8 \overline{)56} \quad 3 \overline{)21}$$

$$7 \overline{)28} \quad 9 \overline{)81} \quad 3 \overline{)9} \quad 3 \overline{)24} \quad 5 \overline{)20} \quad 4 \overline{)32} \quad 3 \overline{)27}$$

$$3 \overline{)6} \quad 5 \overline{)35} \quad 6 \overline{)48} \quad 9 \overline{)36} \quad 7 \overline{)56} \quad 8 \overline{)40} \quad 4 \overline{)8}$$

$$3 \overline{)12} \quad 9 \overline{)54} \quad 4 \overline{)20} \quad 5 \overline{)10} \quad 7 \overline{)21} \quad 6 \overline{)42} \quad 8 \overline{)64}$$

$$5 \overline{)25} \quad 6 \overline{)12} \quad 4 \overline{)28} \quad 3 \overline{)15} \quad 9 \overline{)18} \quad 6 \overline{)18} \quad 8 \overline{)48}$$

$$5 \overline{)40} \quad 4 \overline{)36} \quad 7 \overline{)35} \quad 9 \overline{)45} \quad 4 \overline{)24} \quad 6 \overline{)30} \quad 4 \overline{)16}$$

$$7\overline{)49} \quad 6\overline{)36} \quad 5\overline{)45} \quad 8\overline{)72} \quad 3\overline{)12} \quad 5\overline{)30} \quad 9\overline{)72}$$

Dividing larger numbers

Once you know the basic division facts, you can start to do longer division questions.

Sample: Divide 426 by 6.

Start by writing the question in box form.

$$6\overline{)426}$$

Step One: With longer division questions start working from the left. Look at the first number in the box, which is 4, and see if 6 will divide into it. Since 4 is too small, look at the 4 and the number to its right, to make 42. You recognize that 6 will divide into 42. Put the answer, 7, above the right hand digit in the 42. So far, it looks like this:

$$\begin{array}{r} 7 \\ 6\overline{)426} \end{array}$$

Step Two: Multiply the divisor 6 by the answer to the first step, which is 7. The answer is 42. Write this underneath the 42 you just divided into, draw a line under it and subtract. Write the answer to the subtraction, called the remainder, underneath the line. $42 - 42 = 0$, so write 0 under the line.

$$\begin{array}{r} 7 \\ 6\overline{)426} \\ \underline{42} \\ 0 \quad \text{remainder} \end{array}$$

Step Three: Now bring down the next digit in the dividend, which is 6. Write it beside the remainder of the subtraction. The remainder and the digit you just brought down become the next number that you divide into.

Step Four: Now divide the divisor 6 into the 06 at the bottom of the question, which gives the answer 1. The 1 goes in the answer line next to the 7, above the 6 in the dividend. Multiply this answer by the divisor.

$6 \times 1 = 6$. Write this multiplication answer underneath the number at the bottom of the question (06 in this example) and draw a line under it. Subtract and write your subtraction answer, which is 0, under the line you drew, like this:

$$\begin{array}{r} 71 \\ 6 \overline{)426} \\ \underline{42} \\ 06 \\ \underline{06} \\ 0 \end{array}$$

In this basic example, all the numbers divided evenly, so the remainder was always 0. But in most cases, there are remainders throughout the question. The next level of difficulty is to solve division questions when *the divisor doesn't go evenly into the numbers in the dividend*.

Example: $345 \div 5$

Set up your division: $5 \overline{)345}$

$$\begin{array}{r} 6 \\ 5 \overline{)345} \end{array}$$

1. 5 won't divide into 3 but it will go into 34. Since there is no whole number you can multiply 5 by to get 34, find the closest number **smaller** than 34 that 5 will divide into. Look at the fives column in the multiplication table. 34 falls between the answers 30 and 35. **Since you want the answer closest to 34 that is also smaller than 34, you go to the 30.** The table tells you that 30 divided by 5 is 6.

The 6 is placed in the answer line above the last digit in the number you have been dividing into, which is 34. The 6 is placed in the answer line above the 4.

$$\begin{array}{r} 6 \\ 5 \overline{)345} \\ \underline{30} \\ 4 \end{array}$$

2. Multiply the answer 6 by the divisor 5 to get 30. Put the 30 under the 34, draw a line under it and subtract to get the remainder 4.

$$\begin{array}{r} 6 \\ 5 \overline{)345} \\ \underline{30} \\ 45 \end{array}$$

3. Bring down the next digit in the dividend, which is 5, and place it beside the subtraction answer.

$$\begin{array}{r} 69 \\ 5 \overline{)345} \\ \underline{30} \\ 45 \\ \underline{45} \\ 0 \end{array}$$

4. Divide the 5 into the 45 at the bottom. The answer to $45 \div 5$ is 9. Write 9 beside the 6 in the answer line. Multiply the answer 9 by the divisor 5 to get 45. Write this underneath the 45 already there. Subtract to get the remainder 0.

If there are more digits in the dividend, you continue dividing in the same way. Each step is a separate division, with each answer written in the next space to the right on the answer line.

Remainders from the subtraction in each step are used with the next digit brought down from the dividend. They form the number the divisor is divided into in the next step.

Example: $36533 \div 7$

$$7 \overline{)36533}$$

$$\begin{array}{r} 5 \\ 7 \overline{)36533} \\ \underline{35} \\ 15 \end{array}$$

$$\begin{array}{r} 52 \\ 7 \overline{)36533} \\ \underline{35} \\ 15 \\ \underline{14} \\ 13 \end{array}$$

$$\begin{array}{r} 521 \\ 7 \overline{)36533} \\ \underline{35} \\ 15 \\ \underline{14} \\ 13 \\ \underline{7} \\ 6 \end{array}$$

$$\begin{array}{r} 521 \\ 7 \overline{)36533} \\ \underline{35} \\ 15 \\ \underline{14} \\ 13 \\ \underline{7} \\ 63 \end{array}$$

$$\begin{array}{r} 5219 \\ 7 \overline{)36533} \\ \underline{35} \\ 15 \\ \underline{14} \\ 13 \\ \underline{7} \\ 63 \\ \underline{63} \\ 0 \end{array}$$

1. Start by dividing 36 by 7. The multiplication table shows that 35 is the closest answer smaller than 36 that 7 will divide into. $35 \div 7 = 5$. Place the answer 5 in the answer line above the last digit in the 36 in the dividend, which is the 6.
2. Multiply the answer 5 by the divisor 7 to get 35. Place that answer below the 36 and subtract $36 - 35$, which gives the remainder 1.
3. Bring down the next digit in the dividend, which is 5, and put it beside the remainder 1.
4. Divide the divisor 7 into the 15 at the bottom. Put the answer 2 to the right of the answer 5 on the answer line. Multiply the 2 by the divisor 7. Put the multiplication answer 14 below the 15 at the bottom. Subtract $15 - 14$ to get 1.
5. Bring down the next digit, 3, in the dividend and put it beside the remainder 1.
6. Now divide the divisor 7 into the 13 at the bottom. Put the answer 1 in the answer line beside the 52 already there. Multiply the 1 by the divisor 7 and put the answer 7 below the 13 at the bottom and subtract to get the remainder 6.

Are you starting to see a pattern in the different steps?

7. Bring down the last digit in the dividend, the 3, and place it beside the remainder 6.

8. Divide 63 by 7. It goes 9 times. Put the 9 in the answer line after the 521 to get the final answer 5219. Multiply the 9 by the divisor 7, to get 63, which is written below the 63 at the bottom of the question. Subtracting $63 - 63$ gives the remainder 0.

The next example has a final remainder.

$$\begin{array}{r} \underline{171 \text{ R } 1} \\ 4 \overline{)685} \\ \underline{4} \\ 28 \\ \underline{28} \\ 05 \\ \underline{4} \\ 1 \end{array}$$

The letter R and the remainder are written right after the answer:

Remainder

Here is another example:

$$\begin{array}{r} \underline{776 \text{ R } 1} \\ 6 \overline{)4657} \\ \underline{42} \\ 45 \\ \underline{42} \\ 37 \\ \underline{36} \\ 1 \end{array}$$

Remainder

You might get a question in which the divisor is too large to divide into a remainder. In this case, put a zero in the answer line and bring down the next digit as in this next example.

Example: Divide 351765 by 9.

$$\begin{array}{r} 39 \\ 9 \overline{)351765} \\ \underline{27} \\ 81 \\ \underline{81} \\ 076 \end{array}$$

Since 9 can't divide into 7, put a 0 in the answer line and bring down the next digit (which is 6)

$$\begin{array}{r} 39085 \\ 9 \overline{)351765} \\ \underline{27} \\ 81 \\ \underline{81} \\ 076 \\ \underline{72} \\ 45 \\ \underline{45} \\ 0 \end{array}$$

You can divide 76 by 9, to get the answer 8. This goes in the answer line next to the 0. Multiply the divisor 9 by the 8, to get the answer 72, which is written below the 76. Subtract and continue.

TO DIVIDE WITH A ONE DIGIT DIVISOR

1. Write the question using a division box, with the divisor to the left and the dividend inside the box.
2. Starting at the left side of the dividend, look for the first whole number into which the divisor can divide.
3. If the divisor doesn't divide into that number evenly, find the number that is closest to it and that is also smaller than it that the divisor can divide evenly into.
4. Put the answer to that division step in the answer line above the last digit of the number that you just divided into.
5. Multiply that answer by the divisor and put the multiplication answer below the number in the dividend that you just divided into. Draw a line underneath it.
6. Subtract the two numbers and put the subtraction answer, the remainder, below the line.
7. Bring down the next digit in the dividend and place it beside the remainder.
8. Repeat the steps dividing into the number at the bottom of the question. Place the answer to that step next to the answer already in the answer line.
9. Continue until there are no more digits in the dividend to bring down. The answer in the answer line is your final answer. If there is a final remainder from the last subtraction, the letter R and then the remainder are written right after the final answer in the answer line.

PRACTICE WITH DIVISION: There are a lot of steps in division. Carefully complete each step before going on to the next, working through the whole question. **Answers are on the last page of the skills manual.**

13. Divide the following.

- a) Divide 56 by 8. b) $504 \div 7$ c) $552 \div 6$ d) Divide 1847 by 5.

- e) $6 \overline{)9852}$ f) $3 \overline{)6900}$ g) $4 \overline{)3461}$ h) $9 \overline{)3870}$

14. A 4 wheel delivery truck weighs 1584 kg. How much weight does each wheel bear?

15. A facilities mechanic uses a piece of steel that is 2385 cm long to cut 9 pieces of furnace ducting. What is the average length of each duct?

Division With A Divisor That Has Two Or More Digits

The next operation is division with a divisor that has more than one digit. The basic steps are similar to one digit division but you have larger numbers to divide, multiply and subtract. *The main difference in the first step is the use of a **trial divisor**.*

Read the explanation for each step. Try doing the work for that step on your own before going on to the next. Look back at the correct explanation if you get lost.

Sample: Divide 30038 by 46.

First set up your box:

$$46 \overline{)30038}$$

Step One: You see that 46 is too large to go into 30. You have to start by dividing 46 into 300, but how many times will it go into 300? This is where you need to use a helpful estimate, called a *trial divisor*, to get close to the right answer. Take the first digit in the divisor, in this case 4, and divide it into the first number in the dividend that it will go into, which is 30.

By now you know that 4 won't go evenly into 30 but the next smaller number that it will go into is 28. $28 \div 4 = 7$.

Step Two: Multiply the divisor 46 by the answer 7. Do these multiplications and subtractions on scrap paper. Unlike one digit division, this answer 7 is only an estimate and might not be correct. *The multiplication answer will tell us if we have the right number.*

$46 \times 7 = 322$. If we write the number 322 under the 300 and try to subtract, we see that 322 is too big. Our estimate 7 is close but not the right answer. *Since the **multiplication answer is too big**, we have to choose a **smaller division answer**.*

The next trial answer should be one digit smaller, so we should try 6. (It is easier to erase if you write in pencil.) Now we multiply 46×6 , which equals 276. Put the 276 underneath the 300 and subtract.

$$\begin{array}{r} 6 \\ 46 \overline{)30038} \\ \underline{276} \\ 24 \end{array}$$

We saw that when the trial answer is too big, the multiplication answer will also be too big, so you can't subtract. You can also have a trial answer that is too small. You wouldn't be aware that your answer was too small until you subtracted.

If the **subtraction answer** is bigger than the **divisor**, you need to go back and try a larger trial answer. Always look at the subtraction answer to see if it is smaller than the divisor. In this case, the subtraction answer 24 is smaller than our divisor 46, so we have the right answer.

Step Three: Bring down the next digit in the dividend, which is 3, and put it beside the 24. Our next number to divide into is 243.

$$\begin{array}{r} 6 \\ 46 \overline{)30038} \\ \underline{276} \\ 243 \end{array}$$

Once again we only use the first digit in our divisor, which is 4.

We divide 4 into 24 to get our next trial answer, which is 6.

Step Four: Put 6 in the answer line next to the answer already there and multiply the divisor by the 6.

$$\begin{array}{r} 66 \\ 46 \overline{)30038} \\ \underline{276} \\ 243 \\ \underline{276} \end{array}$$

We already multiplied $46 \times 6 = 276$.

Just write down the multiplication answer worked out before

Once again our multiplication answer is too big to subtract, which means our trial answer is too big.

Erase the last 6. Also erase the 276 underneath 243.

$$\begin{array}{r} 65 \\ 46 \overline{)30038} \\ \underline{276} \\ 243 \\ \underline{230} \\ 13 \end{array}$$

Write down the trial answer 5 instead.

Multiply 46×5 , which equals 230 and write that answer under the 243.

Subtract and check that the subtraction answer isn't larger than the divisor.

The subtraction answer 13 is smaller than the divisor, so we have the right answer to this step.

Step Five: Bring down the last digit in the dividend to get 138, the last number we will divide into. Since the closest answer to $13 \div 4$ is 3, we will use 3 as our next trial answer. Our question now looks like this:

$$\begin{array}{r} 65 \\ 46 \overline{)30038} \\ \underline{276} \\ 243 \\ \underline{230} \\ 138 \end{array}$$

Step Six: Multiply the divisor by 3.

$$\begin{array}{r} 653 \\ 46 \overline{)30038} \\ \underline{276} \\ 243 \\ \underline{230} \\ 138 \\ \underline{138} \\ 0 \end{array}$$

$$46 \times 3 = 138.$$

Put the multiplication answer underneath the 138 at the bottom.

Subtract to get the remainder 0.

Here is an example with a three digit divisor:

$$706 \overline{)55068}$$

We will start by using the first digit of the divisor, 7, as our trial divisor. It will divide into 55, the first two digits of the dividend.

Since the closest answer to $55 \div 7 = 7$, we will use 7 as our first trial answer.

Multiply the divisor 706 by 7.

$$\begin{array}{r} 706 \\ \times 7 \\ \hline 4942 \end{array}$$

This isn't bigger than 5506, so our trial answer isn't too big.

$$\begin{array}{r} 7 \\ 706 \overline{)55068} \\ \underline{4942} \\ 564 \end{array}$$

Write this answer, 4942, under the 5506 and subtract.

Write 7 in the answer line.

Bring the 8 down from the dividend and write it beside the 564 to get 5648.

To get our next trial answer, once again use only the 7 from the divisor and divide it into the number at the bottom. We use the first two digits of 5648 because 56 is the first number 7 will divide into.

$$\begin{array}{r} 78 \\ 706 \overline{)55068} \\ \underline{4942} \\ 5648 \\ \underline{5648} \\ 0 \end{array}$$

Dividing 56 by 7 gives us 8 as a trial answer.

Put the 8 next to the 7 in the answer line, and multiply the whole divisor by 8, which equals 5648.

Put the multiplication answer underneath the number at the bottom and subtract.

Here is an example with a remainder:

$$231 \overline{)188518}$$

Trial divisors used with numbers that have 1 or 2 as their first digit are less reliable. For these numbers, you might have to make educated guesses. Keep adjusting your guess by looking at the multiplication and subtraction answers.

Use 2 as the trial divisor. It will divide into 18, the first two digits in the dividend, which gives the first trial answer 9.

Multiply the divisor by 9.

$$\begin{array}{r} 231 \\ \times 9 \\ \hline 2079 \end{array}$$

We can see that the multiplication answer is bigger than the digits above it, so our trial answer is too big. Try 8 this time.

$$\begin{array}{r} 8 \\ 231 \overline{)188518} \\ \underline{1848} \\ 37 \end{array}$$

The multiplication answer isn't too large and the subtraction answer is smaller than the divisor, so 8 is the correct first answer.

$$\begin{array}{r} 81 \\ 231 \overline{)188518} \\ \underline{1848} \\ 371 \end{array}$$

Bring down the 1. Divide the trial divisor 2 into the 3 from the subtraction answer. $3 \div 2 = 1$.

$$\begin{array}{r} 81 \\ 231 \overline{)188518} \\ \underline{1848} \\ 371 \\ \underline{231} \\ 140 \end{array}$$

Multiply 231 by 1, put the answer beneath the 371 at the bottom of the question and subtract.

$$\begin{array}{r}
 816 \text{ R } 22 \\
 231 \overline{)188518} \\
 \underline{1848} \\
 371 \\
 \underline{231} \\
 1408 \\
 \underline{1386} \\
 22
 \end{array}$$

Bring down the 8 and continue.

The trial divisor 2 divides into the 14, which is the first digit of this subtraction answer.

This gives us the trial answer 7. $231 \times 7 = 1617$. It is bigger than 1408.

The next smaller trial answer is 6: $231 \times 6 = 1386$.

Subtract to get a remainder, 22. Write the remainder in the answer line.

Here is an example with zeros at the end of the dividend:

$$36 \overline{)230400}$$

23 divided by the trial divisor 3 is 7 R 2. Put the trial answer 7 above the third digit in the dividend from the left, which is the first 0.

Multiply the divisor by 7 to get 252. The trial answer is too big. Erase the 7 and put 6 in the answer line as our trial answer.

$$\begin{array}{r}
 6 \\
 36 \overline{)230400} \\
 \underline{216} \\
 144
 \end{array}$$

Multiply 36×6 to get 216. Write it beneath the 230. Subtract to get 14.

Bring down the next digit in the dividend, 4, and put it beside the 14.

$$\begin{array}{r}
 640 \\
 36 \overline{)230400} \\
 \underline{216} \\
 144 \\
 \underline{144} \\
 00
 \end{array}$$

14 divided by the trial divisor 3 goes 4 times. Put 4 on the answer line next to the 6.

Multiply 36 by 4 to get 144. Put this under the 144 at the bottom and subtract to get 0.

Bring down the next digit in the dividend, which is 0. $0 \div 3$ is 0. Put 0 on the answer line next to the 4.

$$\begin{array}{r}
 6400 \\
 36 \overline{)230400} \\
 \underline{216} \\
 144 \\
 \underline{144} \\
 000
 \end{array}$$

Bring down the last 0 from the dividend. $0 \div 3$ is 0, so we put another 0 in our answer line to finish the question.

Example:

$$\begin{array}{r}
 16100 \\
 55 \overline{)885507} \quad \text{R7} \\
 \underline{55} \\
 335 \\
 \underline{330} \\
 55 \\
 \underline{55} \\
 007
 \end{array}$$

Example:

$$\begin{array}{r} 206 \\ 34 \overline{) 7008} \quad R4 \\ \underline{68} \\ 20 \\ \underline{0} \\ 208 \\ \underline{204} \\ 4 \text{ remainder} \end{array}$$

Before you do the two-digit division questions, you might want to go back and redo the division examples on your own. Don't look at the explanations unless you run into difficulties or you get the wrong answer.

TO DIVIDE WITH A DIVISOR THAT HAS TWO OR MORE DIGITS

1. Follow the steps for one digit division with the addition of the following steps.
2. Use a helpful estimate called a trial divisor to get close to the right answer. Use the first digit in the divisor and divide it into the first number in the dividend that it will go into.
3. Multiply the divisor by the trial answer. Place this answer under the dividend.
 - Look to see if it is bigger than the digits above it, making it impossible to subtract. If this is the case, make your trial answer one digit smaller and try again.
 - If the multiplication answer is smaller than the digits above it, subtract and examine your subtraction answer. If the subtraction answer is larger than the divisor, make your trial answer one digit larger and try again. If the subtraction answer is smaller than the divisor, your trial answer is correct.
4. Put that answer on the answer line above the dividend, directly above the ones digit in the multiplication answer. Continue.
5. Bring down the next digit in the dividend and place it next to the subtraction answer. This becomes the next number you divide into. Once again, divide the trial divisor into the first digit or digits of this number. Place the trial answer next to and after the answer already on the answer line. Multiply the divisor by this second trial answer and place it under the number you have just divided into. Check your multiplication answer following the procedure in step 4.
6. When your second trial answer is correct, continue in the same way until there are no more digits in the dividend to bring down. If there is a final remainder, write the letter R and the remainder after your final answer.
7. If at the beginning of Step 5, the number at the bottom is too small for the divisor to divide into, put a zero on the answer line and bring down the next digit in the dividend so it becomes part of the number at the bottom. Continue dividing.

PRACTICE DIVIDING WITH LARGER DIVISORS: The following division questions have more than one digit in the divisors. If you get stuck, go back to the main example, look at all the steps and decide where you are having trouble. Study that step in the example and try to use the same procedure in the question that you are working on. **Answers are on the answer page at the end of the skills manual. Check your answers as you work.**

16. Divide.

a) $567 \div 27$

b) $7383 \div 321$

c) $27044 \div 63$

d) $865 \div 34$

e) $83 \overline{)25232}$

f) $50 \overline{)37450}$

g) $46 \overline{)9915}$

h) $23 \overline{)4669}$

i) $63 \overline{)1260252}$

17. A car travels 1210 km in 11 hours. What is the average km/hr? (To find the average speed, divide the total distance by the number of hours it took.)

18. Each piece of ducting requires a length of metal 240 cm long to be cut from a sheet that is 1 m wide. How many ducts can be made with a piece of metal 8640 cm long?

19. There are 5184 screws in 48 boxes. How many screws are in each box?

ROUNDING OFF NUMBERS

Doing division takes a lot of time. You will most likely use a calculator if you have many calculations to do. However, it is easy to make a mistake when using a calculator, resulting in the wrong answer. You should have a rough idea of what the answer should be so you can tell if the answer on the calculator is reasonable.

In other situations, you might be required to give a quick estimate of the cost of a job. You might be required to give a quick estimate of the cost of materials. To do these calculations in your head, it is easiest to first round off the numbers.

Example: If you need an estimated answer for “318 multiplied by 5”, you could round off 318 by replacing the digits 1 and 8 with zeros to get 300. You can see that it is easier to multiply 300 by 5 than 318 by 5. Just multiply 3×5 to get 15 and keep track of how many zeros (two) to put place after the 15. This gives you 1500 for the estimate.

You need to know how to round off numbers to get a quick answer in your head. It is a useful skill that requires an understanding of place value. Here is the place value chart again so you can refer to it when you are rounding off.

millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones
4	7	8	2	9	5	1

To do an estimate, you decide the most convenient place to round off to. You might be asked to round off to a specific number of places. Rounding off to two places is the same as rounding to the tens place. Rounding off to three places is the same as rounding to the hundreds place.

Once you know the place value you are rounding off to, follow these steps:

1. Look at the digit in the place value to the right of that place.
 - If you are rounding off to the hundreds place, you look at the digit in the tens place, which is one place value smaller than the hundreds place.
 - If you are rounding off to the hundred thousands place, you look at the digit in the ten thousands place. (Look at the chart of place values again.)
2. If the digit you are looking at in the next smaller place value is **five or greater**, **increase the digit in the place value you are rounding off to by one**.
 - To round off to the *hundreds place*, if the digit in the *tens place* is five or greater, **change** the digit in the hundreds place so it becomes one digit larger in value. See Sample 1.

3. If the digit in the next smaller place value is **less than five, the digit in the place value you are rounding off to remains the same.**
 - To round off to the *hundreds place*, if the digit in the *tens place* is less than five, then the digit in the hundreds place **remains as it is**. See Sample 2.
4. After deciding whether to increase the digit in the place value you are rounding off to by one or to leave it as it is, you **replace all the digits to the right of that place value with zeros.**

Sample 1: To round off 6 375 to the hundreds place, follow these steps:

- Look at the digit in the tens place. You see that it is 7, which is greater than 5.
- Now change the digit in the hundreds place, which is 3, to the next higher digit, 4.
- Next replace all the digits to the right of the hundreds place with zeros.

6 375 rounded off to the nearest hundred becomes 6 400.

Sample 2: To round off 18 234, to the nearest thousand follow these steps:

- Look at the digit in the hundreds place. It is less than five.
- Leave the digit in the thousands place as it is. The 8 in the thousands place remains an 8.
- Replace all the digits to the right of the thousands place with zeros.

18 234 rounded off to the nearest thousand is 18 000.

Example: Round off 7 923 538 to the nearest ten-thousand.

- The digit in the ten-thousands place value is 2. The digit to the right in the next smaller place value (the thousands place value) is 3.
- Since the digit 3 is smaller than five, the 2 remains as it.
- All the digits to the right of the ten-thousands place are replaced with zeros.

7 923 538 rounded off to the nearest ten-thousand is 7 920 000.

Example: Round 5 687 to the nearest ten.

- The digit in the tens place is 8. The digit to the right in the next smaller place value (the ones place value) is 7.
- Since 7 is larger than five, the 8 becomes one digit larger. It becomes a 9.
- All the digits to the right of the tens place are replaced with zeros.

5 687 rounded off to the nearest ten is 5 690.

If the digit you are rounding off to is a 9 and the digit in the next smaller place value is five or greater, the 9 becomes a ten. We write 0 and add 1 to the digit in the next place value.

Example: Round off 54 982 to the nearest hundred.

- The digit in the hundreds place value is 9. The digit to the right of the hundreds place (the tens place) is 8, which is greater than five.
- The 9 becomes a 10, so the 9 is replaced by 0; the digit to the left of the 9 in the thousands place, which is 4, becomes 5.
- All the digits to the right of the hundreds place become zeros.

54 982 rounded off to the nearest hundred is 55 000.

Example: Round off 999 503 to the nearest thousand.

- The digit in the thousands place is 9. The digit in the hundreds place is 5.
- The 9 in the thousands place becomes a 10. The 9 is replaced by 0 and the digit in the ten-thousands place increases in value by one.
- Since the digit in the ten-thousands place is also 9, it too becomes a 10. The 9 is replaced by 0 and the digit in the hundred-thousands place increases by one.
- The digit in the hundred-thousands place value is also 9 and so it becomes 10 when it increases by one.
- All the digits to the right of the thousands place become 0.

999 503 rounded off to the nearest thousand is 1 000 000.

If the digit you are rounding off to is a 0 and the digit in the next smaller place value is less than five, the 0 remains as it is and all the digits to its right are replaced with zeros.

If the digit you are rounding off to is a 0 and the digit in the next smaller place value is five or greater, the 0 becomes 1 and all digits to its right are replaced with zeros.

Example: Round off 6 730 479 to the nearest thousand.

- The digit in the thousands place is 0. The digit in the next smaller place value, the hundreds place, is 4, which is less than five.
- The 0 remains as it is.
- All digits to the right of the thousands place are replaced with zeros.

6 730 479 rounded off to the nearest thousand is 6 730 000.

Example: Round off 508 459 to the nearest ten-thousand.

- The digit in the ten-thousands place is 0. The digit in the next smaller place value, the thousands place, is 8, which is greater than five.
- The 0 becomes a 1.
- All digits to the right of the ten-thousands place are replaced with zeros.

508 459 rounded off to the nearest ten-thousand is 510 000.

Here are some questions for you to try.

Round off the following numbers to the nearest hundred:

1. 3 624 becomes _____
2. 870 becomes _____
3. 1 085 becomes _____
4. 76 962 becomes _____

Here are the answers. If you have any difficulties, study the examples to find where you went wrong.

1. 3 600
2. 900
3. 1 100
4. 77 000

Round off the following numbers to the nearest thousand:

1. 6 740 becomes _____
2. 1 040 387 becomes _____
3. 99 657 becomes _____
4. 9 900 becomes _____

Here are the answers.

1. 7 000
2. 1 040 000
3. 100 000
4. 10 000

USING A CALCULATOR

You need to know how to do long division on paper. But if you have to do many division questions with large numbers, using a calculator will save time. You should know how to use a calculator to do all the basic number operations. Most calculators work in a similar way.

Addition: To add using a calculator, key in the first number, then the addition (+) sign, then the next number, then the + sign, etc. Most calculators will add in the preceding number when you key the + sign, but on some you may have to key the equal (=) sign between each number to get it to do the addition.

Subtraction: If you are using a calculator to subtract, you have to key in the number you are subtracting from first, then key in the subtraction sign (–) and, lastly, key in the number you are taking away. Key in the equal sign to get your answer.

Multiplication: When you use a calculator to multiply, it doesn't matter what number you key in first. Key in one number, then the multiplication (x) sign, and then the second number. Key in the equals (=) sign to get your answer.

Division: When you use a calculator to divide, you must key in the dividend first, then the division (\div) sign, then the divisor and lastly the = sign. If you key in the numbers in the incorrect order, you will get the wrong answer. Any remainder in a division question, will show as a decimal. The value of the remainder will be the same, but it will be expressed in a different way.

Finding errors: If you realize you have keyed in a wrong digit, you have to clear the calculator and start over. When you do questions with a calculator, it is a good idea to do each calculation twice. If you get the same answer both times, you likely have the correct answer. If you get different answers, try the question again, paying close attention to every number you key into the calculator.

Try these questions using a calculator. The answers are after each question so you can compare it with the answer you get on your calculator. If your answer is different, key in your numbers again, and make sure you have the correct order when doing subtraction and division.

Remember to clear the calculator before starting the next question.

1. $639 + 795 + 389 = 1\ 823$

2. $79100 + 295044 = 374\ 144$

3. $4392 - 2509 = 1\ 883$

4. $5439 - 248 = 5\ 191$

5. $1493 \times 704 = 1\ 051\ 072$

6. $326 \times 284 = 92584$

7. $4131 \div 51 = 81$

8. $45105 \div 485 = 93$

ANSWER PAGE

ADDITION

1. a) 84 b) 373 c) 469 d) 3717 e) 7310
 f) 41913 g) 8503 h) 18 359 i) 615 j) 11 670
2. 40 hours
3. 187
4. 1637 kilometers

SUBTRACTION

5. a) 33 b) 311 c) 38 d) 224 e) 218 f) 265
 g) 76 h) 4495 i) 185 j) 861 k) 153 l) 347
4. 152 meters
5. 23 centimeters
6. 100 bolts

MULTIPLICATION

9. a) 2 912 b) 3 645 c) 315 d) 24 500
 e) 2 666 f) 24 099 g) 352 308 h) 22 740
 i) 240 000 j) 308 815 k) 3 933 360 l) 366 930
10. 270 lag screws
11. He drives 10 times a week (5 x 2 trips a day). $23 \times 10 = 230$ kilometers
12. 11 900 revolutions

DIVISION

13. a) 7 b) 72 c) 92 d) 369 R 2
 e) 1642 f) 2300 g) 865 R 1 h) 430
14. 396 kg
15. 265 centimeters

DIVISION WITH TWO OR MORE DIGITS IN THE DIVISOR

16. a) 21 b) 23 c) 429 R 17 d) 25 R 15
 e) 304 f) 749 g) 215 R 25 h) 203 i) 20 004
17. 110 km/ hr
18. 36 ducts
19. 108 screws

Here is the completed multiplication table.

Table 3: MULTIPLICATION FACTS

x	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

Here are the basic division answers.

$$\frac{6}{7) 42}$$

$$\frac{7}{9) 63}$$

$$\frac{3}{8) 24}$$

$$\frac{2}{7) 14}$$

$$\frac{9}{6) 54}$$

$$\frac{6}{3) 18}$$

$$\frac{3}{5) 15}$$

$$\frac{7}{6) 42}$$

$$\frac{4}{8) 32}$$

$$\frac{3}{4) 12}$$

$$\frac{9}{7) 63}$$

$$\frac{3}{9) 27}$$

$$\frac{7}{8) 56}$$

$$\frac{7}{3) 21}$$

$$\frac{4}{7) 28}$$

$$\frac{9}{9) 81}$$

$$\frac{1}{3) 9}$$

$$\frac{6}{3) 24}$$

$$\frac{4}{5) 20}$$

$$\frac{8}{4) 32}$$

$$\frac{9}{3) 27}$$

$$\frac{2}{3) 6}$$

$$\frac{7}{5) 35}$$

$$\frac{8}{6) 48}$$

$$\frac{4}{9) 36}$$

$$\frac{8}{7) 56}$$

$$\frac{4}{8) 40}$$

$$\frac{2}{4) 8}$$

$$\frac{4}{3) 12}$$

$$\frac{6}{9) 54}$$

$$\frac{5}{4) 20}$$

$$\frac{2}{5) 10}$$

$$\frac{3}{7) 21}$$

$$\frac{7}{6) 42}$$

$$\frac{8}{8) 64}$$

$$\frac{5}{5) 25}$$

$$\frac{2}{6) 12}$$

$$\frac{7}{4) 28}$$

$$\frac{5}{3) 15}$$

$$\frac{2}{9) 18}$$

$$\frac{3}{6) 18}$$

$$\frac{6}{8) 48}$$

$$\frac{8}{5) 40}$$

$$\frac{9}{4) 36}$$

$$\frac{5}{7) 35}$$

$$\frac{5}{9) 45}$$

$$\frac{6}{4) 24}$$

$$\frac{5}{6) 30}$$

$$\frac{4}{4) 16}$$

$$\frac{7}{7) 49}$$

$$\frac{6}{6) 36}$$

$$\frac{9}{5) 45}$$

$$\frac{9}{8) 72}$$

$$\frac{4}{3) 12}$$

$$\frac{6}{5) 30}$$

$$\frac{8}{9) 72}$$