EVALUATING
ACADEMIC READINESS
FOR APPRENTICE SHIP TRAINING

Revised for

Access to Apprenticeship

MATHEMATICS SKILLS
POWERS AND ROOTS

AN ACADEMIC SKILLS MANUAL
for
The Industrial Maintenance Mechanic Trades

This trade group includes the following trades:
Boiler Maker,
Facilities Maintenance Mechanic & Technician, and
Industrial Mechanic (Millwright)

Workplace Support Services Branch
Ontario Ministry of Education and Training

Revised 2011

In preparing these Academic Skills Manuals we have used passages, diagrams and questions similar to those an apprentice might find in a text, guide or trade manual.

This trade related material is not intended to instruct you in your trade. It is used only to demonstrate how understanding an academic skill will help you find and use the information you need.

MATHEMATICS SKILLS
POWERS AND ROOTS

An academic skill required for the study of the Industrial Maintenance Mechanic Trades

INTRODUCTION

You will need to make calculations using squares, square roots, and other powers, in order to find such things as area and volume To calculate the length of a side of a right triangle, you might need to use Pythagoras' Theorem: $c^2 = a^2 + b^2$. To use the formula, you have to know how to square numbers and how to find the square root.

In this manual, the following topics concerning powers and roots are covered:

- Powers
- Squaring a number, including
  - squares of numbers one to twelve
  - finding the square of a larger number
  - squaring a fraction or a decimal
  - squaring units
  - area
- Finding square roots, including
  - square roots of numbers with units
  - using a calculator

POWERS

*A **power** indicates the number of times that a number is to be multiplied by itself*. The operation 5 x 5 can be written as 5^2. This is read "five squared" or "five times five" or "five to the second power". 2 x 2 x 2 is two to the third power. It can be written 2^3.

5^2 is short-hand for indicating a power.

- The whole number, in this case 5, is called the ***base*** or the factor.
- The small number 2 written after the base and slightly above it is called the ***exponent***.
  - In the expression 2^3, 2 is the base and 3 is the exponent.

In your work, you will most often be multiplying a number to the second power. When a number is multiplied to the second power, we say it is squared. There will be times though when you need to multiply a number by itself five, or six or eight times.

When we multiply any number by itself we call it a ***power****.*

- The number we are multiplying is called the ***base,*** or factor.
- We use the exponent to tell us how many times to multiply a number by itself.

The exponent tells how many times to multiply the base by itself. The number 2^4, written with an exponent, tells us that the base 2 is to be multiplied by itself four times, like this: 2 x 2 x 2 x 2.

The base 8 with the exponent 3, written 8^3, tells us that eight is to be multiplied by itself three times, like this: $8^3 = 8 \times 8 \times 8$

Examples:

5^3 means multiply 5 by itself 3 times	we call it a power of 3	or five to the third power.
5^6 means multiply 5 by itself 6 times	we call it a power of 6	or five to the sixth power
5^{12} means multiply 5 by itself 12 times	we call it a power of 12	or five to the twelfth power

$3^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$ Multiply 3 times itself 6 times.
$= 729$ 3 to the sixth power is 729

$3^{10} = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
$= 30\ 000\ 000\ 000$

The exponent ^{0} is unusual. Any number with the exponent ^{0} is equal to 1.

$8^0 = 1$
$2^0 = 1$
$10^0 = 1$

SQUARING A NUMBER

Instructions asking you to square a number can be worded as *"find the square of a number"* or *"square a number"*. Both instructions ask you to carry out the same calculation. Squaring a number means to multiply that number by itself. The square of the number refers to the answer after you have multiplied it by itself.

The ***square of a number*** *is the number multiplied by itself.*

To find the square of 5, multiply 5 by itself.

5 x 5 = 25

You can also say 5 squared equals 25; it means 5 x 5 = 25.

The square of 10 is 100 (10 x 10).
The square of 6 is 36 (6 x 6)

When the exponent ^{2} follows a number, it indicates that the number is to be squared. Five squared can be written as 5^2. So

$5^2 = 5 \times 5 = 25$

$20^2 = 20 \times 20 = 400$

10 squared (10^2) is 10 x 10 = 100

The square of 9 (9^2) is 9 x 9 = 81

Squares of the Numbers One to Twelve

It is a good idea to memorize the squares of the numbers up to 12. You are probably familiar with most of them. Here is a list:

1 x 1 = 1	7 x 7 = 49
2 x 2 = 4	8 x 8 = 64
3 x 3 = 9	9 x 9 = 81
4 x 4 = 16	10 x 10 = 100
5 x 5 = 25	11 x 11 = 121
6 x 6 = 36	12 x 12 = 144

Go over the list to learn these square roots. Cover up the answers and say them to yourself, or write out the list and check your answers. Practice until they are memorized.

Finding the Square of a Larger Number

It is helpful to know and recognize the squares of the numbers to twelve. When working with larger numbers, though, you will have to multiply on paper or use a calculator to find their squares.

Example: Find the square of 25.

```
  25
x 25
 125
 500
 625
```

The square of 25 is 625.

Squaring a Fraction or Decimal

To square a fraction or decimal, do the same thing. Multiply the fraction or decimal by itself.

Example: Square 3/4

3/4 x 3/4 = 9/16

Example: Square 9.1

```
   9.1
 x 9.1
    91
  8190
 82.81
```

When the exponent ^{2} is used to indicate that you are to square a fraction or decimal, you write the number in brackets first.

3/4 squared is written $(3/4)^2$
9.1 squared is written $(9.1)^2$

The brackets indicate that the entire number is to be squared.

- You could also have $\frac{3^2}{4}$, which means only the top part of the fraction, the numerator, is to be squared.
- The fraction $\frac{3^2}{4}$ would become 9/4 when the 3 is squared.

Remember: *To do questions involving several mathematical operations including exponents, do the operations in order. The letters which make up the term BEDMAS will give you the correct order of operations.*

BEDMAS
1. **B**rackets → make calculations inside brackets
2. **E**xponents → calculate any numbers with exponents
3. **D**ivision → do any division and
4. **M**ultiplication in the order in which they appear from left to right
5. **A**ddition → do any addition and
6. **S**ubtraction in the order in which they appear from left to right

Squaring Units
When you square a quantity such as 6 meters, which has a measurement unit (meters) attached, the unit (meters) is also squared.

If you multiply 6 m x 6 m, you multiply 6 x 6 but you also multiply the meters times meters. This gives square meters or meters squared.

$6m \times 6m = 36m^2$

Square units can be indicated in more than one way.

- The exponent ^{2} can be written after the unit like this, m^2.
- Squared units can also be written with the abbreviation sq for square like this: *sq ft*.

Area
Units of area are always squared units.

Area is found by multiplying two measurements of length, with their units.

- To find the area of a rectangle, you multiply length times width.
- To find the area of a triangle, you multiply the base times the height times 1/2.
- In each case, the units of the measurements are multiplied by themselves or squared.

Note: *To find area, both measurements must have the same units before they can be multiplied. You can't find the area of a rectangle with a length of 138 centimeters and a width of .75 meters. One of the units must be changed so they are both the same.*

- You could choose to change the units of measure so they are both centimeters (.75m x 100 = 75 cm), then multiply (138 cm x 75cm = 10350 cm^2).
- Or, you could make them both meters (138cm ÷ 100 = 1.38m) then multiply (75m x 1.38m = 103.5m^2).

Example: Square 15 meters

15 m x 15 m
= 225 m^2

Example: Find the square of 100 miles.

100 mi x 100 mi
= 10 000 sq mi or mi^2

Example: Find the area of a rectangle with all its sides measuring 4 ft.

4 ft x 4 ft
= 16 sq ft

You probably recognized that a rectangle with all its sides the same length is known as a ***square*** (another use of that term).

Example: What is the area of a square with sides that measure 1 m?

1 m x 1 m
= 1 m^2

This example seems a little strange but 1 x 1 = 1. The difference in the answer is that the units are now squared.

Note: *Notice the difference between 9 sq meters and 9 meters square.*

- An area of 9 sq meters can have any shape as long as it has a total area of 9 m^2.
- An area 9 meters square has the shape of a square and its length and width are each 9 meters long. The area that is 9 meters square is 9 m x 9 m or 81 m^2.

Usually we have to find the area of a space that is not square. To do this we multiply the length and width to get the numerical answer. *We still have to square the units*.

Example: Find the area of a rectangle that is 3 inches by 20 inches.

3 in x 20 in square the units of measurement
= 60 sq in

Example: Find the area of a rectangle that has a length of 1.2 meters and a width of 70 centimeters.

You have to change one of the measurements so that it has the same unit as the other one. Change centimeters to meters.

70 cm = .7 m

Now multiply.

1.2 m x .7 m
= .84 m^2

ROOTS

Roots are the opposite of powers. *A **root** is the number which, when multiplied by itself a certain number of times will result in a specific answer.*

That means that the square root of a number, such as 144, is the number that is squared to get 144. The square root of 144 is 12. (12x12 = 144)

It means that the fourth root of a number, such as 256, would be the number you multiply by itself 4 times to get 256. The fourth root of 256 is 4. (4 x 4 x 4 x 4 = 256)

Most commonly when we use roots we use square root – the number multiplied by itself to give a known number. For example the square root of 4 is 2 (2 x 2 = 4).

We use the symbol to indicate that we are to find a root.

$\sqrt{\ }$ means square root. If there is no exponent, it is a square root.

$\sqrt[3]{\ }$ means cube root (or third root).

$\sqrt[10]{\ }$ means tenth root, and so on.

FINDING SQUARE ROOT

*The **square root** of a number is that number which when multiplied by itself produces the given number*.

This definition sounds rather wordy. It might be easier to understand the meaning of square root by looking at an example. The square root of 81 is 9 because 9 multiplied by itself equals 81.

The symbol $\sqrt{\ }$ indicates you are to find the square root. So $\sqrt{81} = 9$ is read as "the square root of 81 equals 9."

Finding the square root is the opposite of squaring a number. Squaring a number is multiplying that number by itself. To find the square root of a number, you consider the given number as a square. *You are looking for the number you multiplied by itself to get that square*.

To find the square root of 64:

1. First consider that some number has been squared (multiplied by itself) to get 64.
2. Now you have to find that number.
3. From the table on the first page, you know that 8 x 8 = 64, so the square root of 64 is 8.

It is important to know the squares of the numbers up to twelve so you can recognize the square roots. The square root of 144 is 12, the square root of 100 is 10 and the square root of 16 is 4. Here is the table giving these common square roots:

$\sqrt{1} = 1$	$\sqrt{49} = 7$
$\sqrt{4} = 2$	$\sqrt{64} = 8$
$\sqrt{9} = 3$	$\sqrt{81} = 9$
$\sqrt{16} = 4$	$\sqrt{100} = 10$
$\sqrt{25} = 5$	$\sqrt{121} = 11$
$\sqrt{36} = 6$	$\sqrt{144} = 12$

You will often have to use these square roots, so it is a good idea to memorize them.

Some square roots are not so perfect

A perfect square is a number that has a whole number which is its square root. You can see from the chart above that if you were asked for the square root of a number like 40 or 75, the answer would not be a whole number. In fact, you can guess that finding an exact square root of a number like that might be almost impossible.

Still, you might have to find the square root of a number when you don't have a clue what number was multiplied by itself to give that square. Here is a way to find a good approximation, to the nearest tenth, of the square root of a number smaller than 144.

Example: To find $\sqrt{75}$ follow these steps:

Step 1. Find a whole number whose square is close to but smaller than 75. In this case, $8^2 = 64$, so 8 is a first guess.

Step 2. Divide the guessed number into the original number. Carry the division to two decimal places.

$$8\overline{)75.00} = 9.37$$

Step 3. Take the average of the guessed number (8) and the answer to the division question (9.37). You find an average by adding all the numbers together and then dividing the addition answer by however many numbers you added together.

$$\frac{8 + 9.37}{2} = 8.685$$

Check your answer.

```
     8.685
     8.685
     43425
    694800
   5211000
  69480000
 75.429225        That is pretty close.
```

$\sqrt{75}$ is about 8.685

Many of these answers will go on to several decimal places. You can get a good enough approximation by working each answer to two decimal places or to the nearest hundredths. Then round off the final answer to one decimal place or the nearest tenth. The answer above, 8.685, when rounded off to the nearest tenth, becomes 8.7

To check your answer, square it to see if it is close to the square root. They should be almost the same number.

8.7 x 8.7 = 75.69

8.7 is a reasonable approximation of $\sqrt{75}$.

Square root of a number with units

If you are finding the square root of a number with units, the units will be squared units. When you find the square root, the units will no longer be squared.

Example: Find the square root of 121 m^2.

$\sqrt{121\text{m}^2} = 11\text{m}$

Using a Calculator

Finding the square root of a number larger than 144 requires a long procedure. It makes sense to find the square root of larger numbers by using a calculator.

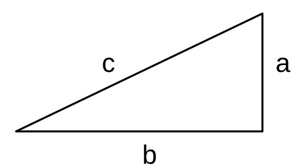
Key in the number you need to find the square root of, then press the square root symbol.

If you have to do problems that involve square roots without a calculator and the square root isn't one that is easily recognized, you can leave the answer under the square root symbol.

Example: Find the length of c in the right angle triangle below if *a* is 6 cm and *b* is 7 cm.

$a^2 + b^2 = c^2$
$6^2 + 7^2 = c^2$
$c^2 = 36 + 49$ (reverse the equation)
$c^2 = \underline{85}$

$c = \sqrt{85}\text{ cm}$

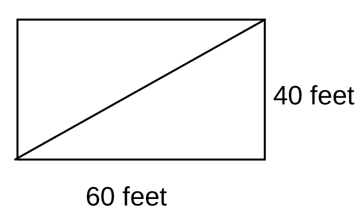


The answer can be left as $\sqrt{85}$ cm

However, if you are using this formula to check if a rectangular foundation is laid out so the corner angles are true 90° angles, you need to find the numerical value of the square root. Use a calculator to do this.

Example: If the length of the foundation is 60 feet and the width is 40 feet, what should the distance across the diagonal be if the corners are true 90° angles?

$c^2 = a^2 + b^2$
$c^2 = 60^2 + 40^2$
$c^2 = 3600 + 1600$
$c^2 = 5200$
$c = \sqrt{5200}$
c = 72.1 ft.



The distance across the diagonal should be 72.1 feet if the foundation is a true rectangle.

Answer the following questions on squares and square roots. Use a calculator if you wish. The answers are at the end of this manual.

1. Square each of the following numbers.

a) 4 b) 10 c) 12 ft

d) 30 m e) 425 f) .5

g) 6.2 h) 2/3 g i) 8 1/4 in

2. Find the value of the following.

a) 3^2 b) $(15\text{ cm})^2$ c) $(2/5)^2$ d) 1.6^2

3. Find the square root to the nearest tenth.

a) 64 b) 144 c) 100 km^2

d) 16 sq mi e) 31 f) 92

4. Find the value to the nearest tenth.

a) $\sqrt{9}$ b) $\sqrt{100}$ c) $\sqrt{49}$

d) $\sqrt{25\text{ sq m}}$ e) $\sqrt{4}$ f) $\sqrt{1\text{ sq ft}}$

g) $\sqrt{45}$ h) $\sqrt{56}$ i) $\sqrt{110}$

ANSWER PAGE

1. a) 16 b) 100 c) 144 sq ft d) 900 m^2

   e) 180 625 f) .25 g) 38.44 h) 4/9 g^2

   i) 64 1/16 sq in

2. a) 9 b) 225 cm^2 c) 4/25 d) 2.56

3. a) 8 b) 12 c) 10 km d) 4 mi

   e) 5.6 f) 9.6

4. a) 3 b) 10 c) 7 d) 5 m e) 2

   f) 1 ft g) 6.7 h) 7.5 i) 10.5