

**EVALUATING
ACADEMIC READINESS
FOR APPRENTICESHIP TRAINING**
Revised for
ACCESS TO APPRENTICESHIP

**MATHEMATICS SKILLS
RATIO AND PROPORTION**

**AN ACADEMIC SKILLS MANUAL
for
The Industrial Maintenance Mechanic Trades**

This trade group includes the following trades:

Boiler Makers,
Facilities Maintenance & Technicians, and
Industrial Mechanics (Millwrights)

*Workplace Support Services Branch
Ontario Ministry of Education and Training*

Revised 2011

In preparing these Academic Skills Manuals we have used passages, diagrams and questions similar to those an apprentice might find in a text, guide or trade manual.

This trade related material is not intended to instruct you in your trade. It is used only to demonstrate how understanding an academic skill will help you find and use the information you need.

MATHEMATICS SKILLS

RATIO AND PROPORTION

*An academic skill required for the study of the
Industrial Maintenance Mechanic Trades*

INTRODUCTION

Comparing Numbers

Numbers are compared in a variety of ways. One way to compare numbers is to note their difference.

Example: If one car sells for \$36,000 and another car sells for \$18,000, we say the first car costs \$18,000 more than the second. In this comparison, we subtract to find the difference.

We can also compare the cost of the two cars by division.

Example: If we divide the cost of the first car by the cost of the second ($\$36\,000 \div \$18\,000 = 2$), we find that the first car costs twice as much as the second.

We can compare by using a *ratio*. A ratio also compares numbers in a form that indicates division. Usually the numbers in a ratio are reduced to lowest terms but not actually divided.

Ratios give useful information about the relationship between numbers. Ratios can be used to describe such things as the relationship between teeth on gears and speed ratios on pulleys, chains and belts. They are also important in interpreting measuring tools and blueprints.

Example: If a length of PVC pipe weighs 10 kilograms and the same length of steel pipe weighs 50 kilograms, you say the PVC is 5 times lighter than steel. This comparison could help you to decide what pipe to use on an installation, especially if the installation has to be done in a tight space from a ladder.

We also use ratio to solve problems by proportion and to read blueprints and scale drawings.

This skill manual looks at the following topics concerning *ratio, proportion, and scale*:

- ◆ Ratio, including
 - finding ratios from given information
 - rates
- ◆ Proportions, including
 - solving a proportion when 3 out of 4 terms are known
 - direct and inverse proportions
- ◆ Scale

RATIO

Comparing two numbers by writing a ratio: Say one length of wire is 5 meters long and another is 10 meters. You want to compare the two lengths by writing them as a ratio.

There are several ways to indicate this ratio:

- ◆ **By comparing one amount to another**, as when we say 6 out of 10.
- ◆ **By putting a colon between the numbers.** The ratio is written 6 : 10. We read this as “*the ratio of six to ten*”.
- ◆ **By writing the ratio as a fraction.** The first number being compared becomes the numerator, which is placed over the second number, the denominator. The fraction is usually written in lowest terms. So 6 out 10 becomes $\frac{6}{10}$ and can be reduced to $\frac{3}{5}$.

When you write a ratio, you don't actually do the division unless you want one of the terms of the ratio to be 1.

Lowest terms: The ratio 3:4 is already in lowest terms. The ratio 8 to 32 is not in lowest terms. When this ratio is reduced to lowest terms, it is written as 1 to 4. A ratio, like a fraction, is usually, but not always, written in lowest terms.

To reduce a fraction or a ratio to lowest terms:

1. Look for a number (a common factor) that will divide evenly into the numerator and denominator of the fraction or the terms of the ratio.
2. Divide the common factor into the numerator and the denominator or into each term.
3. Continue dividing until there are no more common factors.
4. The last division answers form the fraction or ratio in lowest terms.

Example: The bend radius of a conduit can be expressed as a ratio. If the conduit has an internal diameter of 5.1 centimeters or less, the bend radius must be 6 times the internal conduit diameter.

If the conduit diameter is 3 centimeters, the bend must be 3 times 6 = 18 centimeters.

This gives a ratio of 3 to 18.
The common factor is 3.
Reduced to lowest terms, the ratio is 1 to 6.

If the conduit has a diameter of more than 5.1 cm, the bend radius must be 10 times the internal conduit diameter.

If the diameter is 6 centimeters, the bend must be 6 times 10 cm = 60 cm.
The common factor is 6.
Reduced to lowest terms, the ratio is 1:10.

Notice that there are no units in these ratios. Because we are comparing centimeters to centimeters, the units cancel out. When the numbers being compared have the same unit of measurement, there are no units in the ratio.

Ratios with 1: The ratio 2:1 has the number 1 as one of its terms. The ratio 3:4 does not. Sometimes a ratio like 3:4 is more useful if one of the terms is 1. You could divide both terms by 4 and then express the ratio as .75 to 1, or you could divide both terms by 3 and express the ratio as 1 to 1.33.

Equivalent ratios: Reducing a fraction to lowest terms does not change the value of the fraction, nor will it change the value of a ratio. The fractions $\frac{2}{8}$ and $\frac{4}{16}$ can each be reduced to $\frac{1}{4}$. $\frac{1}{4}$, $\frac{2}{8}$, and $\frac{4}{16}$ are *equivalent fractions*. They each represent the same amount.

In the same way, ratios representing the same amount are called *equivalent ratios*. The ratio 3 to 4 and the ratio .75 to 1 represent the same comparison and are equivalent ratios.

Finding a Ratio from Given Information

Before using ratios to solve problems, we will look at setting up ratios from given information.

Questions that ask you to set up ratios are generally worded in one of two ways.

1. You might need to compare part of an amount to the total amount; or
2. You might be asked to compare two parts to each other.

Situation one: You are asked to compare part of the amount to the total amount. If the total amount isn't given, you first have to find it.

Example: A class of apprentices consisted of 6 women and 24 men. What is the ratio of women to the whole class and the ratio of men to the whole class?

First you have to find the total number of students.

Adding $6 + 24$ gives a total of 30 apprentices in the class.

Now find the ratios:

- a) Ratio of women to the whole class is 6 out of 30, reduced to 1 out of 5, $\frac{1}{5}$ or 1:5.
- b) Ratio of men to the whole class is 24 out of 30, reduced to 4 out of 5, $\frac{4}{5}$ or 4:5.

Situation two: The question asks you to compare one amount to another. This time you don't need to know the total.

Example: Using the class of 6 women and 24 men, what is the ratio of women to men and men to women?

Ratio of women to men is 6 to 24, reduced to 1 to 4, $\frac{1}{4}$ or 1:4.

Ratio of men to women is 24 to 6, reduced to 4 to 1, $\frac{4}{1}$ (or 4:1).

Note: if the denominator is 1 when writing a ratio, you must show it)

General Rules For Reading And Writing Ratios

Rule 1: *When you read or write ratios, compare the terms in the same order as they are given, unless they are part of a table or formula.*

To compare the number of women to the class total, the number of women is stated before the class total.

Ratio of women to class = 6:30
This is reduced to 1:5.

To compare the number of men to women, the number of men is written before the number of women.

Ratio of men to women = 24:6
This is reduced to 4:1.

Rule 2: *If the units of each term in the ratio are the same, they cancel each other out and are not written in the ratio.*

The ratio of 25 centimeters to 1 meter is not 25:1. The ratio has to be written as 25 cm to 1 m or 25cm:1m.

Usually it is easier to work with ratios if there are no units, so make the units the same. If you convert 1 meter to 100 centimeters, the units will be the same. You can then cancel them out. The ratio is then written as 25:100 without any units.

If you can't write the ratio with the same unit for all terms, the units must remain in the ratio.

Example: The compression ratio of an engine is found by comparing the cylinder volume when the piston is at the bottom of the cylinder to when the piston is at the top of the cylinder. What is the compression ratio if the volume at the bottom of the cylinder is 126 cubic inches and the volume at the top is 9 cubic inches?

Divide 129 cubic inches by 9 cubic inches to get 14.
The units, cubic inches, are the same, so they cancel.
The compression ratio is 14:1.

Rule 3: *Ratios without units are usually expressed in lowest terms.*

Example: Find the gear ratio of two meshed gears if gear A has 88 teeth and gear B has 32 teeth.

Divide 88 teeth by 32 teeth to get 2.75.
The units cancel.
The division answer is the first term; the second term is 1.
The gear ratio is 2.75:1.

Ratios can be used to describe such things as the relationship between teeth on gears and speed ratios on pulleys, chains and belts.

Example: Find the ratio of two meshed gears if gear A has 96 teeth and gear B has 48 teeth.

Ratio of gears is 96 : 48. Reduced to lowest terms, the ratio is 2 : 1.

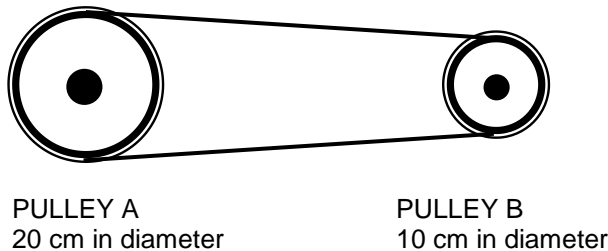
Example: The speed ratio of a roller-chain drive is found by dividing the number of teeth on the driven sprocket by the number of teeth on the drive sprocket.

$$\text{Speed ratio} = \frac{\text{number of teeth on driven sprocket}}{\text{number of teeth on drive sprocket}}$$

What is the speed ratio of a chain drive if the driven sprocket has 75 teeth and the drive sprocket has 25 teeth?

The units, teeth, are the same.
The ratio is 75 : 25.
Reduced to lowest terms the speed ratio is 3 : 1.

Example: What is the speed ratio of pulley A to pulley B in Figure 1?



PULLEY A
20 cm in diameter

PULLEY B
10 cm in diameter

FIGURE 1: Finding The Speed Ratio Of Two Pulleys

Look at Figure 1.

- Pulley B (20 cm) is twice as large as pulley A (10 cm).
- In one turn of pulley A, the belt will move a distance equal to the circumference of pulley A.
- But one turn of pulley A will cause pulley B to make only one half a turn because it is twice as large.
- Therefore, pulley A is turning twice as fast as pulley B.
- The speed ratio of A to B is 2:1.

In meshed gears, the speed ratio of the driven gear to the driving gear is found by dividing the speed of the driven gear in rpm by the speed of the driving gear.

Example: What is the speed ratio if the speed of the driven gear is 60 rpm and the speed of the driving gear is 160 rpm?

Divide 60 by 160 to get .375.
The speed ratio is .375:1.

Example: Certain ratings on compressors are expressed as ratios. The displacement of a compressor is the theoretical amount of air in cubic feet that the compressor can pump in one minute (cfm). The free air rating is the actual amount of free air in cubic feet that the compressor can pump in one minute at working pressure.

The compressor's volumetric efficiency is the ratio of free air delivery to the displacement rating. This ratio is also sometimes expressed as a percent.

If a compressor has a free air delivery of 7 cfm and a displacement rating of 10 cfm, the volumetric efficiency is

$$7 : 10$$

Expressed as a percent, the volumetric efficiency is:

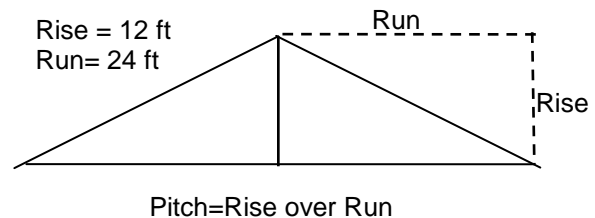
$$7/10 \times 100 = 70 \%$$

Ratios used to express slopes: The slope of a roof is expressed as the ratio of the rise to the span or run.

Example: What is the slope of a roof that has a rise of 12 ft and a run of 24 ft?

The units of both measurements (feet) are the same, so they cancel each other out. The ratio or slope is "rise over run" or 12 : 24. Reduce the ratio to lowest terms. Twelve divides evenly into both 12 and 24, giving the answers 1 and 2.

Slope equals 1 : 2



Example: What is the slope of a roof with a rise of 20 ft and a run of 16 ft 8 in?

The units must be changed so they are the same. The easiest way is to change all the units to inches.

$$20 \text{ ft} = (20 \times 12)\text{in} = 240 \text{ in.}$$

$$16 \text{ ft } 8 \text{ in.} = (16 \times 12)\text{in} + 8 \text{ in.} = 200 \text{ in}$$

Now make the slope ratio: 240 : 200

Reduce to lowest terms by dividing the common factor 40 into 240 and into 200. The answers give the reduced ratio: 6 : 5

The slope is 6 : 5.

Using ratio to make mixtures: If you are given a ratio, you can use it to find out how much of each part is needed to produce a certain amount.

Example: You need to mix paint to fill a 14 liter pail. The paint is mixed using 4 parts base, 2 parts solvent and 1 part colour. The ratio is 4 : 2 : 1.

First you need to find the total number of parts.

$$4 + 2 + 1 = 7$$

Make each part into a fraction of the total parts.

base = $\frac{4}{7}$ of total
solvent = $\frac{2}{7}$ of total
colour = $\frac{1}{7}$ of total

Multiply the fraction that represents each part by the total amount needed.

Amount of base = $\frac{4}{7} \times 14 \text{ L} = 8 \text{ L}$
Amount of solvent = $\frac{2}{7} \times 14 \text{ L} = 4 \text{ L}$
Amount of colour = $\frac{1}{7} \times 14 \text{ L} = 2 \text{ L}$

To make 14 liters of paint, you need to mix 8 L of base, 4 L of solvent and 2 L of colour.

Rates

When the units of the quantities in a ratio are the same, they cancel out and so are not shown.

When the units in a ratio are different, or there is only one unit, the units must be included in the ratio.

Ratios can be used to compare quantities of different types, such as kilometers per hour or cost per kilowatt-hour. These comparisons are called **rates**.

*A **rate** is a quantity or amount of something measured per unit of something else. A rate includes the word “per” which is indicated by the fraction line.*

Usually a ratio is divided so the amount of the unit following the word “per” is 1. If a rate involves two different kinds of units, they must be included in the ratio.

Example: Driving speed is a rate. Say you drove 300 km in 3 hours. The ratio 300 km/3 hr is reduced to 100 km/1 hr or 100 km/hr. Your rate of speed is 100 km per hr.

Rates that involve a cost per unit, such as the rate you pay for electricity, include the dollar sign. Your electrical bill might say that your electrical rate is \$.30/kw-hr. For every kilowatt-hour of electricity you use, you pay \$.30.

Rates can be complex.

Example: A pump is rated by its capacity and its speed at a given height of pressure. For example, a pump might be rated at 500 L/min at 800 rpm and 30 m.

Answer the following questions on ratios. Answers are on the last page.

Express the ratios using a colon between the two quantities and reduce to lowest terms. If the quantities being compared are not in the same units of measurement, convert where possible. Check your answers as you proceed.

- Write as ratios using the colon form. Don't forget to reduce.
 - 1 to 4
 - 5 to 8
 - 6 m to 3 m
 - 20 in to 45 in
 - 15 L to 9 L
 - 3 m to 90 cm
 - 5 in. to 1 ft
 - 2 kg to 125 g
 - 1 m to 50 cm
 - 15 min to 1 hr
 - 5 ft to 6 ft 6 in.
 - one nickel to a quarter
- Find the ratio of two meshed gears if one gear has 72 teeth and the other gear has 36 teeth.
- What is the rate of speed if you travel 400 km in 4 hr?
- What is the cost of gas per liter if you pay \$5.90 for 10 L?
- What is the slope of a roof that has a rise of 8 meters and a run of 4 meters?
- What is the speed ratio of a chain drive if the driven sprocket has 60 teeth and the drive sprocket has 20 teeth?
- If a concrete mix uses cement, sand and gravel in a ratio of 1 : 3 : 4, how much of each is needed to make 8000 lb of concrete?

PROPORTIONS

Two equivalent ratios express the same relationship but are written using different but related terms or numbers. For example, $1/4$ and $2/8$ are equivalent ratios and they represent the same amount. We can say that $1/4$ equals $2/8$. We can write this statement as:

$$1/4 = 2/8$$

*Equivalent ratios written in fraction form with an equal sign between them form a **proportion**.*

- A proportion has four *terms*, or parts.
 - The terms of the proportion above are 1, 4, 2 and 8.
 - When we read the proportion, we name all four terms. $1/4 = 2/8$ is read as “1 to 4 equals 2 to 8.”

Direct and Indirect (or Inverse) Proportions

There are two basic types of proportions: direct proportions and indirect (sometimes called inverse) proportions.

*In a **direct proportion**, as one quantity increases, the corresponding quantity also increases. Similarly, as one quantity decreases, the other one also decreases.*

Example: The relationship between the size of drill bit you choose and the size of hole you drill is a direct proportion.

- The larger the bit, the larger the hole.
- The smaller the bit, the smaller the hole.

This is a direct proportion because as the bit changes in size, the hole changes in size *in the same way*.

*In an **inverse, or indirect, proportion**, as one quantity increases, the corresponding quantity decreases. As one quantity decreases, the other one increases.*

Example: The relationship between the number of teeth on a gear and the speed of the gear is an indirect proportion. The number of teeth in a gear determines the amount of torque or turning force.

- But the more teeth on a gear, the less speed there is available.
- As one quantity (the number of teeth or the torque) increases, the other quantity (speed) decreases.
- You cannot have an increase in both torque and speed in one gear.
- Torque is inversely proportional to speed.

Solving a Proportion When Three of Four Terms Are Known

Proportions such as $1/4 = 2/8$ in the example above don't tell us much. We already know that two ratios or fractions that represent the same amount equal each other. However, if we only know three of the four terms, a proportion can be used to find the fourth term.

The following are the general steps of finding an unknown amount using a proportion. Here are the steps to find the fourth, unknown term:

Here are the steps to find the fourth, unknown term:

- 1. Set up a proportion using a letter to represent the unknown amount** in one of the ratios. The letter can be manipulated (moved around) in an equation just like a number. Write the ratios with an equal sign between them, forming an equation.

Example: Write the proportion using the two ratios n:10 and 8:20.

$$\frac{n}{10} = \frac{8}{20}$$

- 2. Cross-multiply to get rid of the denominators on both sides.** To cross-multiply, multiply the diagonal numbers across the equal sign. In other words, multiply the numerator of one ratio by the denominator of the other ratio.

If an unknown term is represented by a letter, cross multiply in the same way.

Example: Cross-multiply in the equation below to get rid of the denominators.

$$\frac{n}{10} = \frac{8}{20}$$

Notice that n represents the unknown term.

Multiply n by 20 and 10 by 8. Keep the equal sign.

$$20n = 10(8)$$

$$20n = 80$$

- 3. Isolate the unknown term** (get it alone on one side of the equal sign). To do this divide both sides by the number in front of the unknown term.

Example: Isolate n in the following equation.

$$20n = 80$$

$$\frac{20n}{20} = \frac{80}{20}$$

Divide both sides by 20.

$$n = 4$$

Here are some other manipulations that can help isolate the letter representing the unknown term.

- A. If the letter representing the unknown term is on the right side, reverse the equation before dividing. You can reverse an equation without changing its value.

Example: You can reverse:

$$3(15) = 5n \quad \text{to} \quad 5n = 3(15).$$

Both equations have the same value.

- B. You can invert (turn all of the terms upside down) both sides of the equation without changing its value.

Example: You can invert

$$4/s = 5/6 \quad \text{to} \quad s/4 = 6/5.$$

Both equations have the same value.

Note: If you invert one side of an equation, you must invert the other side to keep the equation equal.

Now let's look at some examples of finding an unknown term in a proportion using these steps.

Example: Solve for n in the following proportion.

$$\frac{4}{5} = \frac{n}{15} \quad \text{Set up the proportion}$$

$$4(15) = 5n \quad \text{cross-multiply}$$

$$60 = 5n$$

$$5n = 60 \quad \text{Reverse the equation so that n is on the left side of the equal sign.}$$

$$5n \div 5 = 60 \div 5 \quad \text{Divide both sides of the equation by the number in front of the unknown term.}$$

$$n = 12 \quad \text{The letter is isolated on the left hand side of the equation.} \\ \text{The answer is on the right hand side}$$

Substitute 12 for n to write the complete proportion.

$$\frac{4}{5} = \frac{12}{15}$$

Example: Find the value of n when:

$$\frac{n}{12} = \frac{5}{15}$$

$$15n = 5(12) \quad \text{cross multiply}$$

$$5n = 60 \quad \text{divide by 15 to isolate n}$$

$$\frac{15n}{15} = \frac{60}{15}$$

$$n = 4$$

$$4/12 = 5/15 \quad \text{Substitute 4 for n to write the complete proportion.}$$

Example: Find the value of n when:

$$\frac{n}{8} = \frac{10}{16}$$

$$16n = 10(80) \quad \text{cross multiply}$$

$$16n = 80 \quad \text{divide both sides by 16}$$

$$n = 5$$

Example: Find the value of s.

$$\frac{3}{4} = \frac{9}{s}$$

$$3s = 9(4) \quad \text{cross multiply}$$

$$3s = 36$$

$$3s \div 3 = 36 \div 3 \quad \text{divide by 3}$$

$$s = 12$$

Solving Problems Using Proportions

Proportions can be used to solve problems. You have to figure out what goes with what and then set up your proportion to find the unknown quantity. Notice that when you first set up your ratios, you do not usually reduce to lowest terms.

Method 1: These suggestions are one method to set up a proportion.

- a) Set up the ratios (or fractions) so the same units are over each other.
 - a. Set up minutes over minutes, kilometers over kilometers, or meters over meters.
- b) The units of the two given quantities that form one fraction will cancel out.
 - a. The unit of the third known quantity will be the unit of the unknown quantity
- c) Set up the smaller unit over the larger unit. The proportion will look like this:

$$\frac{\text{small}}{\text{large}} = \frac{\text{small}}{\text{large}}$$

Example: If it takes 4 hours to make 6 elbows, how many will can be made in an 8 hour day?

Set up your proportion.

- Put number of hours it takes over the number of hours in the day.
- Let s equal the unknown number of elbows.

The proportion looks like this:

$$\frac{4 \text{ hours}}{8 \text{ hours}} = \frac{6 \text{ elbows}}{s} \qquad \frac{\text{small}}{\text{large}} = \frac{\text{small}}{\text{large}}$$

This looks like the proportions we already know how to solve. Find the answer by solving for s:

$$\frac{4 \text{ hours}}{8 \text{ hours}} = \frac{6 \text{ elbows}}{s} \quad \text{hours cancel out}$$

$$4s = 6 \times 8$$

$$4s = 48 \qquad \text{cross-multiply}$$

$$s = 12 \qquad \text{divide both sides by 4}$$

He will make 12 elbows in an 8 hour day.

Method 2: You can also set up the two ratios so each is given as a rate. When the ratios are set up as rates, in each ratio, one unit is over the other, different, unit. The following example shows how to set up the proportion.

Example: You travel 25 km in 50 minutes. How long will it take to travel 75 km at that speed?

The first ratio or rate is 50 min/25 km.
The second ratio is *unknown minutes*/75 km.
Set up the proportion by writing the two ratios.
Let m represent the unknown time.

$$\frac{50 \text{ min}}{25 \text{ km}} = \frac{m}{75 \text{ km}} \quad \begin{array}{l} \text{km cancel} \\ \text{you can leave out the other unit, minutes, until the end} \end{array}$$

$$50(75) = 25m \quad \text{cross-multiply}$$

$$\begin{array}{l} 25m = 3750 \text{ min} \quad \text{reverse the equation} \\ m = 150 \text{ min} \quad \text{divide both sides by 25 and put in the unit min} \end{array}$$

It will take 150 minutes to travel 75 km.

Example: If an apprentice takes 4 hours to weld 3 panels, how many similar panels can he weld in an 8 hour day?

We will use the second method, although either method will get the same answer.
The first ratio is 4 hours/3 panels.
The second ratio is 8 hours/unknown number of panels.
Let t represent the unknown number of panels.
Set up the proportion.

$$\frac{4 \text{ hrs}}{3} = \frac{8 \text{ hrs}}{t} \quad \text{cross multiply}$$

$$4t \text{ hrs} = 24 \text{ hrs} \quad \text{divide both sides by 4 hrs}$$

$$t = 8 \text{ panels} \quad \text{put in the unit houses}$$

An apprentice can weld 8 panels in an 8 hour day.

Here are some questions on proportions. Answers are on the last page.

8. Solve for the unknown quantity.

a) $\frac{n}{24} = \frac{1}{2}$

b) $\frac{2}{x} = \frac{10}{10}$

c) $\frac{16}{2} = \frac{s}{3}$

d) $\frac{5}{10} = \frac{12}{n}$

e) $\frac{n}{7} = \frac{3}{21}$

f) $\frac{2}{6} = \frac{n}{7.35}$

9. If it takes 70 minutes to travel 35 km, how long will it take to travel 85 km at the same speed?
10. A gear with a diameter of 20 cm is turning at 400 r.p.m. Find the r.p.m. of a gear with a diameter of 80 cm that is turning with it. (This is an inverse proportion.)
11. If an engine requires a 1 : 20 oil to gas mixture, how much oil has to be added to 70 L of gas?
12. If a quart of paint covers 12 sq ft, how much paint is needed to cover 72 sq ft?
13. Six carpenters can build a garage in 3 days. How long would it take for three carpenters to build the garage?
14. Bolts costs \$32.50 for 100. How much would 360 bolts cost?

SCALE

Using what we know about ratio and proportion, we can draw the things we use in life so that the drawings are to *scale*, or proportional to the real items, in all respects. We use scale to draw and read maps. We also use scale to draw and read drawings of things we are going to build, manufacture, replace, or repair.

In your trades, you will have to know how to read blueprints or scaled drawings. A blueprint is a drawing of an object or part of an object where the lines and symbols are drawn as scaled down representations of the actual dimensions. It would be impossible to manage the drawing of a large object if the actual dimensions were used.

Before you make an object, you have to first read the drawings to find out what parts you need to cut out, what sizes they are and how they will fit together.

The *scale* of a drawing expresses the relationship between the dimensions of the drawing and the actual dimensions of the object it represents. *The scale is the ratio of the drawing size to the actual size of the object.* An equal sign (=) is used instead of a colon when indicating a scale.

A scale uses a smaller unit of measure to represent an actual, larger unit of measure. A scale can be considered as similar to a rate, where the distance on the drawing is compared to a standard unit.

Example: A $\frac{1}{4}$ inch line on a drawing can represent an actual length of 1 foot. The scale of the drawing is $\frac{1}{4} \text{ in} = 1 \text{ ft}$. In this case, a 5 inch pipe on the drawing represents a pipe that is actually 20 feet long.

The word scale is used to indicate the relationship between the drawing and the actual dimensions, as described above.

- ◆ In equations used to convert drawing dimensions to actual dimensions, scale indicates the number with the first, smaller unit.
- ◆ The unit of the second number is called the *standard unit*.
- ◆ So in the example above, the scale is $\frac{1}{4}$.

To convert a length on a drawing to an actual measurement, follow these steps:

1. Divide the length on the diagram by the scale, the first number listed.
2. Use the unit of the standard unit (the larger unit) in the answer.

Example: If the length on a blueprint of an object is $2\frac{1}{2}$ inches and the scale is $1/4$ inch = 1 feet, what is the actual length of the object?

$$\text{length on diagram} \div \text{scale} = \text{actual length}$$

The scale is $1/4$. The standard unit is feet.

$$2\frac{1}{2} \div \frac{1}{4} = 10 \text{ ft} \quad \text{use the standard unit ft}$$

$2\frac{1}{2}$ inches represent 10 feet.

Example: A blueprint is drawn to the scale of $1/4$ inch = 1 inch. If the dimensions of an object are drawn as 6 inches by $7\frac{1}{2}$ inches, what are the actual dimensions?

$$\text{length on diagram} \div \text{scale} = \text{actual length}$$

The scale is $1/4$. The standard unit is inches.

$$6 \div \frac{1}{4} = 24 \text{ inches}$$

$$7\frac{1}{2} \div \frac{1}{4} = 30 \text{ inches}$$

The actual dimensions are 24" by 30".

In metric dimensions, the scale is usually a decimal or whole number, not a fraction.

Example: Find the actual length of a pipe if it is 4 centimeters long on a diagram. The scale is 1 centimeter = 1 meter.

$$\text{length on diagram} \div \text{scale} = \text{actual length}$$

The scale is 1. The standard unit is meters.

$$4 \div 1 = 4 \text{ m}$$

If you know a distance on a diagram and the actual distance, you can find the scale by following these steps:

1. Divide the scale length by the actual length.
2. If the scale is imperial, write the answer as a fraction. If the scale is metric, write it as a decimal.

Example: Find the scale of a blueprint if 10 centimeters on the diagram represents 20 meters.

$$\text{scale distance} \div \text{actual distance}$$

$$= 10 \text{ m} \div 20$$

$$= .5$$

The scale is .5. To express this as a ratio, put the unit of the blueprint length with the scale (cm). The standard unit is 1 followed by the unit of the actual object. The two units are separated by an equal sign.

$$.5 \text{ cm} = 1 \text{ m}$$

Answer the following questions about scale. Answers are on the last page.

15. If the scale of a blueprint is 1 in = 5 ft, what is the actual length of a wall that is 3 inches on the diagram?

16. What are the actual outside dimensions of a house that measures 20 in. by 24 in. on a blueprint with a scale of 1 in = 4 ft?

17. If 2 inches on a blueprint of a house represents 6 feet on the actual house, what is the scale?

19. What is the actual length and width of a building if it is shown as 4 in by 3 in on the blueprints and the scale is $\frac{1}{2}$ in = 4 ft.

20. A blueprint has a scale of $\frac{5}{8}$ in = 2 ft. What are the actual dimensions of a room that measures $5 \frac{1}{4}$ in by $6 \frac{3}{4}$ in on the blueprint?

ANSWERS

RATIOS

1.
 - a) 1 : 4
 - b) 5 : 8
 - c) 2 : 1
 - d) 4 : 9
 - e) 5 : 3
 - f) 10 : 3 (change m to cm)
 - g) 5 : 12 (change 1 ft to 12 in)
 - h) 16 : 1 (change kg to g)
 - i) 2 : 1 (change m to cm)
 - j) 1 : 4 (change hr to min)
 - k) 10 : 13 (change 5' to 60" and 6 ' 6" to 78")
 - l) 1 : 5
2. 2 : 1
3. 100 km/hr
4. \$.59/L
5. 2 : 1
6. 3 : 1
7. total parts = 8
 - 1/8 x 8000 = 1000 lb cement
 - 3/8 x 8000 = 3000 lb sand
 - 4/8 x 8000 = 4000 lb gravel

PROPORTIONS

8.
 - a) 12
 - b) 8
 - c) 24
 - d) 24
 - e) 1
 - f) 27/11
9. 170 min
10. 100 rpm
11. 3.5 L Check the example on page 10, mixing solvent and paint in a ratio of 1 :4.
12. 6 qt
13. 6 days
14. \$117.00

SCALE

15. 15 ft
16. 80 ft by 96 ft
17. 1 in = 3 ft
18. length is 32 ft
width is 24 ft
19. 16 4/5 ft by 21 3/5 ft