EVALUATING
ACADEMIC READINESS
FOR APPRENTICESHIP TRAINING
Revised for
ACCESS TO APPRENTICESHIP

MATHEMATICS SKILLS
MANIPULATION OF VARIABLES

AN ACADEMIC SKILLS MANUAL
for
The Industrial Maintenance Mechanic Trades

This trade group includes the following trades:
Boiler Maker,
Facilities Maintenance Mechanic & Technician, and
Industrial Mechanic (Millwright)

Workplace Support Services Branch
Ontario Ministry of Training, Colleges and Universities

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In preparing these Academic Skills Manuals we have used passages, diagrams and questions similar to those an apprentice might find in a text, guide or trade manual.

This trade related material is not intended to instruct you in your trade. It is used only to demonstrate how understanding an academic skill will help you find and use the information you need.

MATHEMATICS SKILLS
MANIPULATION OF VARIABLES

An academic skill required for the study of the Industrial Maintenance Mechanic Trades

INTRODUCTION

When learning concepts in math and science, you may get a problem that presents a situation, gives some numerical values and asks you to find an answer. You are not told whether to add, subtract, multiply or divide. Instead, you have to decide what operations to do and in what order.

Example: You are asked to find the resistance of a circuit using Ohm's Law.

This problem requires the use of a formula which uses letters, or ***variables*** *to represent amounts.* To find the answer to a problem like this, you have to know the different steps in problem solving and how to ***manipulate*** *(move around)* the parts of the formula.

This skills manual looks at the steps involved in problem solving, including problems that require the use of a formula. It covers the following topics:

- ♦ Basic steps in problem solving
- ♦ Deciding which operation will solve a problem
- ♦ Problems which involve units of measurement
- ♦ Solving problems using variables, including
  - finding an unknown amount using a scientific formula
- ♦ Manipulating variables in a formula
- ♦ Solving problems when the formula is not given

BASIC STEPS IN PROBLEM SOLVING

Some problems are simple; they require a basic mathematical operation like adding or subtracting in order to solve them. Other more complex problems involve the use of formulas. Although each problem is different, some general steps can help you find the right solution:

1. **Read** through the whole problem carefully.
2. **List** the facts and figures that are given in the question.
3. **Decide what needs to be found** or calculated, rereading the question if necessary.
4. **Decide what methods can be used** to find the answer. Also note what order the steps should be done in.
5. **Use the given information to do the calculations** using the steps decided on. Be aware that some information in the question is not needed to solve the problem.

6. **Write your answer** including any units or dollar signs.
7. **Check to see that your answer seems reasonable** and that it provides the answer (the unknown quantity) to the problem.

Example: A water tank has a volume of 6.34 cubic feet. 1 cu ft holds 7.48 gal of water. How many gallons does the tank hold?

Step 1. *Read the problem.*

Step 2. *List what information is provided.*

The tank contains 6.34 cu ft of water
1 cu ft holds 7.48 gal.

Step 3. *Decide what needs to be found.*

6.34 cu ft of water needs to be converted to gallons; the conversion factor is 7.48.

Step 4. *Decide what method to use.*

The numbers given must be multiplied.

Step 5. *Do the operations necessary to solve the problem*.

6.34 cu ft x 7.48 = 47.42.

Step 6. *Write the answer including units.*

6.34 cu ft equals 47.42 gallons.

Step 7. This seems like a ***reasonable*** answer.

DECIDING HOW TO SOLVE A PROBLEM

You need to be sure about what the question is asking you to find out (Step 3) *and* which operation to use in solving the math problem (Step 4). When you are given a problem, look for information that will help you make your decisions. After reading the problem (Step 1) and listing the facts and figures (Step 2), it's time to do some detective work. Identify the clues that tell you what to do.

Decide What the Needs to be Found (Step 3)

Remember, each problem gives you the information you need to find a solution. Make sure you are clear about what the question wants you to answer. Once you recognize which words in the question tell you what to do, solving the problem becomes a step by step process. You need to figure out what formula to use and where to find it. Check for clues.

Identifying the Clues
The clues are usually given by words or phrases like these:

- *how much ...*
- *how many ...*
- *find the ...*
- *what is the ...*
- *calculate ...*

Whatever follows those words is what you need to find out. The question could be worded:

How many holes do you need to drill if ...?
or
Calculate the number of hours it takes to ...

***Decide What Method to Use to Find the Answer* (Step 4)**
Once you are sure what the question is asking, the *next step is to discover how to find the answer*. If you are using a formula to find the answer, the formula will indicate what operations to use. If it is not obvious what math operations to use, ask yourself: "Do I add, subtract, multiply, or divide?"

Remember, the clues as to what operations to use are in the question. Go through the question again, this time to find directions on what operations to use.

Recognizing the right math operations
The following phrases help to indicate which operation to use:

Use addition when the question uses words like the following:

- *find the sum,*
- *more than,*
- *increased by,*
- *what is the total,*
- *find all.*

Use multiplication with words like these:

- *if each .... find all,*
- *so many hours at .... so much per hour,*
- *find the total,*
- *find all.*

Note: Since "what is the total", "find the total" and "find all" can indicate either addition or multiplication, you will have to decide based on the other facts in the problem.

Use subtraction with words like:

- *the difference between,*
- *how much more, or larger, or greater,*
- *how much less,*
- *how many fewer,*
- *take away,*
- *subtract,*
- *decreased by.*

Use division with words like:

- *how much is each if ...*
- *find the cost or rate per ...*
- *divided by,*
- *divided into.*

Do the Calculations
(**Step 5**)

This step is straightforward. Just be careful to get the correct answer. If you are using a calculator, do the calculations twice to make sure you didn't make an error when punching in the numbers.

Finish the Problem
(Steps 6 and 7)

Finish the problem by writing out your answer, usually in sentence form, with any needed units or dollar signs **(Step 6)**. Then look at your answer once more to make sure it is reasonable and it answers the question asked **(Step 7)**.

Here are some examples. We use all seven steps in the examples, but when you become familiar with the process, you will make many of the decisions in your head.

Example: A facilities mechanic earns $25.85 per hour for regular hours and $36.35 for overtime. If he works 40 hours at the regular rate and then works 8 hours of overtime in one week, how much does he make in total?

Step 1 Read the question carefully.

Step 2 List facts and figures given.

Rates of pay and hours worked:
$25.85 per hour for 40 hours
$36.35 per hour for 8 overtime hours

Step 3 Decide what the question wants you to find.

The words "How much does he make..." tell you to figure out what he should be paid.

Step 4 Decide what method to use.
The words “per hour” tell you to multiply; the words “and” along with “in total” tell you to add. Multiply to find the two amounts and then add them to find the total amount earned.

Step 5 Do the calculations.

$25.85 x 40 = $1034.00 find the wages
$36.35 x 8 = $290.80 find the overtime wages

$1034.00
+ $290.80 add them together
$1324.80

Step 6 Write your answer.

The facilities mechanic makes a total of $1324.80

Step 7 Does that seem about right? It does.

Note: *If the problem deals with money, the answer should show two decimal places (if there are cents) and the $ sign should be included.*

In the following examples, the separate steps used to solve problems aren’t listed since most of the steps involve making decisions in your head, but the steps will all be used.

Example: A 170 centimeter steel bar needs to be attached by clips every 10 centimeters. The apprentice installing the bar has 20 clips. How many clips will be left when the job is done?

The steps to solve this problem are not clearly stated. First you must find how many clips are used to attach the bar. Then, subtract this number from 20 to find out how many clips are left.

170 ÷ 10 = 17 clips
20 − 17 = 3 clips left over

Example: Kevin bought a truck worth $18,900. He made a down payment of $4,300 and he will pay the rest in 50 monthly installments of $320 each. How much interest will he be paying?

You probably need to think for a moment about what the question is asking. To find the amount of interest paid, you have to find the difference between the selling price of the truck and the total amount paid in down payments and installments.

Amount paid in installments.
$320 x 50 = $16,000

Total amount paid by Kevin.
$16,000 + $4,300 = $20,300

Difference between selling price and amount paid. $20,300 - $18,900 = $1400

The amount of interest paid is $1400.

PROBLEMS INVOLVING UNITS OF MEASUREMENT

Many of the previous examples involved quantities with units of measurement such as meters, gallons or dollars. When quantities in a question have units, they must be included in the calculations and the answer. Here are the rules for working with numbers that have units.

Rule 1. ***Units*** of similar measure, such as length or weight, ***must be the same*** before they can be added, subtracted, multiplied, or divided. If two units of measure are not the same, one unit must be converted so it is the same as the other.

Example: To find the perimeter (distance around) a rectangle, add together 2 times its length and 2 times its width.

perimeter = 2 *lengths* + 2 *widths*

If the units for length are in centimeters and the units for width are in meters, you can't add them until they are in the same units. The units for length can be converted to meters or those for width can be converted to centimeters so that all the units are the same.

Example: What is the perimeter in centimeters of a rectangle .15 m long and 10 cm wide?

perimeter = 2 *lengths* + 2 *widths*

The measurements given are in different units. Change the meters to centimeters before using the measurements in the formula.
.15 m = 15 cm

Now substitute the given numbers for length and width.
perimeter
= (2 x15 cm) + (2 x 10 cm)
= 30 cm + 20 cm
= 50 cm

Your answer could be in meters. Change centimeters to meters before using the formula. The answer will be expressed in meters instead of centimeters, but it will still be correct.

Rule 2. If two measurements with the same units are multiplied together, the units become squared units. If three measurements with the same units are multiplied together, the units become cubed units.

Example: Find the area of a rectangle if the length is 5 cm and the width is 2 cm.

To find the area of a rectangle, you multiply the length by the width. The formula for area is:

Area = length x width

Substitute the given values in the formula.
Area = 5 cm x 2 cm
= 10 cm^2 (or sq cm)

Example: Find the volume of a solid figure that is 5 cm long, 2 cm wide, and 4 cm high.

Volume equals length times width times height. The formula is:

Volume = length x width x height

Substitute the given values in the formula.
Volume = 5 cm x 2 cm x 4 cm
Volume = 40 cm^3 (or cu cm or cc)

Rule 3. If the same unit appears above and below a fraction line, they cancel each other out, eliminating the unit. But if there is only one unit, it remains in the answer.

Note: *The fraction line is a way of indicating division. 10/5 is the same as 10 ÷ 5. Division in a formula is usually shown as a fraction.*

Example:

$$\frac{45\ \cancel{\text{ml}}}{15\ \cancel{\text{ml}}} = 3 \quad \text{the units cancel}$$

Example:

$$\frac{80\ \text{g}}{40} = 2\ \text{g} \quad \text{the unit remains}$$

Example: If 1 can of paint covers 10 sq ft, how many cans are needed to cover 50 sq ft?

$$\text{No. of cans} = \frac{50\ \cancel{\text{sq ft}}}{10\ \cancel{\text{sq ft}}} \qquad \text{units cancel}$$

No. of cans = 5

Example: Find the length of a rectangular piece of metal if the area is 48 cm^2 and the width is 6 cm. The formula for area of a rectangle is:

Area = length x *width* rearrange the formula so *l* is alone on the left side
length = Area / width (we look at how to do this later)
length = 48cm^2 / 6cm

When a squared unit is divided by the same unit which isn't squared, this unit cancels one of the squared units.

$$\frac{48\ \text{cm}\ (\cancel{\text{cm}})}{6\ \cancel{\text{cm}}} = 8\text{cm}$$

The length of the rectangular piece of metal is 8 centimeters.

Rule 4. When you work with units of different types such as gallons and miles, or distance and time, the units must be carried through the calculations and be shown in the answer. Here are some things to remember:

- Units of different types cannot be added or subtracted. You cannot add miles to gallon or kilometers to seconds.
- The number values of units of different types can be multiplied or divided but the units cannot cancel each other out.

Example:

$$\frac{100\text{ km}}{2\text{ liters}} = 50\text{km/l}$$

Example: A truck travels a distance of 100 km in 2 hours. What is its speed?

The formula for speed is:

Speed = distance/time

Substitute the numerical values with their units for the variables in the formula.

$$\text{Speed} = \frac{100\text{km}}{2\text{ hr}}$$

When you do the numerical division the units remain, resulting in the unit of km/hr.

$$\text{Speed} = \frac{50\text{km}}{1\text{ hr}}$$

$$= 50\ \frac{\text{km}}{\text{hr}} \quad \text{or} \quad 50\text{ km/hr}$$

The speed is 50 km/hr.

Example: A car traveling 220 km uses 20 liters of gas. How many kilometers does it travel on 1 liter of gas?

Divide number of kilometers by number of liters.
220 km ÷ 20 L = 11 km/L

The car travels 11 kilometers on 1 liter of gas.

The numbers can be divided, but since the units measure different types of quantities, they don't cancel out and both must remain in the answer.

SOLVING PROBLEMS INVOLVING VARIABLES

Formulas

*A **formula** (or an **equation**) shows the relationship of two or more quantities by using numbers, variables represented by letters, and operating symbols.*

A formula is a mathematical shorthand that uses letters, or ***variables*** to stand for different quantities. ***Variables** are letters which we use to represent amounts we do not know.*

We use variables in mathematics for the following reasons

1. We can treat these variables just as if they were numbers; and
2. We can use the numbers we know in an equation to find the value of a variable.

Formulas were used in many of the previous examples that showed how to organize units.

In a formula we use variables and indicate that they are to be added, subtracted, multiplied, or divided with operating symbols.

Addition and subtraction: We show addition and subtraction with the plus sign, +, and the minus sign −.

Multiplication: We show multiplication in several ways:

- by writing the variables side by side,
  - cd means c times d
- by writing a constant in front of a variable,
  - 2c means 2 times c
- by writing the constants and variables side by side with brackets.
  - 2(c) means 2 times c

Division: We show division by writing the variables and constants as fractions
2/d means 2 ÷ d, and
d/c means d ÷ c.

The variables used in a formula represent different, specific quantities.

Example: In the formula, **A =** ***lw,***
A always stands for area,
l stands for length, and
w stands for width.

In a problem using the formula **A =** ***lw***, the actual amount of each quantity represented by a letter could be any amount before we are told what value to assign to it.

In one question, the length can be 5 m and the width 2 m, while the area is unknown. Another time you might be told that the area is 30 sq ft and the width 5 ft; this time the length is unknown.

Using Variables in a Formula

1. Every quantity that is not a ***constant*** number in a formula can vary.

   Example: The formula for the surface area of a cube is A= 6(*lw*).
   - In this formula the values for the length and width of the sides will vary depending on the size of the cube.
   - But, the 6 will always be the same; *it is a constant*.

2. Once the amount of any variable in a formula is given in a question, it then has a specific value that doesn't change. *The variable we are not given a value for is called an* ***unknown.***

   Example: Find the area of a rectangle that is 30 feet in length and 5 feet in width using the formula ***A= l w***
   - At this point, the values for *l* and *w* are the known quantities,
   - The value for the variable *A is the unknown*.
   - The value for *A* can be determined using the known quantities for *l* and *w*.

3. To solve a question using a formula, you are usually given the formula and the amounts of all but one quantity. You are required to find the unknown quantity. ***The value of the unknown can be determined using the known quantities*** .By substituting the given quantities for their letters in the formula, you can find the unknown amount.

4. If the unknown variable is isolated (alone) on one side of the equal sign, you can fill in the known quantities and do the calculation to find the value of the unknown quantity.

   Example: If you are asked to find the area of a rectangle and are give its length (say 30 cm) and width (say 20 cm), and, you can use the formula **A = *lw*** just as it is.

A = l w	substitute 30 cm for *l* and 20 cm for *w*
A = 30 cm x 20 cm	multiply
A = 600 cm^2	

5. Sometimes you have to ***manipulate*** (move around) the letters of a formula before you substitute the given amounts in the formula. However, you can start by manipulating (rearranging) the variables in the formula to isolate the unknown variable. Then you can replace the known variables with their given quantities and solve the equation.

   Example: If you are given the width (say 30 cm) and area (say 600 cm^2) of a rectangle and asked to find the length, you still use the formula, **A = *lw***.

A = l w	isolate *l*
l = $\frac{A}{w}$	substitute 600 cm^2 for *A* and 20 cm for *w*
l = $\frac{600^2}{20\text{ cm}}$	divide
l = 30 cm	

This manipulation of letters involves the basic rules of algebra.

Algebra *is a way of finding an unknown number or quantity using an* ***equation or formula****.* An equation or formula usually consists of these parts:

- an unknown quantity (to be found) that is represented by a letter;
- other quantities that are represented by letters or given as numbers;
- a statement called an equation or formula that shows the relationship between all the quantities. It usually includes an equal sign.

Variables and the Letters Representing Them
Some quantities in a formula are represented by letters called variables:

- ♦ The actual value of a variable can be any amount.
- ♦ Its value is not known until it is given in a question, or its value is found using the formula.

Letters representing variables in a formula can be treated exactly as if they were numbers:

1. They follow the same rules for adding, subtracting, multiplying and dividing.

2. They follow the same rules for order of operations.

3. If variables have units attached, the units must be included when the letters representing the variables are manipulated.

4. All the rules you learned for organizing units in problems still apply.

Remember*, when you manipulate variables, you also move their units. The units must remain with the variables unless they are canceled out.*

Finding an Unknown Amount Using a Scientific Formula
We have been using formula in many of the examples shown so far. Now we will look at how to write and solve some of them.

Example: To find the perimeter of a rectangle, add two times the length of the rectangle to two times its width.

We write the formula using the variables p for perimeter, l for length and w for width. So:

$p = 2l + 2w$

Example: The formula for area of a rectangle, area equals length times width.

It substitutes the variables A for area, l for length and w for width.

$A = lw$

Solving an equation
Let's look at the steps in solving an equation when the variable representing the unknown quantity is isolated (alone) on the left.

To find an unknown quantity represented by a variable isolated on the left side of a formula:

1. First write the formula.
2. Substitute the given amounts of the known variables in the formula.
3. Carry out the necessary mathematical operations which are on the right side to arrive at an answer.
4. Make sure the answer has all the required units.

Example: Find the pressure when force is 20 kg and area is 2 m^2.

Pressure of a solid is defined as the force exerted over a certain area. The letter P always stands for pressure, but the amount of pressure varies depending on what the specific quantities of force and area happen to be. Force is represented by F and area by A. The formula for pressure is: *P = F/A*

P = F/A	Always write the formula first. The variable for pressure (P) is already isolated on the left side.
F = 20 kg A = 2 m^2	Next, list the value of the known quantities.
P = 20 kg/2 m^2	Now substitute the known quantities for their letters.
P = 10 kg/m^2	Divide 20 kg by 2 m^2. Units do not cancel

The pressure is 10 kg/m^2.

MANIPULATING VARIABLES IN A FORMULA

If the letter of the unknown quantity is not by itself on the left side, the situation is more complicated. To solve for the value of an unknown quantity, the letter standing for that quantity must end up by itself on the left. We rearrange, or ***manipulate***, the formula so the letter representing the unknown quantity is by itself on the left side. The quantity on the right side of the equation is the answer we are looking for.

Terms in a Formula or Equation
In order to properly manipulate a formula, we need to understand the concept of terms.

All individual letters and numbers, groups of letters, and groups of letters and numbers in an equation which are separated by a ***plus or minus*** *sign are called* ***terms.***

A term can be:

- an individual letter or number such as 9 or V
- several letters and/or numbers combined together as in *lwh* or 2*w*
- letters and/or numbers written as a fraction as in F/A or d/2
- one or more letters with one or more numbers attached, as in the term 2πr

We can manipulate terms in an equation so that an unknown term is alone (isolated) on one side of an equation or formula.

> **Example:** If, in the question on pressure, you had been given amounts for the pressure and the force, and been asked to find area, you would have to manipulate the letters so "A" was by itself on the left side.

There are rules for manipulating variables in an equation.

The most important rule for manipulating an algebraic equation is as follows: ***If we carry out any mathematical operation on one term in an equation, we must do the same operation to all the terms on both sides***.

Because there is an equal sign separating the two sides of an equation, both the left and the right sides must have the same numerical value. They are equal. *If you perform any operation (add, subtract, multiply, or divide) so that you change the value of the numbers on one side of the equal sign, you must do exactly the same thing on the other side of the equal sign.* This will keep the two sides of the equation equal to each other.

Think of an equation as a balanced scale. If you make a change to anything on one side of it, you must make the same change to the other side or the scale will be out of balance.

In the same way, if you carry out a mathematical operation on one side of an equal sign in an equation, you must do exactly the same operation on the other side. If you don't, the two sides will not be equal; the equation will not be correct. See Figure 1.

When both sides are equal the scale is balanced

If you add or remove anything from one side, the scale will be out of balance

To keep the scale balanced, whatever you do to one side you must also do to the other side

FIGURE 1: An Equation Is Like A Balanced Scale

Example: $20 \times 4 - 60 = 20$ **True**.

Now add 10 to only the left side of the equation.

$20 \times 4 - 60 + \boldsymbol{10} = 20$

$30 \neq 20$ **Not true**. The two sides of the equation are not equal.

Add 10 to both sides.

$20 \times 4 - 60 + \boldsymbol{10} = 20 + \boldsymbol{10}$

$30 = 30$ **True** The two sides of the equation are equal.

Reversing and Inverting Equations

The most important ways of manipulating equations involve adding, multiplying and dividing terms. However, before looking at these ways of rearranging equations, we will mention two other methods used to isolate the unknown variable on the left side.

1. We can reverse an equation without changing its value.

$V = IR$

is the same equation as

$IR = V$.

2. We can invert an equation (turn it upside down) without changing its value.
   The equation $3/5 = 6/x$ can be inverted.

$3/5 = 6/x$

$5/3 = x/6$

Now we will look at how to rearrange equations. By adding, subtracting, multiplying and dividing terms we can isolate the letter representing the unknown quantity on the left side.

Manipulating Terms by Adding and Subtracting

- *To move a term from one side of an equation to the other, we can add its opposite term to both sides.* The result is that the term disappears from the one side and reappears *as its opposite* on the other.

- *The opposite of a positive term like 6x is its negative version,* − 6x. The opposite of a negative term like −7n is its positive version, 7n. (Negative terms always show the − sign but positive terms don't usually show the + sign. If no sign is showing, the number is assumed to be positive.)

- *If any term is added to its opposite, the answer is 0.* That is why you can remove a term from where it is unwanted by adding it to its opposite. If you add the opposite of 6x (which is − 6x) to 6x, you get 0.

    - $6x - 6x = 0$

- *Of course, whatever you do to one side of an equation, you have to do to the other.* So if you add the opposite of a term to one side, you must add that opposite to the other.

Example: Solve the following equation by isolating the variable, x, on the left side of the equation.

$x + 6 = 24$

$x + 6 - 6 = 24 - 6$ Add the opposite of 6, which is - 6, to both sides.

$x = 18$

Example: Solve the following equation.

$4n - 7 = 3n + 14$

$4n - 7 + 7 = 3n + 14 + 7$ First, isolate 4n by adding 7 to each side.

$4n = 3n + 21$

$4n - 3n = 3n - 3n + 21$ Now, move the 3n by adding -3n to each side.

$n = 21$

Manipulating Terms by Multiplying or Dividing

First we will look at some general information on multiplying. Then we will look at using multiplication to manipulate terms in an equation. We will do the same with division.

When we multiply or divide one term by a letter or number, we must multiply or divide ***every term in the equation*** *by that same letter or number.* (Remember, a term is any letter or number or joined groups of letters and/or numbers separated by a + or − sign.)

Multiplying terms

- Usually the times sign (x) is not written to indicate multiplication in a formula.
- When two letters are to be multiplied, they are often written right beside each other, as in IR, which means the same as I x R.
- Brackets are also used to indicate multiplication. 5(P) indicates that 5 and P are to be multiplied.
- The brackets after the 6 in the expression 6(3s + 45) indicates that both terms inside the brackets are each to be multiplied by 6. So 6(3s + 45) is the same as 6 x 3s + 6 x 45, which can be multiplied to get 18s + 270.

Multiply to remove a denominator

When a term in an equation is in a fraction form, we usually multiply to remove the denominator, the bottom number of the fraction.

Any fraction can become a whole number if you multiply it by its denominator. If we multiply the fraction 4/7 by 7, we get the whole number 4.

4/7 x 7 = 4

We have eliminated the denominator.

If we multiply the variable 6/A by A, we eliminate the A.

6/A x A = 6

Of course, in an equation, *if we multiply one variable by a number to remove the denominator, we have to multiply all the other **terms** in the equation by that number*.

Example: Find the value of d (distance) in the formula:
speed = d/t
when speed equals 100 km/hr and t (time) equals 5 hours.

First reverse the equation.
d/t = s

Now, remove the denominator *t* so *d* is isolated on the left side. Multiply each term in the equation by *t*.

d/t(t) = st   d/t x t = d   t is canceled on the left side and s times t = st
d = st   t is now on the right hand side and d is isolated on the left

To finish solving for *d*, substitute the values for *s* and *t* and multiply them together.

d = st
d = 100 km/hr x 5 hr   100 x 5 = 500

The units hr, on opposite sides of the fraction line, cancel out, leaving the unit km.

d = 100 km/~~hr~~ x 5 ~~hr~~
d = 500 km

Dividing terms

The division sign is seldom used in a formula to indicate division. Instead, *the fraction form is used to show division*, as in

$\frac{a}{2}$ (read a over 2, or a divided by 2)

or

p/A (read *p* divided by A)

When one number or variable is written under another number or variable separated by a fraction line (as in 10n/5) it indicates that you are to divide.

- 10n/5 tells us to divide the numerator (the top number) 10n by the denominator, 5.

If we divide 3v by 3, we are left with v. The 3 is eliminated.

3v ÷ 3 = v

or

$\frac{\cancel{3}v}{\cancel{3}} = v$

If we wish to eliminate a number or variable from a term in an equation, we can divide by that number or variable. For example, to eliminate the *l* in the term *lw*, we divide by *l*.

$\frac{\cancel{l}w}{\cancel{l}} = w$

Remember, if we divide a term in an equation by a letter or number, we have to divide *every other* ***term*** *in the equation by that letter or number*.

In these examples, we isolate the letter representing the unknown variable before substituting the known quantities. There are times, however, when you might prefer to substitute the given quantities first and then isolate the unknown variable. Either method will get the correct answer.

Example: Find the width if *A* equals 200 m^2 and *l* equals 20 m.

First reverse the equation.

A = *l*w

to

*l*w = A

To isolate *w* on the left side, we want to remove the *l* from that side. We divide by *l* in order to eliminate it, but we must also divide any other terms by *l*.

$\frac{\cancel{l}w}{\cancel{l}} = \frac{A}{l}$ *l*w ÷ *l* = *w*

The *l* is removed from the left side.

$$w = \frac{A}{l}$$

To complete finding the value of *w*, we substitute the values of A and *l* in the formula and divide.

$$w = A / l$$
$$= 200\ m^2/20\ m$$
$$= \frac{200\ \not{m}(m)}{20\ \not{m}}$$

$= 10\ m$ m^2 divided by m = m

SOLVING PROBLEMS WHEN THE FORMULA IS NOT GIVEN

When solving problems in class, you are usually given the formula and the known quantities. On the job, you might have to decide yourself what formula to use to find an unknown quantity. The steps are the same except that you have to know where to find the formula.

1. *Decide what quantities you know and what you are required to find*.
2. *Decide what formula is needed* to solve the problem, looking it up if necessary in a textbook or manual. Write it down.
3. *Rearrange the formula if necessary* so the letter representing the quantity you are looking for is by itself on the left.
4. *Fill in the known quantities and their units.*
5. *Solve the equation to find the required quantity*, including the units needed in the answer.

Example: Find the volume of a cylinder 10 cm high with a radius of 5 cm. ($\pi = 3.14$)

1. The formula for volume of a cylinder is not given.
2. You will have to find the formula, $V = \pi r^2 h$.
3. V is already isolated on the left, so it is not necessary to rearrange the formula
4. Substitute given values in the formula:

   $V = 3.14 \times (5\ cm)^2 \times 10\ cm$

5. $V = 785\ cm^3$

The volume of the cylinder is $785\ cm^3$.

Example: A circular piece of steel with a radius of 5 cm is to be cut from a rectangular piece that is 11 cm by 15 cm. Find the amount of steel left over after the circular piece is cut out and removed.

1. We need to find the area of the rectangular piece and the area of the circular piece that is to be cut out.
2. Two formula are required.
   Area of a rectangle:
   $A = lw$
   Area of a circle:
   $A = \pi r^2$
3. In both cases, the unknown, A, is already isolated on the left. We do not have to manipulate the formula to isolate A.
4. Fill in the known quantities.
   $A = lw$
   $A = 11 \text{ cm} \times 15 \text{ cm}$
   $= 165 \text{ cm}^2$
   and
   $A = \pi r^2$
   $A = 3.14 \times 5^2$
   $= 78.5 \text{ cm}^2$
5. To get the final answer, we have to subtract the area of the circle from the area of the rectangle.
   $165 \text{ cm}^2 - 78.5 \text{ cm}^2 = 86.5 \text{ cm}^2$
   The amount left over is 86.5 cm^2.

Example: Find the area of a square building if its perimeter is 32 m.

1. We are given the perimeter, 32m, and we need to find the area

2. We need to first find the length of one side of the building. Use the value of the perimeter to find the length. Then we can find the area.

3. The formula for perimeter is:
   $P = 2l + 2w$

   Because the building is square, $l = w$.
   $P = 4l$

   The formula for area of a square is:
   $A = lw$ or,
   $A = l(l)$or use the exponent,,
   $A = l^2$

4. We want to find l, so we rearrange the formula.
   $P = 4l$ reverse the equation
   $4l = P$ divide by 4 to isolate l
   $l = P/4$

5. $l = P/4$ substitute 32 m for P
   $= 32 \text{ m}/4$
   $= 8 \text{ m}$
   $A = l^2$ substitute 8 m for l
   $= 8 \text{ m} \times 8 \text{ m}$
   $= 64 \text{ sq m}$

Solve the following problems. Some will involve the use of scientific formula. Choose the formula you need from the ones in the box. **Answers are on the last page.**

Formula:	P = F/A	p = 2 *l* + 2 *w*	Speed = d/t
	A = lw	$^oC = 5/9(^oF - 32^o)$	V = lwh

1. Gordy went on a business trip for three days. On the first day he traveled 379 km, on the second he traveled 402 km and on the third 265 km. What is the total distance he traveled?

2. What is the height of a 3 storey building if each storey is 12.55 ft?

3. If each cell of a 6 cell storage battery gives a voltage reading of 1.95 volts, what is the total voltage of the battery?

4. A planer takes .25 inches from the side of a board. If the board was 2 inches wide before planing, what is the width after it is planed?

5. Find the pressure (P) exerted on the floor by a box with an area (A) of 25 cm^2 if the force is 50 kg.

6. A car is traveling for 5 hours. It covers a distance of 300 kilometers. What is its speed?

7. Find the area (A) of a rectangle that is 4 meters long (*l*) and 16 cm wide (*w*). (Make sure your units are the same.)

8. Find the volume of a rectangular solid 10 in long, 5 in wide and 3 in high.

9. The temperature of a shop is to be set at 77 ^{o} F. If the thermostat only has ^{o} C, what should the thermostat setting be?

10. Tape is to be placed around the outside edge of a window that measures 1.5 m by 80 cm. How many centimeters of tape will be needed?

11. The chassis of a truck is 4.5 meters long. Its body extends 1.3 meters past the end of the frame. If there is 2.8 meters space between the truck and the end wall of the garage, and 1.2 meters space between the truck and the garage door, what is the length of the garage?

12. If one third the cost of a project is for material and two thirds is for labour, how much will you pay for materials on a bill of $108.99? How much will you pay for labour?

13. Steel crossmembers weigh 3.5 kg per meter of length. What is the total weight of two pieces, one 3 meters long, the other 1.2 meters long?

14. If 16 machines are installed by a crew in 8 hours, what is the average time spent on each machine? What is the time in minutes?

15. How many sheets of 1/2 in plywood are in a stack 1 ft high?

16. A piece of drainpipe measuring 26.5 meters costs $53. How much does it cost per meter?

17. A room measuring 8.5 meters by 6.5 meters is to be tiled. If the tile costs $28.95 per sq meter, what will it cost to tile the room?

18. to the theoretical output expressed as a percent. It is calculated using the formula:

$$\text{Volumetric efficiency} = \frac{\text{actual output}}{\text{theoretical output}} \times 100\%$$

What is the volumetric efficiency of a pump if the theoretical output is 250 liters/min but it actually pumps out 225 liters/min?

19. The rim speed of a belt is calculated by using the formula:

rim speed = pulley diameter (mm) x rpm x π/1000

The answer is expressed in meters/min. If the pulley diameter of a belt is 300 mm and the rotational speed is 2250 rpm, what is the rim speed of the belt?

ANSWER PAGE

a. Add - 1046 km

b. 12.55 ft x 3 = 37.65 ft

3. 1.95 v x 6 = 11.7 v

4. 2 in - .25 in = 1.75 in.

5. P = F/A = 50 kg/25 cm^2
P = 2 kg/cm^2

6. Speed = d/t
= 300 km/5 hr
= 60 km/hr

7. If you change both units to cm,
A = l w
= 400 cm x 16 cm
= 6400 cm^2
If you change both units to meters,
A = 4 m x .16 m = .64 m^2

8. V = lwh
= 10 in x 5 in x 3 in
= 150 cu in

9. °C= 5/9(°F- 32°)
= 5/9(77°F - 32°)
= 5/9(45°F)
= 25°C

10. P = 2l + 2w
= 2(150 cm) + 2(80 cm)
= 300 cm + 160 cm
= 460 cm

11. Add - 9.8 meters

12. Multiply: 1/3 x $108.99 = $36.33 Cost of materials is $36.33
Subtract: $108.99 - $36.33 = $72.66 Cost of labour is $72.66

13. Add: 3 m + 1.2 m = 4.2 m
Then multiply 4.2 m x 3.5 kg/m = 14.7 kg

14. 8 hours ÷ 16 = .5 hr per machine
    60 minutes in 1 hour
    Multiply 60 x .5 = 30 minutes

15. 12 inches in 1 foot
    12 in ÷ 1/2 in/sheet
    = 12 x 2/1 sheets
    = 24 sheets

16. $53 ÷ 26.5 m = $2 per meter

17. A = lw
    = 8.5 m x 6.5 m
    = 55.25 m^2
    1 sq meter costs $28.95
    55.25 m^2 costs 55.25 m^2 x $28.95/$m^2$
    = $1 599.49

18. $\frac{225 \text{ L/min}}{250 \text{ L/min}}$ x 100%
    = 90 %

19. Rim speed = 300 mm x 2250 rpm x 3.14/1000
    = 675 000 x .00314
    = 2119.5 m/min