EVALUATING
ACADEMIC READINESS
FOR APPRENTICESHIP TRAINING
Revised for
ACCESS TO APPRENTICESHIP

MATHEMATICS SKILLS
OPERATIONS WITH INTEGERS

AN ACADEMIC SKILLS MANUAL
for
The Industrial Maintenance Mechanic Trades

This trade group includes the following trades:
Boiler Maker,
Facilities Maintenance Mechanic & Technician, and
Industrial Mechanic (Millwright)

Workplace Support Services Branch
Ontario Ministry of Education and Training

Revised 2011

In preparing these Academic Skills Manuals we have used passages, diagrams and questions similar to those an apprentice might find in a text, guide or trade manual.

This trade related material is not intended to instruct you in your trade. It is used only to demonstrate how understanding an academic skill will help you find and use the information you need.

MATHEMATICS SKILLS:
OPERATIONS WITH INTEGERS

An academic skill required for the study of the Industrial Maintenance Mechanic Trades

INTRODUCTION

Integers *include the positive whole numbers and also all the negative opposites of the positive whole numbers.* The number +5 (read positive five) has a negative opposite –5 (read negative five). Every positive whole number has a negative opposite. On a really cold day, the weatherman might say it is minus 20° C, or –20°. The negative sign indicates that the number is opposite in value to + 20.

Integers consist of all the positive whole numbers, their negative opposites, and zero. Operations with integers are most often used when doing algebra and using certain gauges. Working with integers provides a different way of thinking about subtraction. Understanding negative numbers on the number line is useful if you have to read a measuring tape both forwards and backwards.

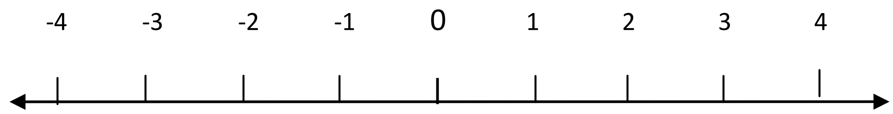
This skills manual covers the following operations with integers:

- Integers on a number line
- Rational numbers
- Operations with positive and negative numbers, including
    - addition of integers
    - subtraction of integers
    - removing brackets
    - multiplication of integers and rational numbers
    - division of integers and rational numbers
- Simplifying equations with integers

INTEGERS ON A NUMBER LINE

All whole positive and negative numbers, including zero, make up the set of numbers called ***integers***. Integers can be arranged in order as points on a ***number line***. The line extends from zero in both a positive and a negative direction. The points on the line to the right of zero are positive and the points to the left of zero are negative.

Here is what the number line looks like:



Although only the integers from -4 to +4 are shown here, the arrows indicate that the numbers extend endlessly in both directions.

- *The positive sign is not usually written in front of a positive integer*.
  - It is assumed that if no sign is shown in front of a number then the number is positive.
  - If the + sign is written in front of an integer, it indicates that the number following the sign is to be added when you simplify the expression.

- *The negative sign is written in front of a negative number every time.*

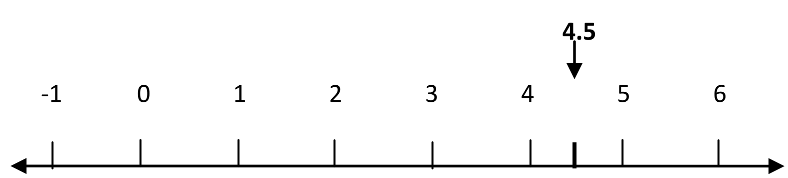
Understanding a number line is useful in reading certain gauges. On these gauges there is a zero in the centre of the numbering. When the dial is pointed to the right of the zero, the numbers are positive. When the dial is pointed to the left, the numbers are negative.

RATIONAL NUMBERS

All integers (whole numbers) fractions and decimal numbers, are called ***rational numbers****.*

1. Between each integer on the number line is an infinite number of fractional numbers.
   - For example, the fractions $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, and $\frac{2}{3}$ along with countless other fractions and decimals, can all be found as points on a number line between any two integers.

2. Any fraction can also be written as a decimal and so any decimal number has a corresponding point on the number line.
   - For example, the decimal number 4.5 (which can also be expressed as the mixed number 4 ½) can be found on the number line half way between 4 and 5.



   - There is an endless number of fractions so we only show a few if we are drawing a number line that includes fractional or decimal numbers.

3. To find the distance between two integers or two rational numbers on the number line, start from the point that is furthest to the left and count the number of spaces to the second point.

   - For example, if you wanted to find the distance from –3 to 2, you could count from point –3 on the number line to point 2. The distance equals 5 units.

   - You can find the distance without actually counting spaces on the line, if you follow the rules for subtracting integers an operation we will look at in the next section.

OPERATIONS WITH POSITIVE AND NEGATIVE NUMBERS

Addition of Integers

Doing operations with integers involves keeping track of the signs of the numbers you are working with along with figuring out the basic operations. There are five different situations we will look at involving addition. In each case, we have to consider what the sign of the answer will be.

1. *To add two positive numbers,* find the sum of their values and write the answer with a positive sign.

Add:
$$\begin{array}{r} 4 \\ \underline{7} \\ 11 \end{array}$$
Or $4 + 7 = 11$

2. *To add two negative numbers,* add the value of the numbers together and write the answer with a negative sign in front.

Add:
$$\begin{array}{r} -8 \\ -\underline{14} \\ -22 \end{array}$$
Or $-8 + (-14) = -22$

3. *To add a negative and a positive number,* subtract the smaller number from the larger. Give the answer the sign of the larger number.

Add:
$$\begin{array}{r} -8 \\ \underline{5} \\ -3 \end{array} \qquad \begin{array}{r} 10 \\ \underline{-7} \\ 3 \end{array} \qquad \begin{array}{r} 8 \\ \underline{-9} \\ --1 \end{array}$$

$-8 + 5 = -3$ $10 + (-7) = 3$ $8 + (-9) = -1$

To Add Integers

(+) + (+) = +
(–) + (–) = –
(+) + (–) = the sign of the larger number

4. *The sum of any positive number and its negative opposite is zero.*

Add:	8	–12
	–8	12
	0	0

5. *To add three or more numbers with positive and negative signs, add all the positive numbers following step 1, add all the negative numbers following step 2 and then add the two answers following step 3.*

Add: 3 + (–5) + (–10) + 7 + (–6) + 4
= 14 + (–21)
= –7

- *You can also add each integer one at a time to the current total, following rules 1 to 3,* depending on the signs of the numbers you are adding. Using this method, the example above would be simplified like this:

Add: 3 + (–5) + (–10) + 7 + (–6) + 4
= –2 + (–10) + 7 + (–6) + 4
= –12 + 7 + (–6) + 4
= –5 + (–6) + 4
= –11 + 4
= –7

Once you are familiar with this method, you probably won't write down all these steps. Instead you will carry the total after each new addition in your head and add the next number to it, writing only the final total.

Note: *The brackets and the + signs between the brackets enclosing negative numbers often are not written* when you are asked to solve equations like the example above,.

- The positive numbers have a + sign in front of them and
- the negative numbers have a – sign in front of them.

The question will look like this:

Add: 3 –5 – 10 + 7 – 6 +4
= 14 – 21
= –7

Follow the same addition rules to find the answer.

Subtraction of Integers

♦ *Every number on the number line has an opposite.* A pair of numbers such as +8 and –8 are called opposites. In other words, opposites have the same value but opposite signs.

The opposite of –1/2 is +1/2.

♦ In the operations with positive and negative numbers (integers), you follow the rules of algebra, which approach the basic operations in a slightly different way.

♦ *Subtracting an integer gives the same answer as adding its opposite.* Subtracting becomes a two step process:

1. Change the sign of the number to be subtracted to its opposite sign
2. Then add the number with its opposite sign
   - If the number has a positive sign, its sign changes to negative and the number, now with a negative sign, is then added.
   - If the number to be subtracted has a negative sign, the sign changes to positive and this number, now with a positive sign, is then added.

The operation “9 subtract –5” requires you to change the sign of the number to be subtracted, which is –5, to its opposite +5. Add the +5 to 9. “9 minus –5" is the same as 9 plus 5 as the negative sign is changed to a plus and the +5 is then added to the 9.

Basic rules for subtracting numbers with signs:

1. Change the sign of the number to be subtracted to its opposite.
2. Add the number with the changed sign to the other number, **using the rules for addition of integers.**

Example: Subtract: $-4-(6)$.

1. Change the number to be subtracted, which is 6, to its opposite, which is -6. Then change the subtraction sign to an addition sign.

   The question is now written:

   $-4+(-6)$

2. Add, following the addition rules. In this case, use addition rule # 2. Add the two negative numbers together and put a negative sign in front of the answer.

   $-4+(-6)$
   $= -10$

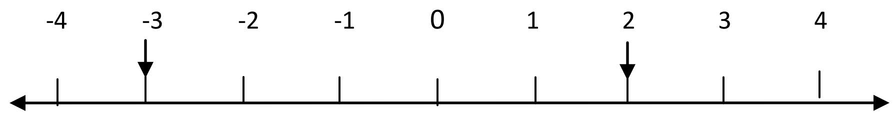
Here is another example:

$2-(+5)$

$=2+(-5)$ Use addition rule # 3

$=3$

The rules for subtracting integers can be used to find the distance between two points on the number line. The number to the left on the number line is subtracted from the number to the right to find the distance between them.



To find the distance between two integers or two rational numbers on the number line, you could start from the point that is furthest to the left and count the number of spaces to the second point

To find the distance between -3 and 2 on the number line, we subtract the number on the left, which is -3, from the number on the right, which is 2. The problem is written like this:

$2 - (-3)$

We change the sign of the number to be subtracted, which is ⁻ 3, to its opposite, which is 3. The minus sign changes to an addition sign. The problem is now written:

2 + 3 (We use addition rule # 1.)
= 5

This gives us the same answer as when we counted the number of points between the two integers. Using the rules of subtraction to find the distance between two points on the number line is helpful when the points indicate decimal numbers. It is difficult to count the distance between the points -5.8 and 6.7, so we use the rules for subtracting positive and negative numbers.

We subtract the number furthest to the left, which is ⁻ 5.8, from the number on the right, 6.7. To add decimals, write them so the decimal points fall under each other Write the question like this:

$$\begin{array}{r} 6.7 \\ \underline{-(-5.8)} \end{array}$$

Follow the rules for subtracting. The number to be subtracted, which is - 5.8, changes to its opposite, which is 5.8 and the minus sign changes to an addition sign.

$$\begin{array}{r} 6.7 \\ \underline{+\ 5.8} \\ 12.5 \end{array}$$

The distance from -5.8 to 6.7 is 12.5 units.

To simplify it, you can consider subtraction of integers as addition of negative numbers. Then you only need to use the rules for adding positive and negative numbers. Most of the time when you are using positive and negative numbers, you are given an expression or equation and asked to simplify or solve it.

To simplify an expression such as $7 + 5 - 6 - 11 + 21 - 3 - 15$, think of the + and − signs not as indicating addition or subtraction but as the positive or negative signs attached to integers.

Then think of the operation to be done as addition of integers, following the rules already given.

To Subtract Integers

1. Change the signs of numbers to be subtracted
2. Add, using the rules for adding integers

Removing Brackets

An expression to be simplified may have brackets with signs in front them and also signs in front of the numbers inside the brackets. *It is easier to simplify the expression if these brackets are removed and there is only one sign in front of each number.*

To determine the sign to leave in front of each number when you remove the brackets follow these rules:

1. If there is both a – sign and a + sign, the sign becomes – .
2. If there are two + signs, the sign stays +.
3. If there are two – signs, the sign becomes +.

You used these rules to remove brackets in subtraction but took several steps to do it. Now, you can remove the brackets right away and simplify the expression using the rules for addition. This eliminates the need to use the rules for subtracting integers.

Example:

$(+8) - (-4)$ the two signs become a plus sign

= 8 + 4

=12

Example:

8 + (9)

= 8 + 9 the brackets are removed, leaving the + sign

Example:

$-4-(+5)$ the − and + signs become a − sign

= − 4 − 5

= − 9

Example:

$-6-2$ the − and + signs become a − sign

= – 6 – 2

= – 8

Example: Simplify: $8+(-5)-(-7)+(+6)-(+8)-(-3)$

= 8 – 5 + 7 + 6 – 8 + 3 First remove the brackets following the three rules that determine signs.

= 24 – 13 Next simplify using the rules for addition of integers.

= 11

Multiplication of Integers and Rational Numbers

There are several ways to indicate multiplication when working with rational numbers. Multiplication is indicated in the following ways:

- 6 x –5
- 6 · (–5)
- 6 (–5)

Multiplication of integers is usually indicated by the third example. The two numbers to be multiplied are written next to each other, with the second number enclosed in brackets.

- *If there are operations to be done inside the brackets, these are done first, before the answers are multiplied.*

- *If there are two sets of numbers within brackets to be multiplied, then the X sign is usually shown to clearly indicate the calculations to be done. In the complex example* $(25\div 5)\times[36\div 9-20(4)]$ *the X sign is used to indicate multiplication.*

- Multiplication is done in the usual way but the sign of the answer must be considered.

To determine the sign of the answer to a multiplication question follow these rules:

1. When two positive numbers are multiplied, the answer is positive.
2. When two negative numbers are multiplied, the answer is positive.
3. When a positive and a negative number are multiplied, the answer is negative.

To Multiply Integers
$(+) \times (+) = +$
$(-) \times (-) = +$
$(+) \times (-) = -$

Example: When two positive integers are multiplied, the answer is positive.

9 (8) = 72

Example: When two negative numbers are multiplied, the answer is positive.

–4 (–6) = 24

Example: When a positive number and a negative number are multiplied, the answer is negative. If the first number is negative, it is often also enclosed in brackets:

(–7) (5) = –35

Example: When a positive number and a negative number are multiplied, the answer is negative.

12 (–2) = –24

The rules for multiplying integers also apply to all rational numbers.

Example:

1.5 (–3.3) = –4.95

Division of Integers and Rational Numbers
Division of integers is usually indicated algebraically in an equation by writing the number as a fraction, with the number you are dividing into (the dividend) written on top and the number you are dividing by (the divisor) written on the bottom.

So $-36 \div 9$ is written: $-36/9$ *or* $\frac{-36}{9}$ in an algebraic expression.

In a complicated formula, the division sign, like the x sign, might still used.

To determine the sign of the answer to a division question follow these rules:

1. When the dividend and the divisor are both positive, the answer is positive.
2. When the dividend and the divisor are both negative, the answer is positive.
3. When the dividend is negative and the divisor is positive, the answer is negative.
4. When the dividend is positive and the divisor is negative, the answer is negative.

To Divide Integers

$(+) \div (+) = +$
$(-) \div (-) = +$
$(+) \div (-) = -$

Example: When two positive numbers are divided, the answer is positive.

$$\frac{56}{8} = 7$$

Example: When two negative numbers are divided, the answer is positive.

$$\frac{-46}{-7}$$

Example: When either the dividend or the divisor is negative, the answer is negative

$\frac{16}{-8} = -2$ and $\frac{-16}{8} = -2$

Rational numbers follow the same rules.

Example:

$$\frac{3.2}{-4} = -2$$

SIMPLIFYING EQUATIONS WITH INTEGERS

To simplify an equation with negative and positive numbers, you do the operations required following in the correct order of operations.

Brackets
Exponents
Division
Multiplication
Addition
Subtraction

Determine the correct sign for each answer, depending on the rules for that operation. You also determine the resulting sign of the number when any brackets are removed.

Example: Simplify $5(-6) + (-12 \div -3) - 7(-3)$

= –30 + 4 + 21
= –5

Example: Simplify $-8 \div 2 + (-9)(5) - (-66/3)$

= –4 – 45 + 22
= –27

Example: Simplify $(25 \div 5) \text{ x } [36 \div 9 - 20(4)]$

= 5 x (4 – 80)
= 5(–76)
= – 380

Example: Without going into details about what the letters in the formula stand for, let's solve the formula for correcting misalignment with shims.

$S_1 = (B \div A) \text{ x } [(D1 + D2) \div 2] - (D1 \div 2)$

Find S_1 where:
A = 5.250"
B = 10.5"
C = 30.75"
D_1 = –0.012"
D_2 = +0.018"

$S_1 = (B \div A) \text{ x } [(D1 + D2) \div 2] - (D1 \div 2)$

For each letter, substitute the known amount. The formula becomes:

$S_1 = (10.5 \div 5.25)\, x\, [(-.012 + .018) \div 2] - (-.012 \div 2)$

Simplify inside of each set of brackets first.

S_1 = 2(.006 ÷ 2) – (– .006) — Negative sign before bracket.
= 2(.003) +.006 — Change the sign and remove bracket
= .006 + .006
= + .012"

(In this case, the answer is positive, so .012" shims would be added to correct the misalignment.)

SUMMARY

Here are the rules for determining the sign of an answer when you are doing calculations with integers and rational numbers.

Addition

1. If you add positive number plus a positive number you get a positive answer.
2. If you add a negative number plus a negative number you get a negative answer.
3. If you add a positive number plus a negative number, subtract the smaller number from the larger number. The answer has the sign of the larger number.

Remember

$(+) + (+) = +$
$(-) + (-) = -$
$(+) + (-) =$ the sign of the larger number

Subtraction

Follow these steps:

1. Change the sign of the number to be subtracted to its opposite.
2. Add the numbers ***following the rules for addition***.

Remember: To Subtract Integers

1. Change the signs of numbers to be subtracted
2. Add, using the rules for adding integers

Multiplication

1. Positive times positive gives a positive.
2. Negative times negative gives a positive.
3. Positive times negative gives a negative.
4. Negative times positive gives a negative.

Remember

$(+) \times (+) = +$
$(-) \times (-) = +$
$(+) \times (-) = -$

Division

1. Positive divided by positive gives a positive.
2. Negative divided by negative gives a positive.
3. Positive divided by negative gives a negative.
4. Negative divided by positive gives a negative.

Remember

$(+) \div (+) = +$
$(-) \div (-) = +$
$(+) \div (-) = -$

Simplify the following equations. You will need to use all the different rules given on operations with integers. **Answers are on the last page of the skills manual.**

1. $5(-8 + 4) + (-36 \div 9) - (-2)$

2. $-16 \div (-2) + (-8)(-7 + 5) - 12$

3. $42 \div (-6) + 8 - 3\ (-5)$

4. $(-6)(5) + 54 \div 9 - [-8 + (-3)] + 4(7 - 1)$

ANSWER PAGE

1. $5(-8 + 4) + (-36 \div 9) - (-2)$
$= 5(-4) + (-4) + 2$
$= -20 - 4 + 2$
$= -22$

2. $-16 \div (-2) + (-8)(-7 + 5) - 12$
$= 8 + (-8)(-2) - 12$
$= 8 + 16 - 12$
$= 12$

3. $42 \div (-6) + 8 - 3(-5)$
$= -7 + 8 + 15$
$= 16$

4. $(-6)(5) + 54 \div 9 - [-8 + (-3)] + 4(7 - 1)$
$= -30 + 6 - (-11) + 4(6)$
$= -30 + 6 + 11 + 24$
$= 11$