EVALUATING
ACADEMIC READINESS
FOR APPRENTICESHIP TRAINING
Revised for
ACCESS TO APPRENTICESHIP

MATHEMATICS SKILLS
OPERATIONS WITH PERCENTAGES

AN ACADEMIC SKILLS MANUAL
for
The Hairstylist Trade

Workplace Support Services Branch
Ontario Ministry of Training, Colleges and Universities

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In preparing these Academic Skills Manuals we have used passages, diagrams and questions similar to those an apprentice might find in a text, guide or trade manual.

This trade related material is not intended to instruct you in your trade. It is used only to demonstrate how understanding an academic skill will help you find and use the information you need.

MATHEMATICS SKILLS
OPERATIONS WITH PERCENTAGES

An academic skill required for the study of the Hairstylist Trade

INTRODUCTION

When you read the information handed out in your course, you will come across statements such as these:

> "The cortex, the major component of the hair structure, accounts for up to 90% of its total weight."
> "Hair with good elastic qualities can be stretched 20% of its length without breaking."

> "If a person with 50% grey wants to colour her hair, the hair colour chosen should be one level lighter than the desired level."

All these statements use a mathematical shorthand called percent. This skills manual looks at different ways to use percent. It covers the following topics:

- The meaning of percent
- Writing a number as a percent
  - changing whole numbers, decimals and fractions to percent,
  - rounding off
- Changing a percent to a decimal
- Problems involving percent, including
  - finding a percent of a number
  - finding what percent one number is of another
  - finding a number when a percent of it is known

THE MEANING OF PERCENT

*A **percent** is an amount expressed out of a total of one hundred.*

- 25% means 25 parts out of a total of 100 parts.
- When we say that 75% of the population has dark hair, we mean that 75 people out of every 100 people will have dark hair.
- If a person has 20% grey hair, it means that 20 hair strands out of a total of every 100 strands on his head are grey.
- A 10% increase in heating costs this winter means that for every dollar you spent on heating costs last winter, you now need to spend $ .10 more.
- A type of hair straightening called hard press removes 100% of the curl in a person's hair. This means that all the curl is removed.

Percent is often used to calculate financial amounts.

Examples:
A 10% increase in heating costs this year means that for every dollar you spent on heating costs last year, you now need to spend $.10 more.

You pay 13% HST on all parts and materials which you forward on to your customers when you calculate the bill.

Your clients also pay 13% HST which you add to labour as you calculate the bill.

If you want to start your own business and you need a loan, the amount of interest on the loan is given as a percent such as 6.5%. To find out how much interest you will pay on an $85,000 loan at 6.5% interest per year, you need to know how to do calculations with percent.

A Percent is a Special Kind of Fraction
A percent expresses an amount out of a total of 100 hundred equal parts. Since the 100 parts represent one whole amount, any percent less than 100 is a partial amount. A partial amount can be written as a percent, a fraction or a decimal.

Fraction: A fraction tells how many equal parts a whole amount is divided into; it also tells you how many of those parts you have.

Example: In the fraction 19/100, the bottom number tells you how many parts, in this case 100, that the total amount is divided into. The top number tells you how many of these parts, in this case 19, you have.

- The top part of the fraction, representing the partial amount 19, is called the *numerator*.
- The bottom part of the fraction, representing the total amount 100, is called the *denominator*.
- The numerator and denominator are divided by the *fraction line*.

Percentages are fractions too: A percent is a kind of fraction whose denominator, like the fraction 19/100, is 100.

- When we write a percent, we only write the numerator.
- The numerator is written with the percent (%) symbol after it.
- The % symbol represents the fraction line and indicates that the unwritten denominator is 100.

Example: To express the fraction 19/100 as a percent:

- The numerator 19 is written with the % sign after it.
- The denominator 100 is not shown.
- Nineteen percent is written as 19%.

WRITING A NUMBER AS A PERCENT

Your course description might state that you have to attend 90 hours out of a total of 100 course hours in order to pass. Or it might state that you must attend 90% of the course. 90 out of 100 is the fraction 90/100. The fraction 90/100 and 90% both indicate the same time spent in class, but they are expressed in different ways.

To convert any fraction, whole number or decimal number to a percent use the following rule:

- To write a number as a percent, multiply it by 100 and add the % sign.

Remember: *A quick way to multiply a number by 100 is to move the* ***decimal point*** *two places to the* ***right.*** *If the decimal point isn't shown at the end of a whole number, it is assumed to be after the last digit in the number. (Digits are the symbols we use to write numbers. The digit for four is 4.)*

We will look at changing a whole number, a decimal number and a fraction to a percent. There is one method each for converting whole numbers and decimals to percent and two methods for changing fractions to percent.

To Change a Whole Number to a Percent

1. Multiply the number by 100.
2. Add the percent sign.

Example: Express 3 as a percent.

3 x 100
= 300 write the % sign
= 300%

To Change a Decimal Number to a Percent

1. Multiply the number by 100.
2. Move the decimal point two places to the right, using zero as a place holder if necessary.
3. Add the % sign.

Example: Express .725 as a percent.

.725 x 100 move the decimal point two places to the right
= 72.5
= 72.5% write the % sign

Example: Express .5 as a percent.

.5 x 100 move the decimal point two places to the right, using one zero as a place holder
= 50
= 50% write the % sign

Changing a Fraction to a Percent
There are two methods for changing fractions to percentages.

Method One: (This method is handy if the denominator of the fraction divides evenly into 100.)

1. Multiply the fraction by 100.
2. Reduce the answer to its lowest terms
3. Write the percent sign with the answer.

Example: Express 2/5 as a percent.

$\frac{2}{5_1} \times \frac{\cancel{100}^{20}}{}$	5 divides evenly into 100 we can cross out before multiplying
= 40	
= 40%	write the % sign

Method Two: (This is the most common way to convert a fraction to a percent. This method is handy if the denominator of the fraction does not divide evenly into 100.)

1. Change the fraction to a decimal.
    a. Divide the denominator of the fraction into the numerator. The answer is a decimal number.
2. Multiply the decimal by 100 and write the percent sign.

Example: Express 3/8 as a percent.

3/8	
= 3 ÷ 8	change the fraction to a decimal
= .375	
.375 x100	multiply by 100
= 37.5	
= 37.5%	write the percent sign

Methods One and Two *result in the same answer. You can use either method with any fraction to convert it to a percent.*

Examples:

a) Express .25 as a percent.
.25 x 100 = 25%

b) Express 1 as a percent.
1 x 100 = 100%

c) Express 1/4 as a percent.
1/4 x 100 = 25%

d) Express 65 out of 100 as a percent.
65/100 x 100 = 65%

e) Express 5/8 as a percent.
5 ÷ 8 = .625 *4*
.625 x 100 = 62.5%

f) Express1/3 as a percent.
1/3
= 1 ÷ 3
= .33...
= .33... x 100
= 33.33... %

To round off an answer:

1. Look at the digit one place to the right of the place you are rounding off to.
2. If that digit is five or more, change the digit you are rounding off to one more than it is.
3. If the digit to the right is less than five, leave the digit you are rounding off the same.
4. Discard any digits after the one you have rounded off.
5. If the number you are rounding off to is 9 and the digit one past it is five or greater, the 9 becomes 10 and the place value where the 9 was becomes 0. The one from the 10 is added to the digit to the left of the 9, which becomes one larger in value.

Example: Round .6769 to two decimal places.

We look at the digit one past the 7, the second decimal place, which is a 6. Since it is five or larger, the 7 becomes 8 and the other digits are dropped.

6769 rounded to two decimal places is .68

Example: Express 1/3 as a percent with the percent rounded to two decimal places.

1/3 = 33.3333...%. expressed as a percent
= 33.33 % rounded to two places

Example: Express 2/3 as a percent.

2/3 = 66.666..%. expressed as percent
= 66.67% rounded off

Sometimes you are told how many places to round off to in the question and sometimes you are not. If you are not told, answers are generally rounded off to two places, although there are often exceptions. One eighth (1/8) is 12.5% as a percent and .125 as a decimal number. When a decimal number ends evenly in three or four places, we don't round it off.

CHANGING A PERCENT TO A DECIMAL

Imagine you have been told that insulating the walls of your shop will result in an energy savings of 10% on your heating bill. Before you decide whether to insulate the shop, you might want to calculate the actual monthly savings. You convert 10% to a decimal and then multiply the

decimal number by your average monthly heating bill. To do this calculation, you first need to know how to change a percent to a decimal.

Decimals Are Also a Type of Fraction

Like percent, decimals are a kind of fraction. *A decimal is the numerator of a fraction which has a denominator that is a power of ten.* (A power of ten is a number that starts with 1 followed by any number of zeros such as 10, 100, 1000.) However, decimals show no denominator. A decimal point in front of the numerator replaces the denominator and the fraction line.

You have already learned how to convert a decimal number to a percent: multiply by 100 and add the % sign. To change a percent to a decimal, reverse the process.

To change a percent to a decimal:

1. Remove the percent sign and divide by 100.

Remember: *To divide by 100 quickly, move the decimal point two places to the left. When dividing by 100, the decimal point moves in the opposite direction than when multiplying by 100. If there are empty spaces between the decimal point and the digits of the number after you have moved the decimal point to the left, fill the spaces with zeros.*

Example: Divide 3.67 by 100 the quick way.

Move the decimal point two places to the left and fill the empty space with a zero. The answer is .0367

Example: Change 5% to a decimal.

5% = .05 move the decimal 2 places to the left, use zero as a place holder and remove the % sign

Example: Change 33.3% to a decimal.

33.3% = .333 move the decimal 2 places to the left and remove the % sign

Example: Express 10% as a decimal.

10% = .10 or .1 move the decimal point two places to the left and remove the % sign

Example: If you can save 10% in energy costs if you insulate your shop how much money you would save if you insulted the shop.

10% = .1

If the average monthly heating bill is $250, 10% saving would be .1 x $250 = $25. The savings would be $25 a month. You can now decide if it is worthwhile insulating.

TO WRITE A NUMBER AS A PERCENT

1. Multiply the number by 100 using one of the methods shown. Use zeros as place holders when needed. If the decimal point is not shown in a whole number, it is assumed to be after the last digit.
2. Write the % sign.

Express the following as a percent. **Answers are at the end of this skills manual.**

1. a) 10 out of 100 b) 1/5 c) 100 out of 100

   d) 2/1 e) .85 f) 2.75

   g) .05 h) 1/3 i) 1

TO CHANGE A PERCENT TO A DECIMAL

1. Remove the percent sign.
2. Divide the number by 100 by moving the decimal point **two** places to the **left**. Use zeros as place holders when needed.
3. If the decimal point is not shown, it is assumed to be after the last digit in the number.

Change the following percents to decimal numbers.

2. a) 28% b) 56.7% c) 7%

   d) 100% e) 8.5% f) 250%

   g) 15% h) .2% i) 62.5 %

PROBLEMS INVOLVING PERCENT

Sometimes you have to solve problems which involve percent. You may be given information expressed as a percent and asked to find an unknown amount by using the given information. For example, you use percent to find the amount of sales tax owed or to find the interest on a loan.

Note: *When doing percent problems, a calculator can do the calculations accurately, but you first have to decide what steps to use and in what order. Once you have figured out what you are asked to find and what method to use, you can use your calculator.*

We will look at three main types of percent problems so you will know what method to choose for different situations. The three types of problems are:

- Finding the percent of a number.
- Finding what percent one number is of another.
- Finding a number when you know a percent of it.

Problem Type #1: To Find a Percent of a Number
The most common kind of percent problem involves finding a percentage of a number.

> **Example:** In Ontario, you pay 13% HST for many goods and services you buy. If you want to know the total price of an object or service before buying it, you need to calculate the amount of tax (13%) and add that to the cost. If the price of a truck is $22 000, you have to add 13% harmonized sales tax to find the total cost.

To find the percent of a number:
1. Change the percent to a decimal.
2. Then multiply the number by the decimal.

We looked at how to change a percent to a decimal in the last section. Drop the percent sign and move the decimal point two places to the left.

> **Example:** Find 5% of 20. .

5% = .05	1. Change 5% to a decimal. When you move the decimal point two places to the left, you will need to use a zero as a place holder
20 x .05 100 00 1.00	Now multiply .05 x 20.

5% of 20 = 1

Example: Find 25% of $150.

$150 Change 25% to the decimal .25 and multiply.
x .25
7500
3000
$37.50

25% of $150 = $37.50

Most problems that ask you to find a percentage of a number are worded something like this:

- If an electric hair dryer costs $49.99 and the sales tax is 15%, what is the total amount you will have to pay?
- Find the amount of interest you must pay in a year on a loan of $3000 if the interest rate is 12% per annum (per year).
- Find the total amount you pay for a car that sells for $22 00 if you pay 13% tax.

In each case, you must first find the percentage of a given number. In the first problem, you find 15% of $49.99, in the second, 12% of $3000 and in the third problem, 13% of $22,000.

Example 1: If an electric hair dryer costs $49.99 and the sales tax is 15%, what is the total amount you have to pay?

First find the amount of tax paid. Change 13% to the decimal .13 and multiply.

.13 x $49.99
= $6.4987 rounded to $6.50

Add the amount of tax paid to the cost of the drill.

$49.99 + $6.50
= $56.49

The total amount paid is $57.49.

Example 2: Find the amount of interest you must pay in a year on a loan of $3000 if the interest rate is 12% per annum.

Change 12% to the decimal .12 and multiply.

.12 x $3000
= $360

The yearly interest paid is $360.

Example 3: Find the total cost of a car that sells for $22,000 after you pay 13% HST.

Change 13% to .13 and multiply by $22,000 to find the amount of tax you have to pay.

$22,000 x .13 = $2860

Add $2860 to the cost of the truck.

$22000 + $2860 = $24 860

The total cost for the car is $24,860.

TO FIND A PERCENT OF A NUMBER

1. Change the number with the percent to a decimal by dropping the percent sign and moving the decimal **two** places to the **left.**
2. Multiply the two numbers together.

Solve the following questions. Round off money values to two decimal places. **Answers are at the end of this skills manual.**

3. a) Find 50% of 200. b) Find 33.3% of 150. c) Find 4.25% of 30.

   d) Find 12% of $48.50. e) Find 15% of $1.99. f) Find .25% of 12.

   g) What is 100% of 75? h) What is 150% of 68? i) What is 9% of $10.75?

4. If the sales tax is 15%, what is the tax on a set of curling irons that costs $45.98? What is the total cost?

5. 80% of the students in a course passed their test. If there are 40 students in the class, how many failed the test? (Hint: If 80% passed, what % failed?)

6. How much interest will you pay in a year on a loan of $6,000 if the interest rate is 6% per year?

Problem Type #2: To Find What Percent One Number Is of Another

When you know what percent one number is of another it shows a relationship between the numbers in a way that is easy to picture.

> **Example:** Say you are asked to decide which brand of shampoo your salon should order. You find that Brand A, the shampoo the salon is now using, costs $.50 per shampoo compared to $.35 per shampoo for Brand B. Both shampoos seem to do a similar job cleaning hair. Your report to the manager might require you to calculate the percent of the difference in cost between the two brands savings over the cost of Brand A.
>
> $\frac{35}{50} \times 100 = 70\%$
>
> To express the difference in cost as a percentage, you say that Brand B costs 70% of Brand A.

To make that calculation you have to know how to write one number as a percent of another.

Questions that require you to do this can be worded in two ways:

"36 is what percent of 400?" or "What percent of 400 is 36?"

To find what percentage one number is of another, follow these three steps:

First step: Make a fraction. When one number is expressed out of another, it can be written as a fraction.

> **Example:** 150 out of 500 is the fraction 150/500.

You have to know which number is the numerator and which number is the denominator to make the fraction.:

- The number representing the total, or *the number following "of", is the* ***denominator***. It forms the bottom part of the fraction.
- The other number is the ***numerator***. It forms the top part of the fraction.

> ***Remember:*** *To make a fraction, write "is" over "of", or is/of.*

Second step: Make the fraction into a decimal number. (Nvide the *numerator by the denominator*.

Note: *You make the fraction in order to see what part is the numerator and what part is the denominator. This lets you correctly divide the numerator by the denominator. You can divide without making a fraction if you know to divide the number associated with "is", the numerator, by the number following "of", the denominator.*

Third step: Change the decimal answer to a percent:

- Multiply it by 100.
- Add the percent sign.

Remember*: To multiply by 100 move the decimal point two places to the right.*

Example: 150 is what percent of 500?

$\frac{150}{500}$ Write the fraction. Remember *is* over *of*

150 ÷ 500 = .3 Change the fraction to a decimal.
.3 x 100 = 30% Change the decimal to a percent.

In Brief

To find what percent one number is of another number:

1. Make a fraction by writing the number with the word *is* over the number with the word *of*.
2. Convert the fraction to a decimal by dividing the numerator by the denominator.
3. Change the decimal to a percent by multiplying by 100 and adding the % sign.

If you are dividing with a calculator, always key in the numbers so that the dividend (the number being divided into) is keyed in first, then the division sign and then the divisor (the number you divide by). To divide 75 by 25 using a calculator, key in the dividend 75 first and then the divisor 25.

Example: 620 is what percent of 1000?

$\frac{620}{1000}$ Write the fraction. (*is* over *of*)

620 ÷ 1000 = .62 Change the fraction to a decimal by dividing.

.62 x 100 = 62% Change the decimal to a percent by multiplying and adding the % sign.

620 is 62% of 1000

Example: What percent of 50 is 25?

25/50
= 25 ÷ 50
= .5
.5 x 100 = 50%

Rounding off: If the division answer doesn't come out evenly or is a repeating decimal, round off the answer. If the answer will be in percent, round off to four places so you have two decimal places in your percent.

Example: 1 is what percent of 3?

1 ÷ 3
= .3333... round off to four places

.3333 x 100
= 33.33%

Example: 200 is what percent of 300?

200 ÷ 300
= .6666... rounded to four places is .6667

.6667 x 100 = 66.67%"

TO FIND WHAT PERCENT ONE NUMBER IS OF ANOTHER

1. Make a fraction. Use the number representing the total or the number following "of" as the denominator. Make the other number the numerator.
2. Change the fraction to a decimal number by dividing the numerator by the denominator.
3. Multiply this division answer by 100 and add the percent sign.

Answer the following questions. Answers are at the end of this skills manual.

7. a) 3 is what percent of 12? b) 28 is what % of 32? c) What percent of 36 is 9?

   d) What % of 40 is 15? e) What % of 1 is 1? f) 250 is what % of 200?

   g) 25 is what % of 75? h) What % of $3.99 is $1.10?

8. If the normal price of a hairdressing chair is $500 and you only pay $440, what % savings do you get? (Your savings are the amount you don't pay. The discount is $60. What % is 60 of 500?)

9. If an average of 9 out of 100 people have light and medium blonde hair, what percent of the population has light hair?

10. You read in a professional hairstyling magazine that the national average of people with grey hair who colour their hair is 40%. If you have around 250 clients with grey hair, and about 100 of them get their hair coloured, are you close to the national average?

Problem Type #3: Find a Number When a Percent of It Is Known

To find a number when a percent of it is known follow these steps:

1. Change the percent to a decimal number by dropping the percent sign and moving the decimal point two places to the left.
2. Divide the given number by the decimal number. The division answer is the unknown number.

Example: In the bill that you are charged for re-wiring your salon, 60% of the cost is for labour. If the labour charge comes to $900, how much is the total bill?

Change 60% to a decimal.

60% = .6

Divide the cost for labour by the decimal number. The division answer is the cost of the total bill.

$900 ÷ .6 = $1500

The total bill is $1500.

This type of question can be worded in two different ways, but each is solved in the same way.

Example: 45% of what number is 90?

Change 45% to .45
90 ÷ .45 = 200

The unknown number is 200.

Example: 12 is 24% of what number?

Change 24% to .24
12 ÷ .24 = 50

The unknown number is 50.

Example: A hairstyling textbook recommends that a salon spend 3% of its total expenses on advertising. If the advertising expenses add up to $3,600 for one year, what are the total yearly expenses of the salon?

Change 3% to a decimal.

3% = .03

Divide the cost of advertising by the decimal number. The division answer is the total expenses for the year.

$3600 ÷ .03 = $120,000

The total yearly expenses are $120,000.

TO FIND A NUMBER WHEN A PERCENT OF IT IS KNOWN

1. Change the percent to a decimal number by removing the % sign and moving the decimal point two places to the left.
2. Divide the given number by the decimal number to get the number you are looking for.

Answer the following questions. The answers are at the end of this skills manual.

11. a) 2% of what number is 10?

b) 18 is 36% of what number?

c) 44 is 55% of what number?

d) 25% of what number is 6?

e) 100% of what number is 70?

f) 40% of what number is 12.6?

12. You need to get some wiring done in your salon. 25% of the electrician's bill is for materials and the rest is for labour. What will the total bill be if material costs are $50?

ANSWER PAGE

WRITING PERCENT

1 a) 10% b) 20% c) 100% d) 200% e) 85% f) 275%
g) 5 % h) 33.33% i) 100%

CHANGING PERCENT TO A DECIMAL

2. a) .28 b) .567 c) .07 d) 1 e) .085 f) 2.5
g) .15 h) .002 i) .625

FINDING A PERCENT OF A NUMBER

3. a) 100 b) 49.95 c) 1.275 d) $5.82 e) $.30 f) .03
g) 75 h) 102 i) $.97

4. Sales tax is .15 x 45.98 = $6.90 The total cost is $45.98 + 6.90 = $52.88.

5. 20% of the students didn't pass the test, so .2 x 40 = 8. 8 students failed.

6. .06 x 6,000 = $360. You will pay $360 interest.

FINDING WHAT PERCENT ONE NUMBER IS OF ANOTHER

7. a) 25% b) 87.5% c) 25% d) 37.5% e) 100% f) 125%
g) 33.33% h) 27.57

8. 60 ÷ 500 = .12
.12 x 100 = 12%
The saving is 12%.

9. 9 ÷ 100 = .09
.09 x 100 = 9%

10. 100 ÷ 250 = .4.
.4 x 100 = 40%. The number of clients you have who colour their hair is identical to the national average.

FINDING A NUMBER WHEN A PERCENT OF IT IS KNOWN

11. a) 500 b) 50 c) 80 d) 24 e) 70 f) 31.5

12. $50 ÷ .25 = $200
The total bill will be $200.