EVALUATING
ACADEMIC READINESS
FOR APPRENTICE SHIP TRAINING

Revised for

Access to Apprenticeship

MATHEMATICS SKILLS
PROPERTIES OF ANGLES

AN ACADEMIC SKILLS MANUAL
for
The Horticulture Trades

Arborist. and
Horticulturist

Workplace Support Services Branch
Ontario Ministry of Training, Colleges and Universities

Revised 2011

In preparing these Academic Skills Manuals we have used passages, diagrams and questions similar to those an apprentice might find in a text, guide or trade manual.

This trade related material is not intended to instruct you in your trade. It is used only to demonstrate how understanding an academic skill will help you find and use the information you need.

MATHEMATICS SKILLS
PROPERTIES OF ANGLES

An academic skill required for the study of the Horticulture Trades

INTRODUCTION

Wherever two lines meet, they form an angle. In your work as a horticulturist or arborist, you constantly work with angles. Before you prune a tree, you look at the angles formed by the branches leaving the trunk. You might open up the tree by leaving only the branches that come out of the trunk at a 45° angle. You might put an angled shape at the top of an arbor so it will shed water. You use tools that measure angles to see if a rectangular bed is level and true, checking to see if the corners form right angles. When you use a level to determine if a post is straight, you are checking to see if it makes a perpendicular angle with the ground.

A basic understanding of angles is essential to completing these tasks successfully, so some knowledge of geometry becomes important. Geometry is the study of figures and the relationships that the different parts of the figures have to each other.

This skills manual on angles covers the following topics:

- Basic terms used to describe angles
- Definition of an angle
- Different types of angles
- Angles in geometric figures
- Angles formed by a pair of intersecting lines
- Angles formed when two parallel lines are intersected by another line

BASIC GEOMETRIC TERMS

Specific terms are used to describe the parts of an angle and the different types of angles. **A line** is described as a set of points. A **straight line** is the shortest distance between two points. When we talk about a line, we assume it is a straight line. Otherwise it is called **a curved line**. A highway consists of a series of connected straight and curved lines.

A line can extend indefinitely or it can be limited by one or two **endpoints**. Points on a line and endpoints are named by a letter such as point A or point X. A **ray** is a line that has one endpoint.

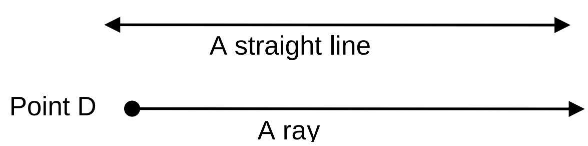


FIGURE 1: A Straight Line and a Ray

Parallel lines are lines that run side by side and never intersect or meet. When a truck is moving in a straight line, the front wheels are parallel to each other.

Intersecting lines are lines that cross each other.

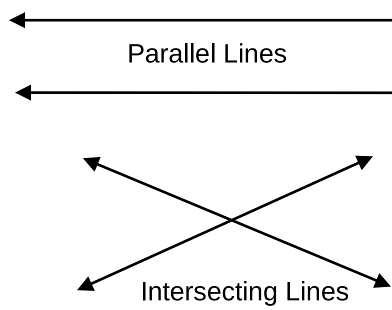


FIGURE 2: Parallel Lines and Intersecting Lines

DEFINITION OF AN ANGLE

*An **angle** is formed by two rays having the same endpoint.* The endpoint of an angle is also called the vertex. When we name an angle, we use the word "angle" or the symbol for angle, ∠.

Angles can be identified by one letter, usually a capital, written at the vertex. See Figure 3, Angle A.

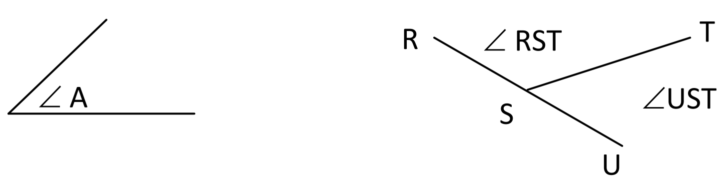


FIGURE 3: Two ways of naming angles

Angles can also be identified by three letters, one for a point on each ray and one for the vertex. Look at Figure 3 and ∠RST and ∠TSU.

The vertex is always named second if three letters are used. If two angles share the same vertex, three letters must be used to name the angles so there is no confusion as to which angle is being referred to.

When instructed to use a certain angle while working, you don't always have two distinct lines to consider.

> **Example:** If you are told to hold the welding torch at an angle of 45 ^{o}, you need to know what actually forms the angle. In this case, the steel surface forms the bottom line. The torch in your hand forms the other line. If one end of the torch is almost touching the surface, you

can move the other end up or down to change the angle the torch makes with the surface. To make a 45° angle, you want to hold the torch so it is halfway between flat and straight up and down.

Measuring Angles
Angles are divided into units of measurement called **degrees (°)**. To get a picture of the size of a degree, think of the outside of a circle divided into 360 equal parts with each division marked.

- Lines drawn from the centre of the circle to each of the 360 dividing marks form 360 equal angles, each measuring 1 degree.

- To determine the size of any angle, say a 45 ° angle, you could draw a line from the centre of the circle to one of these degree marks and then counted 45 more marks. If you draw another line from the centre of the circle to this second mark, the two lines would be 45° apart.

- We use a circular compass that is divided into degree marks to draw an angle of a certain size.

- The angle is drawn with two rays that meet at a common endpoint. The length of the rays does not affect the size of the angle.

In Figure 4, a circle is divided into angles each measuring 15 degrees. One complete rotation around the circle measures 360 degrees. One quarter of a circle measures 90 °. One half of a circle measures 180 °. Three quarters of a circle measures 270 °.

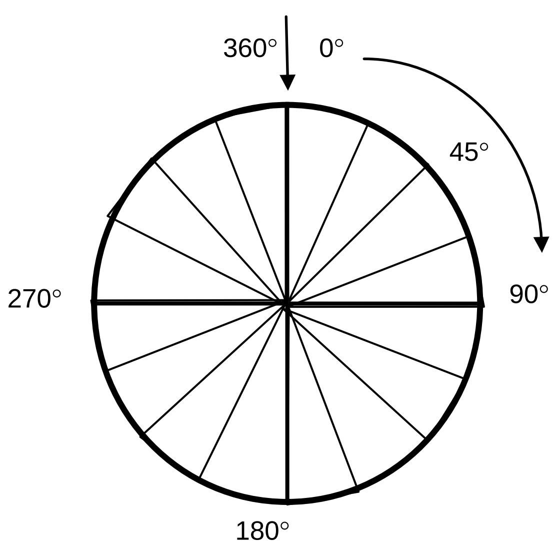


FIGURE 4: Degrees in a Circle Some of the angles formed by dividing a circle into 360°

Example: When the second hand on a clock rotates through 360°, it makes one full turn or rotation. If you followed one point on the end of the second hand, you would see the mark turn a complete circle or 360° for one rotation. Two complete rotations are 720°.

Although the circle was originally used to determine the size of a degree, we do not show the outside of the circle when we draw an angle. We draw only the two rays and the endpoint, using a protractor to determine the correct size of the angle. The length of the rays does not affect the size of the angle.

Usually the size of an angle is written inside the angle near the vertex, as in Figure 5. Notice the small arc near the endpoint in Angle JKL. It is sometimes written to indicate an angle measurement. In the 90° angle shown in angle A, the angle measurement is indicated by two small, straight lines instead of an arc.

TYPES OF ANGLES

Right angle: A 90° angle is called a ***right angle***. Angle A in Figure 5 is a right angle. The term 90° is often used to describe the position of an object.

Figure 5: Indicating The Number Of Degrees In An Angle.

> **Example:** When working on with air pressured equipment, you might be told that a 90° elbow in a service line restricts air flow the same amount as 7 feet of straight line. You should have a clear picture of what a 90° bend looks like.

A line drawn through the centre of a 90° angle forms two 45° angles. A 45° angle, Angle JKL, is shown in Figure 5.

Straight angle: The 180 ° angle formed by one half of a complete rotation of a circle is called a ***straight angle.*** It is an important angle because it forms a straight line.

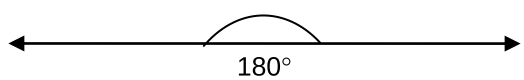


Figure 6: **A STRAIGHT ANGLE**

Other common angles you should be familiar with include the 30° angle and the 60° angle.

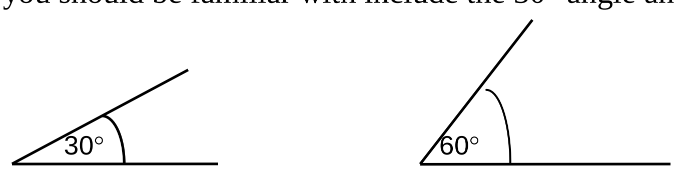


FIGURE 7: A 30° and a 60° Angle

Perpendicular lines: When one straight line extends out from another straight line in such a way that two right angles (90°) are formed, the two lines are ***perpendicular***.

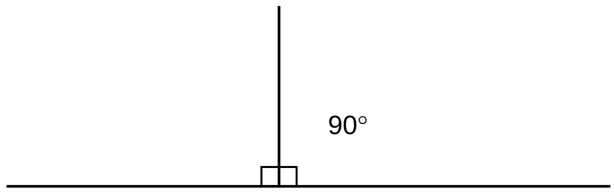


FIGURE 8: Perpendicular Lines

Some angles are named by their size relative to right and straight angles.

Acute angle: An acute angle is greater than 0° but less than 90°. See Figure 5, ∠ JKL, and Figure 3, ∠ A.

Obtuse angle: An obtuse angle is greater than 90° but less than 180°. See ∠ XYZ in Figure 8 and ∠ ABD in Figure 11.

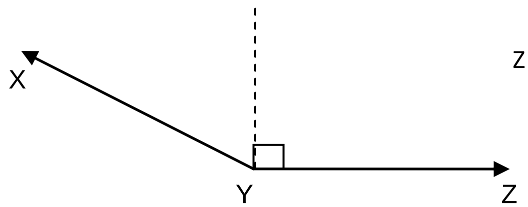
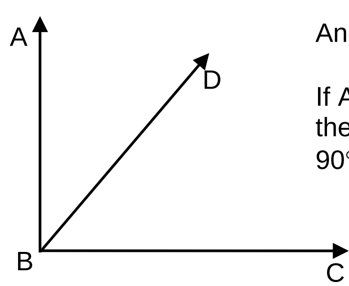


Figure 9: Obtuse Angle

Complementary Angles

If the sum of the measurements of two angles is 90°, the angles are called ***complementary angles.***

If there are two angles within a right angle and you know the measurement of one of them, you can find the measurement of the unknown angle. You do this by subtracting the value of the known angle from 90°.



Angle ABC is a right angle.

If Angle ABD = 35°
then Angle DBC equals
90° - 35° = 55°

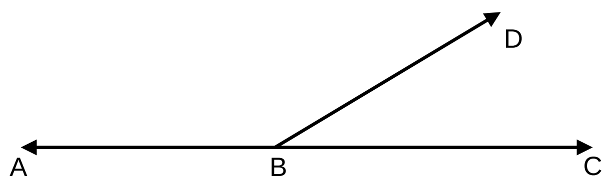
Figure 10: Finding An Unknown Complementary Angle When The Other One Is Known.

Example: If you have two complementary angles and one angle measures 40°, what is the measure of the other angle?

Complementary angles add up to 90°.
90° - 40° = 50°
The other angle measures 50°.

Supplementary Angles

If the sum of the measurements of two angles adds up to 180°, they are ***supplementary angles (Figure 11).*** If you have two angles contained within a straight angle, you can find one angle if you know the other. Subtract the known angle from 180°.



Angle ABC is a straight angle equal to 180°.
If ∠ ABD is equal to 130° then
∠CBD is equal to 180° - 130°
180° - 130° = 50°
∠CBD = 50°

FIGURE 11: **Finding An Unknown Supplementary Angle When The Other Angle Is Known**

Example: ∠ RST and ∠ TSU are supplementary angles. If ∠ RST is 120°, what is ∠ TSU?

Since supplementary angles add up to 180°, subtract the known ∠ RST from 180° to find ∠ TSU.
180° - 120° = 60°
∠ TSU is 60°.

ANGLES FOUND IN GEOMETRIC FIGURES

A closed geometric figure is made up of straight lines that are joined at distinct endpoints. Where two lines meet at an endpoint, angles are formed. For this reason, descriptions of common figures such as triangles and rectangles include information about the angles at the endpoints of the figures. (We actually use the word *angle* in the names of these figures.) We will look at the angles in rectangles and triangles.

Rectangles: A rectangle is a four-sided figure with equal opposite sides that are parallel. The sum of the angles of any four-sided figure is equal to **360°**. If you draw a rectangle and measure the four angles in the corners, you will find that each angle in a rectangle is a right angle measuring 90° and the sum of the four angles equals 360°.

Triangles: A triangle is a closed, three-sided figure formed by three connected line segments. At each of the three endpoints (or vertices), an angle is formed by the lines of the adjacent (next to each other) sides. See Figure 12.

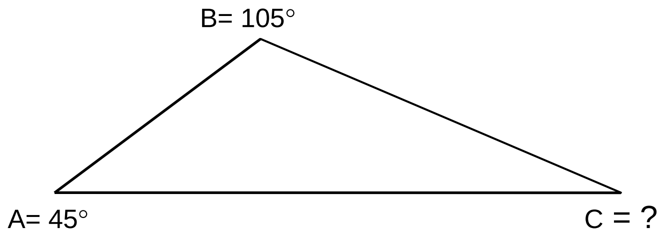


FIGURE 12: The Sum Of The Angles Of A Triangle Is 180°

If you draw any triangle and measure its three angles, the value of the three angles adds up to **180°**. Because of this relationship, if you know the value of two angles in a triangle, you can find the value of the third. If you add the values of the two, known angles and subtract the answer from 180° you will have the value of the third angle.

Example: If two of the angles in a triangle are 45° and 105°, what is the value of the third angle?

Find the sum of angles A + B.
45° + 105° = 150°

Subtract that sum from 180°.
180° - 150° = 30°

The third angle, C, is equal to 30°.

Right Angle Triangles

A ***right triangle*** has one right, or 90°, angle and two acute angles. The side opposite to the right angle is called the ***hypotenuse***. The hypotenuse is always the longest side. The side that the triangle rests on is called the ***base***. The vertical side is called the ***altitude*** or ***height***. There are some special relationships between the angles and sides of a right triangle.

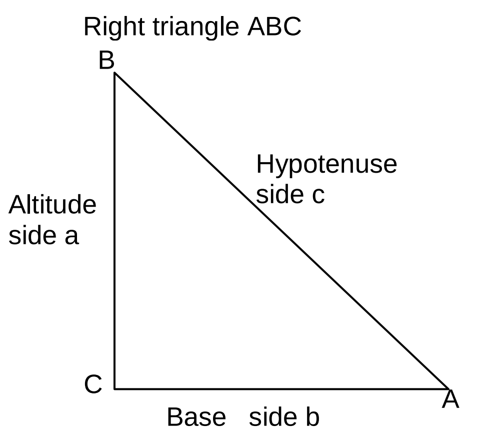


FIGURE 13: Parts of a Right Triangle

Pythagoras' Theorem

You can find the length of an unknown side of a right triangle by using Pythagoras' theorem. The theorem states that in a right triangle:

The square of the hypotenuse is equal to the sum of the squares of the other two sides.

The formula expressing this theorem for Δ ABC is:

$c^2 = a^2 + b^2$ where c is the hypotenuse, a is the altitude and b is the base.

To calculate c, find the square root of both sides.

$$c^2 = \sqrt{a^2 + b^2}$$

Using a calculator lets you find the square root of a number easily and accurately.

Example: Find the hypotenuse of a right triangle if the altitude is 4 cm and the base is 3 cm.

a = 4 cm
b = 3 cm
c = ?

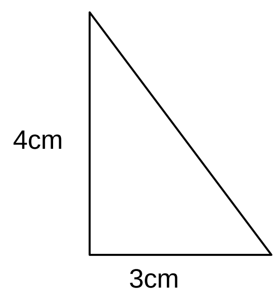
$c^2 = a^2 + b^2$

$c = \sqrt{a^2 + b^2}$

$c = \sqrt{4^2 + 3^2}$

$c = \sqrt{25}$

c = 5 cm



ANGLES FORMED BY A PAIR OF INTERSECTING LINES

When two lines intersect, four angles are formed. The following diagram, Figure 13, shows two intersecting lines forming four angles. A pair of intersecting lines provides us with some interesting information about the angles they form.

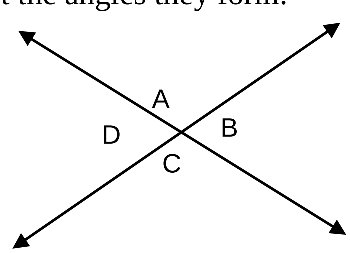


FIGURE 14: Four Angles ∠A, ∠B, ∠C, and∠D Are Formed By Intersecting Lines

Supplementary angles formed by intersecting lines: ***Supplementary angles*** *are two angles, that together will form a 180° angle, (a straight line).*

There are several interesting relationships between the angles in Figure 14. The intersecting lines are straight lines. We know that lines form straight angles measuring 180°. Therefore, there are four sets of angles that are supplementary angles whose measurements add up to180°.

- angles A + B = 180°,
- angles B + C= 180°,
- angles C + D= 180°, and
- angles A+ D= 180°,

Opposite angles formed by intersecting lines: *When two straight lines intersect, the* ***opposite angles*** *are* ***equal****.* We can see this important relationship in Figure 14. The pairs of angles that are opposite each other are equal.

In Figure 14, the opposite angles B and D are equal; the opposite angles A and C are also equal.

The relationships between supplementary and opposite angles formed by intersecting lines enables us to find the value of unknown angles.

Example: In Figure 14, if ∠A is 110° what is the value of ∠D?

Angles A and D are supplementary angles whose value adds up to 180°.

∠ A + ∠ D = 180°
∠ D = 180° - ∠ A
∠ D = 180° - 110°
∠ D = 70°

Example: In Figure 14, if ∠A is equal to 110°, what is the value of ∠C?

∠A and ∠C are opposite so they are equal.
Both measure 110°.

Example: If ∠A and ∠C both measure 110° and ∠D equals 70°, what does ∠B equal?

∠B has the same value as ∠D because they are opposite angles.
∠B equals 70°.

ANGLES FORMED BY TWO PARALLEL LINES INTERSECTED BY ANOTHER LINE

Two straight lines are parallel if the distance between them always remains the same. If another line cuts across the parallel lines, eight angles are formed as in Figure 15. The two sets of four angles formed by the intersecting line are identical. Angles A, B, C, and D form one set. Angles E, F, G, and H form the second set.

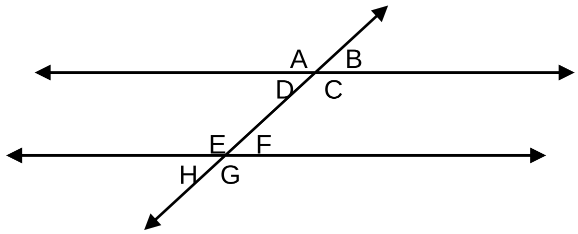


FIGURE 15: Sets Of Angles Formed By A Line Intersecting Two Parallel Lines

Remember: *any two angles next to each other in the diagram are supplementary angles and their combined value is 180°. Also, angles opposite each other are equal.*

Corresponding angles: In Figure 15 the two lines that have been intersected are parallel. Some special relationships exist between angles that are in corresponding (similar) positions in each of the two sets. Look at ∠A and ∠E. Each is on top of its intersected line and on the left side of the intersecting line. *∠A and ∠E are in similar positions and are identical to each other. They are called* ***corresponding angles.***

Look at Figure 15 as you read the following:.

1. A and ∠ C are opposite and equal, as are ∠ E and ∠ G.

2. The four angles, ∠ A, ∠ E, ∠ C and ∠ G, all have the same value.

3. Since ∠ C and ∠ G are equal corresponding angles:
    a. their supplementary angles, ∠ B and ∠ F, are also equal to each other.
    b. D and ∠ H are opposite and equal to ∠ B and ∠ F.

Therefore, ∠ B, ∠ D, ∠ F and ∠ H all have the same value

You only need to know the value of ***one*** *angle out of the* ***eight*** *angles formed by a line intersecting two parallel lines in order to find the value of the rest of the angles.*

As long as you know one angle and you keep straight what the corresponding and supplementary angles are, you can find all the other angles.

Note: *All the information above applies no matter in what directions the parallel lines and the intersecting lines run.*

Example: Use Figure 15. If ∠A is 120°, what is the measurement of ∠B? What are the values of the other angles?

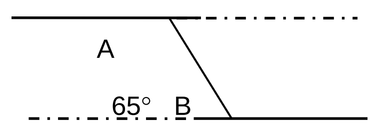
Angles A and B are supplementary. Therefore,
∠B = 180° - 120° = 60°

∠A equals ∠E, ∠C and ∠G. So, they all equal 120°.
∠E = 12°0
∠C = 120° and,
∠G = 120°

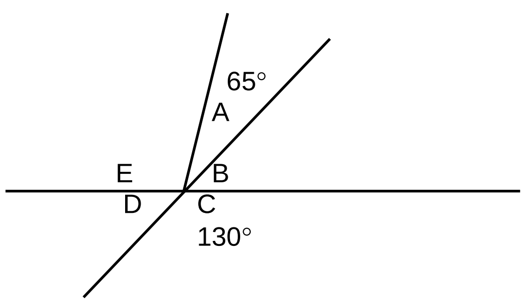
∠B equals ∠D, ∠H and ∠F. They all equal 60°.
∠D = 60°
∠H = 60° and,
∠F = 60°

Example: Use the diagram below for this question. If ∠A equals ∠B, what is the value of ∠A?

∠B + 65° **=** 180° (supplementary angles)
∠B = 180° - 65° = 115°
∠B = ∠A = 115°



Example: Use the diagram below for this problem. In the drawing, ∠ A and ∠ E added together form an opposite angle to angle C. If ∠ A = 25°, what is the value of ∠ E?



∠ A + ∠ E = ∠ C
so ∠ E = ∠ C - ∠ A
if ∠ C = 130°
and ∠ A = 25°
then ∠ E = 130° - 25°
∠ E = 105°

Here is another way to solve this problem:

For ∠ B
∠ D = ∠ B
∠ D + ∠ C = 180°
∠ D = 180° - 130°
∠ D = 50°
∠ B = 50°
and for ∠ E
∠ A + ∠ B = 25° + 50°
= 75°
∠ A + ∠ E + ∠ B = 180°

∠ E = 180° - 75°
= 105°

To sum up, here are some angle facts to remember:

- The sum of a pair of complementary angles is 90° .
- The sum of a pair of supplementary angles is 180°.
- The sum of the angles in a triangle is 180°.
- The sum of the angles in a rectangle is 360°.
- When two straight lines intersect, the opposite angles are equal.
- Corresponding angles formed when a line intersects two parallel lines are equal.

Answer these questions on angles and their relationships. Look back at the examples for guidance. **Answers are on the last page.**

1. A complete rotation of the circumference of a circle is equal to _______ degrees.

2. A right angle measures ________ degrees.

3. A straight angle measures ________ degrees.

4. The sum of the three angles in a triangle adds up to _______ degrees.

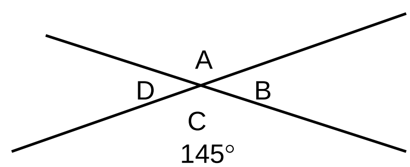
5. The sum of the angles in a four-sided figure adds up to _______ degrees.

6. If two angles in a triangle are equal to 55° and 80°, what is the value of the third angle?______

7. If a line extends at a right angle from another line, the two lines are __________________ to each other.

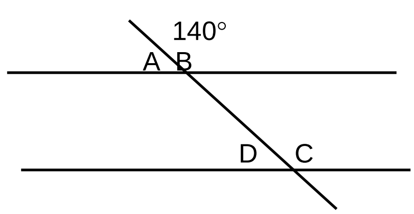
8. When two straight lines intersect, the ___________________ angles are equal.

9. In this drawing ∠ C is 145°:
   Answer the following:
   ∠ A is equal to ____________.
   ∠ B is equal to ____________.
   ∠ D is equal to ___________.

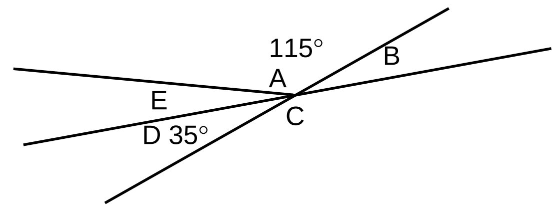


10. If two parallel lines are intersected by another line, two sets of four angles are formed. In each set, opposite angles are _____________ and the value of any two angles next to each other adds up to _________ degrees.

11. Use the drawing below. If ∠ B is equal to 140°, what is the value of ∠ A? __________
    What is the value of ∠ D? ________



12. Use the drawing opposite to answer these questions.
$\angle$ A = 115 ^{o} and $\angle$ D = 35 ^{o}



What is the value of $\angle$ E?
($\angle$ A + $\angle$ D +$\angle$ E = 180 ^{o}.) __________

What is the value of $\angle$ B? _________

What is the value of $\angle$ C? ($\angle$ C is equal to
$\angle$A + $\angle$ E) _____________

ANSWER PAGE

1. 360°

2. 90°

3. 180°

4. 180°

5. 360°

6. angles in a triangle = 180°
   ∠1 + ∠2 = 55° + 80° = 135°
   ∠3 = 180°- 135° = 45°

7. perpendicular

8. opposite

9. 145° ∠ A is opposite angle to 145°
   35° ∠B is supplementary to ∠A 180° - 145° = 35°
   35° ∠D is opposite to ∠B, and is supplementary to ∠A

10. equal, 180°

11. 40° ∠ A is supplementary to B
    40° ∠D is corresponding angle to ∠A

12. 30° ∠'s A, E, and D are supplementary.
    35° ∠'s D and B are complementary.
    145°