

**EVALUATING  
ACADEMIC READINESS  
FOR APPRENTICESHIP TRAINING**  
Revised for  
**ACCESS TO APPRENTICESHIP**

**MATHEMATICS SKILL  
PROPERTIES OF CIRCLES**

**AN ACADEMIC SKILLS MANUAL  
for  
The Horticulture Trades**

This trade group includes the following trades:  
Arborist, and  
Horticulturist

*Workplace Support Services Branch  
Ontario Ministry of Training, Colleges and Universities*

*Revised 2011*

In preparing these Academic Skills Manuals we have used passages, diagrams and questions similar to those an apprentice might find in a text, guide or trade manual.

**This trade related material is not intended to instruct you in your trade. It is used only to demonstrate how understanding an academic skill will help you find and use the information you need.**

---

# MATHEMATICS SKILLS

## PROPERTIES OF CIRCLES

---

*An academic skill required for the study of the  
Horticulture Trades*

### **INTRODUCTION**

As a horticulturist or arborist, you deal with different shapes when you prune trees, when you design garden beds and walkways, and when you figure out watering or spraying patterns. Some of these shapes are formed by curved lines, such as a curved path, the rounded top of a Globe Catalpa tree, a circular patio design or the area watered by a rotating spray head. You have to do calculations with circles to decide how much edging to order to surround a circular bed or how many paving stones are needed to construct a round patio.

The circle is the most important shape formed from a curved line. A **circle** is a simple, closed figure consisting of points on a curved line that are all the same distance from the center. This skill sheet looks at the properties of circles. It covers the following topics:

- ◆ Terms used to describe a circle, including
  - degrees in a circle
  - circumference of a circle
  - diameter and radius
  - meaning of pi
- ◆ Finding the circumference of a circle
- ◆ Finding the area of a circle

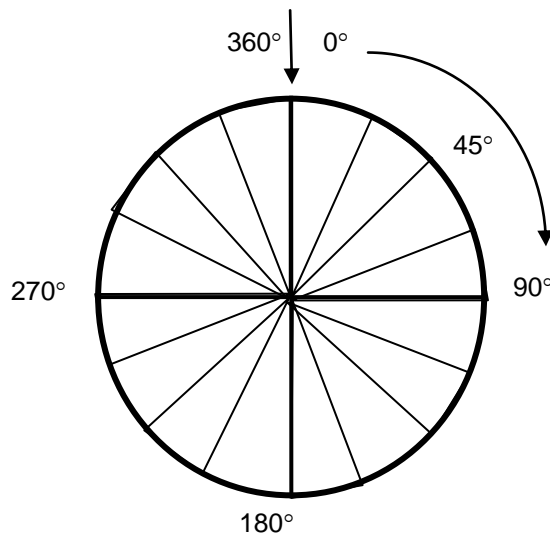
### **TERMS USED TO DESCRIBE A CIRCLE**

#### ***Degrees in a Circle***

The face of a clock is usually in the shape of a circle. When the minute hand of a clock makes a complete circle around the face, moving from 12 around to 12 again, it has travelled 60 minutes or 1 hour. The clock is divided into 60 divisions of one minute each.

In the same way that a clock is divided into different sections:

- ◆ A circle is divided into **degrees**.
- ◆ *A circle is divided into 360 equal parts that measure 1 degree each.*
- ◆ This means that a degree is 1/360th of the outside boundary or the *circumference* of a circle.
- ◆ To indicate the unit degree, we write the symbol  $^{\circ}$
- ◆ Each degree is further divided into 60 equal parts called minutes.



**FIGURE 1: Degrees in a Circle**

Some of the angles formed by dividing a circle into 360°

### ***Circumference of a Circle***

*The circumference (C) of the circle is the outside boundary of the circle.*

- ◆ Any partial section of the circumference is called an **arc**.
- ◆ A circle has 360 arcs of equal size, each measuring one degree.
- ◆ *The actual circumference of a circle in meters, inches, centimeters, etc, varies with the size of the circle.*
  - A larger circle will have a greater distance around its circumference than a smaller circle.

Examine Figure 1, and note the following:

- One-fourth of the circumference of a circle is equal to an arc of 90°.
  - If you draw lines from the end points of a 90° arc to connect in the centre of the circle, the lines form a 90° (right) angle.
- One-half the circumference of a circle is equal to an arc of 180°.
  - If you draw lines from the end points of a 180° arc to connect in the centre of the circle, the lines form an angle of 180°, which is also a straight line.
- An arc covering three-quarters of the circle is equal to 270°.
  - Lines drawn from the ends of this arc to the centre form an angle of 270°.
- If you travelled twice around the outside of a circle, you would travel  $360^\circ \times 2 = 720^\circ$ .
  - If you travelled three times around, you would travel  $360^\circ \times 3 = 1080^\circ$ .

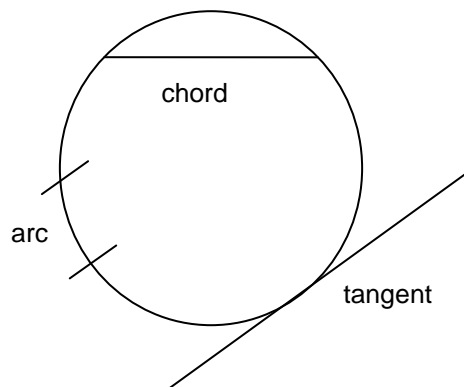
Every circle is divided into 360 degrees around its circumference. However, *the actual distance in meters, inches, centimeters, etc, around the outside of a circle varies with the size of the circle.*

A larger circle will have a greater distance around its circumference than a smaller circle.

- ◆ This distance could be measured by placing a flexible measuring tape around the circumference of the circle; however, it is difficult to measure accurately this way.
- ◆ We calculate circumference using the length of the diameter or the radius because they can be measured accurately.

### ***Chord and Tangent of a Circle***

A line drawn through the circle from one point on the circumference to another point on the circumference is called a **chord**. A **tangent** to a circle is a line which touches the circle at only one point on the circumference.

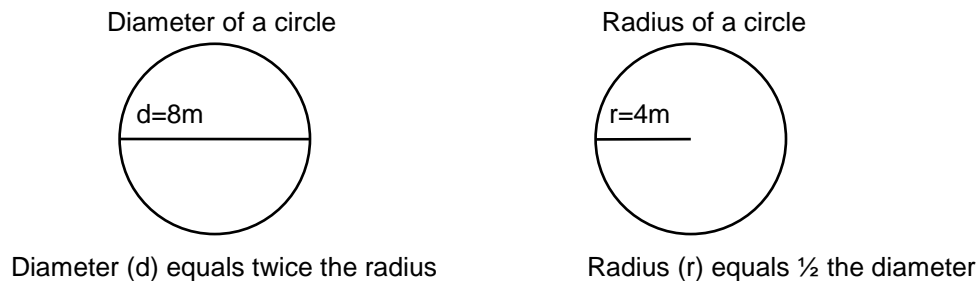


**FIGURE 2:** Arc, Chord And Tangent Of A Circle

### ***Diameter and Radius***

The **diameter** ( $d$ ) of a circle is a line that goes from one edge of the circle to the other edge and passes through the centre.

- It is easy to measure the diameter of a circle accurately because it is a straight line, as you can see in Figure 3.



**FIGURE 3:** The Diameter And Radius Of A Circle

The **radius** ( $r$ ) of a circle is a line that goes from the centre of the circle to the circumference.

- It is equal to one half the length of the diameter.

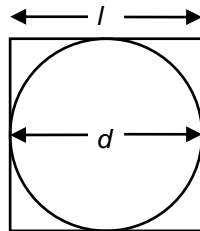
If you know either the diameter or the radius, you can find the other:

$$r = \frac{1}{2}d \quad \text{and} \quad d = 2r$$

**Example:** If the diameter is 14 inches, what is the radius?

$$\begin{aligned} r &= \frac{1}{2}d \\ &= \frac{1}{2} \times 14 \\ &= 7 \text{ in} \end{aligned}$$

If you place a circle inside a square so that the edges of the circle touch the square in four places, the length of the side of the square is equal to the diameter of the circle. So if you know the length of a side of the square, you also know the diameter of the circle.



**FIGURE 4: The Length Of A Square Is The Same As The Diameter Of A Circle Placed Inside It.**

Conversely, if you know the radius of the circle, you can find the length of the sides of the square. Multiplying the radius by two gives the diameter. We know that the diameter is equal to the length of the sides. See Figure 4.

**Example:** If the length of the side of a square is 16 cm, what is the diameter of a circle that is inside the square and touching all four sides, as in the diagram above? What is the radius?

The diameter is the same as the length of the side of the square, which is 16 cm.  
The radius is one-half of the diameter,

$$\frac{1}{2} \times 16 = 8 \text{ cm}$$

**Pi** is written using the symbol  $\pi$

- ◆  $\pi = 22/7$  when it is written as a fraction.
- ◆  $\pi = 3.14159\dots$  When it is written as a decimal, the decimal places go on forever.
  - it is usually rounded off to two decimal places.
  - $\pi = 3.14$  rounded off to two decimal places

---

## ***FINDING THE CIRCUMFERENCE OF A CIRCLE***

The circumference is the distance around the boundary of a circle. To find the circumference, you must know either the diameter or the radius.

*Circumference (C) is equal to  $\pi$  times the diameter (d).* The formula for finding circumference is:

$$C = \pi d$$

or, since the diameter is twice the radius (r),

$$C = 2 \pi r$$

**Example:** Find the circumference of a circle with a radius of 32 cm.

$$\begin{aligned} C &= 2 \pi r \\ C &= 2 \times 3.14 \times 32 \text{ cm} \\ C &= 201 \text{ cm} \end{aligned}$$

**Example:** Find the circumference of a circle with a diameter of 8 m.

$$\begin{aligned} C &= \pi d \\ &= 3.14 \times 8 \text{ m} \\ &= 25 \text{ m} \end{aligned}$$

**Example:** A piece of wire is to be put around the outside of a can that has a diameter of 20 cm. How much wire will be used?

The amount of wire used will be equal to the circumference of the can.

$$\begin{aligned} C &= \pi d \\ &= 3.14 \times 20 \\ &= 62.8 \text{ cm} \end{aligned}$$

62.8 cm of wire are needed to circle the can.

## ***FINDING THE AREA OF A CIRCLE***

The area of a circle is the amount of space enclosed within the boundary of the circle. **Area (A)** is equal to  $\pi$  times the radius squared. The formula for finding the area of a circle is:

$$A = \pi r^2$$

**Squaring a number:** To find the area of a circle, you need to know how to square a number. *To square a number, multiply it by itself.* For example, to square 11, multiply 11 x 11 to get 121.

If you square a number that has units attached, the units are also squared. For example, the square of 5 meters (5m x 5m) is 25 meters squared. To show that meters, or any units, have been squared, we write a small two after the unit and slightly above it, like this: m<sup>2</sup>. (The small raised 2 is called an exponent.) 5 m squared is equal to 25 m<sup>2</sup>.

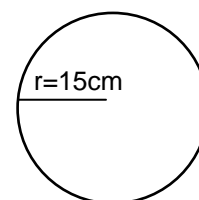
To square a number and its unit of measurement, we put brackets around *both of them* and write the exponent <sup>2</sup> after the brackets. So (9 m)<sup>2</sup> means you square 9 m by multiplying 9 m by 9 m.

$$(9 \text{ m})^2 = 9 \text{ m} \times 9 \text{ m} = 81 \text{ m}^2$$

Now let's use the formula  $A = \pi r^2$  to find the area for the following circle.

**Example:** Find the area of a circle with a radius of 15 cm.

Known  
 $r = 15 \text{ cm}$   
 $\pi = 3.14$



We need to know:

- the area of the circle

We use the formula  $A = \pi r^2$

$A = \pi r^2$	write the formula
$A = 3.14 \times (15 \text{ cm})^2$	substitute the values for known quantities
$A = 3.14 \times 225 \text{ cm}^2$	solve for A
$A = 706.5 \text{ cm}^2$	

**Note** that the units are squared in the answer. Whenever you find the area of a circle, a square, a rectangle etc, the units are squared.

**Example:** Find the area of a circle with a diameter of 10 m.

Known  
radius = diameter  $\div$  2  
 $r = 10 \text{ m} \div 2$   
 $= 5 \text{ m}$

Find the area:

$$\begin{aligned} A &= \pi r^2 \\ &= 3.14 \times (5 \text{ m})^2 \\ &= 3.14 \times 25 \text{ m}^2 \\ &= 78.5 \text{ m}^2 \end{aligned}$$

**Using  $A = \pi r^2$  to find the radius:** If you know the area of a circle, you can find its radius by manipulating or changing around the position of the variables (the letters and symbols) in the formula. Rewrite the formula so that  $r^2$  is by itself on the left. The letters in equations can be moved around just like numbers as long as you follow the rules of basic algebra.

---

**Example:** Find the radius of a circle with an area of  $78.5 \text{ cm}^2$ . First change the formula so that  $r^2$  is by itself on the left:

Known  
 $A = 78.5 \text{ cm}^2$   
 $\pi = 3.14$

Find the radius

$$\begin{aligned} \text{Area} &= \pi r^2 && \text{reverse the equation} \\ \pi r^2 &= A && \text{divide by } \pi \text{ to isolate } r^2 \\ r^2 &= 78.5 \text{ cm}^2 \div 3.14 \\ r^2 &= 25 \text{ cm}^2 \\ r &= 5 \text{ cm} && \text{Find the square root of } r^2 \text{ and } 25 \text{ cm}^2 \end{aligned}$$

**Finding the volume of a circular bed or pond:** To find the amount of water that will fill a circular pond, you need to know the area and the depth of the pond. Multiply the area by the depth. The resulting units will be cubic units.

**Example:** Find the amount of water needed to fill a circular pond with a diameter of 2 meters and a depth of 80 centimeters.

Known  
 $d = 2 \text{ m}$   
 $r = 1 \text{ m}$   
 $\text{depth} = 80 \text{ cm}$       units must be the same  
 $= .8 \text{ m}$

Find

1. The area of the surface  
Use  $A = \pi r^2$
2. The volume of water it will hold  
Use Area x depth

1. Find the area:  
 $A = \pi r^2$   
 $A = 3.14 \times (1 \text{ m})^2$   
 $A = 3.14 \times 1 \text{ m}^2$   
 $A = 3.14 \text{ m}^2$

2. Find the depth

Area x depth:  
 $3.14 \text{ m}^2 \times .8 \text{ m} = 2.5 \text{ cubic meters}$        $\text{m}^2 \times \text{m} = \text{m}^3$

The volume of the water in the pond is  $2.5\text{m}^3$ .

---

**Answer the following questions about the circle. Answers are on the last page.**

1. Find the radius of a circle with a diameter of 24 inches.
2. Find the diameter of a circle with a radius of 3 meters.
3. Square the following numbers:  
a) 7      b) 31      c) 4 km      d) 15 in      e) 2 cm
4. Find the circumference of a circle that has a diameter of 12 cm.
5. How much edging is needed to go around a circular bed with a diameter of 3.5 m?
6. What is the cost of putting a strip of Tanglefoot (designed to stop insects climbing up a tree) around a tree with a diameter of 48 centimeters if the strip costs \$6.50 per meter?
7. A ball on a string that is 4 meters long is swung around in a circle two and a half times. How far did the ball travel? (Find the circumference and multiply it by  $2\frac{1}{2}$ )
8. Find the area of a circle with a radius of 28 cm.
9. Find the area of a circle that has a diameter of 14 m.
10. A circular bed has an area of  $28.26\text{ m}^2$ . Find the radius, using 3.14 as  $\pi$ .



---

## 14. ANSWER PAGE

1.  $d \div 2 = r$   
 $r = 12 \text{ in}$
2.  $r \times 2 = d$   
 $d = 6 \text{ m}$
3. a) 49 b) 961 c)  $16 \text{ km}^2$  d)  $225 \text{ in}^2$  e)  $4 \text{ cm}^2$
4.  $C = \pi d = 37.68 \text{ cm}$
5. Edging needed = circumference of circle  
 $C = \pi d$   
 $C = 3.14 \times 3.5$   
 $C = 10.99 \text{ m}$
6.  $d = 48 \text{ cm} = .48 \text{ m}$   
 $C = 1.5 \text{ m}$   
 $\text{Cost} = 1.5 \times \$6.50 = \$9.75$
7.  $r = 4 \text{ m}$   
 $C = 2 \times 3.14 \times 4 \text{ m}$   
 $= 25.12 \text{ m}$   
Don't forget that the ball went around not once, but 2.5 times,  
so multiply the circumference of 25.12 m by 2.5  
 $25.12 \text{ m} \times 2.5 = 62.8 \text{ m}$ . The ball traveled 62.8 m.
8.  $A = \pi r^2$   
 $= 3.14 \times (28 \text{ cm})^2$   
 $= 3.14 \times 784 \text{ cm}^2$   
 $= 2461.76 \text{ cm}^2$
9. Radius = diameter  $\div$  2  
 $= 7 \text{ m}$   
Area =  $3.14 \times 7^2$   
 $= 153.86 \text{ m}^2$
10.  $r^2 = \text{area} \div \pi$   
 $= 28.26 \text{ m}^2 \div 3.14$   
 $= 9 \text{ m}^2$   
 $r = \sqrt{9 \text{ m}^2}$   
 $r = 3 \text{ m}$
11. Area =  $78.5 \text{ m}^2$   
 $\frac{1}{2}$  of area =  $39.25 \text{ m}^2$

12. The entire paved area is  $5\text{m} \times 5\text{m} = 25\text{ m}^2$

$$\begin{aligned}\text{Area of circle} &= 3.14 \times 2 \\ &= 6.28\text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Area of rectangular paving stones outside the circle} &= 25\text{ m}^2 - 6.28\text{ m}^2 \\ &= 18.72\text{ m}^2\end{aligned}$$

13.  $25\text{ cm} = .25\text{ m}$

$$A = 3.14 \times 4.2\text{ m} = 13.19\text{ m}^2$$

$$V = 13.19\text{ m}^2 \times .25\text{ m} = 3.3\text{ m}^3$$

$$\text{Cost} = 3.3 \times \$10.95 = \$36.14$$